

Introduction to (Formal) Logic

Spring 18, M R 12–1:50 DCC308

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version 0208180830NY

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1 General Orientation

This course is an accelerated, advanced introduction, within the “LAMA” paradigm,¹ to deductive formal logic (with at least some brief but informative pointers to both *inductive* and heterogeneous formal logic).² The phrase we use to describe what the student is principally introduced to in this class is: *beginning deductive logic, advanced* (BDLA). After this class, the student can proceed to the intermediate level in formal deductive logic, and — with a deeper understanding and better prepared to flourish — to various areas within the *formal sciences*, which are all based on formal logic. The formal sciences include e.g. theoretical computer science (e.g., computability theory, complexity theory, rigorous coverage of programming and programming languages), mathematics in its traditional branches (analysis, topology, algebra, etc), decision theory, game theory, set theory, probability theory, mathematical statistics, etc. (and of course formal logic itself).

We have referred above to “the LAMA paradigm.” What is that? This question will be answered in more detail later, but we do say here that while the LAMA paradigm is based upon a number of pedagogical principles, first and foremost among them is what can be labelled the Driving Dictum:

If you can’t prove it, you don’t get it.

Turning back to the nature of formal logic, it can accurately be said that it’s the science and engineering of reasoning,³ but even this supremely general slogan fails to convey the flexibility and enormity of the field. For example, all of classical mathematics can be deductively derived from a small set of formulas (e.g., **ZFC** set theory, which you’ll be hearing more about, and indeed experimenting with in the Slate system) expressed in the formal logic known as ‘first-order logic’ (FOL, which you’ll *also* be hearing more about), and, as we shall see and discuss in class, computer science emerged from and is in large part based upon logic (for cogent coverage of this emergence, see Glymour 1992, Halpern, Harper, Immerman, Kolaitis, Vardi & Vianu 2001). Logic is indeed the foundation for *all* at once rational-and-rigorous intellectual pursuits. (If you can find a counter-example, i.e. such a pursuit that doesn’t directly and crucially partake of logic, I would be very interested to see it.)

¹‘LAMA’ is an acronym for ‘Logic: A Modern Approach,’ and is pronounced to rhyme with ‘llama’ in contemporary English, the name of the exotic and sure-footed camelid whose binomial name is *Lama glama*, and has in fact been referred to in the past by the single-l ‘lama.’

²Sometimes ‘symbolic’ is used in place of ‘formal,’ but that’s a bad practice, since — as students in this class will see — formal logic includes the representation of and systematic reasoning over *pictorial* information, and such information is decidedly *not* symbolic. For a discussion of the stark difference between the pictorial vs. the symbolic, and presentation of a formal logic that enables representation of and reasoning over both, see (Arkoudas & Bringsjord 2009), which directly informs Chapter 8 of our textbook.

³Warning: Increasingly, the term ‘reasoning’ is used by some who don’t *really* do anything related to reasoning, as traditionally understood, to nonetheless label what they do. Fortunately, it’s easy to verify that some reasoning is that which is covered by formal logic: *If the reasoning is explicit, links declarative statements or formulae together via explicit, abstract reasoning schemata or rules of inference (giving rise to at least explicit arguments, and often proofs), is surveyable and inspectable, and ultimately machine-checkable, then the reasoning in question is what formal logic is the science and engineering of.* In order to characterize *informal* logic, one can remove from the previous sentence the requirements that the links must conform to explicit reasoning schemata or rules of inference, and machine-checkability. It follows that so-called informal logic would revolve around arguments, but not proofs. An excellent overview of informal logic, which will be completely ignored in this class and its textbook, is provided in “[Informal Logic](#)” in the Stanford Encyclopedia of Philosophy. In this article, it’s made clear that, yes, informal logic concentrates on the nature and uses of argument.

2 Assistance to Bringsjord

The TAs for this course are: Rini Palamittam; email address: rinipalamittam@gmail.com; and Meiqi “Jessie” Cao; email address caomq1@gmail.com. Rini will hold office hours on Tues 1230–230 in Carnegie 301A (and by appointment). Jessie will hold office hours on Wed 300–500 in Carnegie 301A (and by appointment). Some guest lectures may be provided by researchers working in the RAIR Lab, a logic-based AI lab.

3 Prerequisites

There are no formal prerequisites. However, as said above, this course introduces *formal* logic, and does so in an accelerated, advanced way. This implies that — for want of a better phrase — students are expected to have a degree of logico-mathematical maturity. You have this on the assumption that you understood the math you were supposed to learn to make it where you are.⁴ For example, to get to where you are now, you were supposed to have learned the technique of *indirect proof* (= proof by contradiction = *reductio ad absurdum*). An example of the list of concepts and techniques you are assumed to be familiar with from high-school geometry can be found in the common-core-connected (Bass & Johnson 2012). An example of the list of concepts and techniques you are assumed to be familiar with from high-school Algebra 2 can be found in the common-core-connected (Bellman, Bragg & Handlin 2012). It’s recommended that during the first two weeks of the class, students review their high-school coverage of formal logic, which includes at minimum the rudiments of the propositional calculus.⁵

4 Texts

Students will purchase an inseparable combination of the e-text *Logic: A Modern Approach; Beginning Deductive Logic, Advanced via Slate and HyperGrader* (LAMA-BDLA) and the Slate software system; both will be available in the RPI Bookstore. Full logistics of the purchase, and the contents of the CD that holds this pair (and other files), will be explained the second class meeting, Jan 22 2018, and subsequently, as needed. The first use in earnest of Slate will happen in class on Feb 1, so by the start of class on that day students should have the textbook-software combo, and be able to open it on their laptops in class. Updates to LAMA-BDLA, and additional exercises, will be provided by listing on the course web page (and sometimes by email) through the course of the semester. You will need to manage many electronic files in the course of this course, and e-housekeeping and e-orderliness is of paramount importance. You will specifically need to assemble a library of completed and partially completed proofs/arguments/truth-trees etc. so that you can use them as building blocks in harder proofs; in other words, building up your own “logical library” will be crucial.

⁴If you happen to be a student reading this as one wanting to be introduced to formal logic, from outside RPI, please examine your own case realistically. If you are not in command of the traditional high-school-level content for algebra, geometry, trigonometry, and at least some (differential and integral) calculus, you will need to go with a standard, non-advanced introduction to logic in the LAMA paradigm, or in some other paradigm. Specifically, if in the LAMA paradigm, you will need the LAMA-BDL textbook, *not* LAMA-BDLA. The ‘A’ in ‘LAMA-BDLA’ is for ‘Advanced.’ Check which textbook you have!

⁵Sometimes referred to as ‘sentential logic’ or ‘zeroth-order logic.’ If you are at all confused about how these terms were used before reaching the present course, please discuss with me.

Please note that Slate is copyrighted software: copying and/or distributing this software to others is strictly prohibited. You will need to submit (via hard copy in person, or email to Selmer.Bringsjord@gmail.com) to S Bringsjord a signed hard-copy version of a Software License Agreement (a pdf is included on the aforementioned CD). This agreement will also cover the textbook, which is copyrighted as well, and as an ebook cannot be copied or distributed.

In addition, occasionally papers may be assigned as reading. Two background ones, indeed, are hereby assigned: (Bringsjord, Taylor, Shilliday, Clark & Arkoudas 2008, Bringsjord 2008).

Finally, slide decks used in class will contain crucial additional content above and beyond LAMA-BDLA and Slate, and will be available on the web site for course for study.

5 Schedule

The course is basically divided into first a motivating stretch (during which we show that the logically untrained have great trouble reasoning well), and then six additional parts. In the first three of these remaining parts we'll focus on the **propositional calculus**; in the second on **first-order logic** (= FOL), with a brief look at **second-order logic** and beyond; and in the third, we'll cover **modal logic** (in the form, specifically, of four closely related modal logics: **T**, **S4**, **D** (= **SDL**), and **S5**). Emphasis will be on learning how to construct proofs in each system. The fourth part of the course looks at formal axiom systems, or as they are often called in mathematical logic, **theories**. The fifth part of the course looks at formal *inductive* logic, and to a degree at logics for reasoning over visual content (e.g., diagrams). The sixth and final part of the course is a synoptic look at some of the astonishing work of perhaps the greatest logician: Kurt Gödel.

A more fine-grained schedule now follows.⁶

⁶Note that the Rensselaer Academic Calendar is available [here](#).

5.1 Why Study Logic?; Its History

- **Jan 18:** *General Orientation, Logistics, Mechanics*. The syllabus is reviewed in detail. It's made clear to the students that, in this class, there is a very definite theoretical position on formal logic and the teaching thereof, and that in lockstep with this position the LAMA-BDLA textbook, Slate software system, and HyperGrader, are used. Students wishing to learn under the "Stanford" paradigm are encouraged to take PHIL 2140 in its alternating spot (i.e., Fall semester, annually). Students may also find it helpful to consult the content available for another, related S Bringsjord course: [Are Humans Rational?](#).
- **Jan 22:** *Motivating Puzzles, Problems, Paradoxes, \mathcal{R} , \mathcal{H} , Part I*. Here we among other things tackle problems which, if solvable before further learning, obviate taking the course. We also discuss Bringsjord's "elevated" view of the human mind as potentially near-perfectly rational, and specifically capable of systematic and productive reasoning about the infinite.
- **Jan 25:** *Motivating Puzzles, Problems, Paradoxes, \mathcal{R} , \mathcal{H} , Part II*. A continuation of Part I; the problems in question get harder.
- **Jan 29:** *Whirlwind History and Overview of Formal Logic (in intimate connection with computer science and AI), From Euclid to today's Cutting-Edge Computational Logic*. In one class meeting we surf the timeline of all of formal logic, from Euclid to the present. A particular emphasis is placed on Leibniz, the inventor of modern formal logic. Aristotle is cast as the inventor of formal logic in its original form (syllogistic deduction). The crucial time-point of the discovery of the unsolvability of the *Entscheidungsproblem* by Turing-level computers figures prominently, and supports a skeptical position on The Singularity.

5.2 Propositional Calculus & "Pure" Predicate Calculus

- **Feb 1:** *Review from High School: Variables & Connectives; Propositional Calculus I*. This meeting will tie up any loose ends on the history side of things. Students by this point should have Slate running on their laptops, have

their codes registered, and have submitted their signed SLA.

- **Feb 5:** *Propositional Calculus II: The Formal Language, & First Rules of Inference/Inference Schemata*. Application to some of the original problems used to motivate the course (meetings Jan 18 & 22). Simple proofs settle these problems.
- **Feb 8:** *Propositional Calculus III: Remaining Rules of Inference/Inference Schemata*. Here we discuss the "harder" inference schemata; e.g. proof-by-cases/disjunction elimination. More substantive proofs achieved. In addition, indirect proof is introduced in earnest.
- **Feb 12:*** *Propositional Calculus III continued*. Some harder proofs obtained. This class meeting will be the first time Slate is used in conjunction with HyperGrader. Demonstrations will be given. By this time students should be set up to use HyperGrader with their code. PhD student Rini Palamittam will give a guest lecture on Herb Simon's forgotten dream, and Logic Theorist and GPS, the first /bona fide/
- **Feb 15:*** *The Pure Predicate Calculus*. What kind of logic do we get if we add to the Propositional Calculus machinery for relation symbols, function symbols, and identity (=)?
- **Feb 19:** No class (President's Day Holiday)
- **Feb 20** (a Tuesday): Review, taking stock, prep for first test (next class), sustained Q&A.
- **Feb 22:** Test #1



5.3 First-Order Logic (FOL); Glimpse of SOL, TOL

- **Feb 26:** *The Need for Quantification, and the Centrality Thereof in Human Thought and Communication*.
- **Mar 1:** *New Inference Schemata in FOL, I*.
- **Mar 5:** *New (Harder) Inference Schemata in FOL, II*.
- **Mar 8:** *Proofs/Problems in FOL, I*.
- **Spring Break:** Mar 12–16
- **Mar 19:** *Proofs/Problems in FOL, II*.

- **Mar 22:** *The Liar; Russell's Paradox; Skolem's Paradox.*
- **Mar 26:** *ZFC; Second-Order Logic (SOL), Third-Order Logic (TOL), and Beyond (e.g. Type Theory).*
- **Mar 29:** **Test #2**



5.4 Theories (= Axiom Systems)

- **Apr 2:** (GM Week) *Theories of Arithmetic I (e.g., EA).*
- **Apr 5:** (GM Week) *Theories of Arithmetic II (e.g., PA).*



5.5 Deontic Logic and Killer Robots

- **Apr 9:** *Modal Logic: What and Why.*
- **Apr 12:** *The System D = SDL.*
- **Apr 16:** *The Threat of "Killer" Robots.* Here I present the "PAID" problem: artificial agents/robots that are **p**owerful, **a**utonomous, and **i**ntelligent, are **d**angerous (if not capable of **d**estroying us).
- **Apr 19:** *Logic Can Save Us; Here's How.* After taking note of the fact that *Star Trek* (original) teaches us that logic can save us, This class includes coverage of quantified multi-modal logic, and use of the computational version thereof, implemented, to adjudicate thorny ethical dilemmas.



5.6 Beginning Heterogenous Logic & Beginning *Inductive Logic* (BIL): Glimpses

- **Apr 23:** *Heterogeneous Logic; Whirlwind History & Overview of the Modern Approach to Beginning (Formal) Inductive Logic (LAMA-BIL).* A solution to the Lottery Paradox is provided, and recent work in the RAIR Lab devoted to solving the St Petersburg Paradox will also be covered.



5.7 Gödel

- **Apr 26:** *Gödel's Completeness Theorem & First Incompleteness Theorem.* We seek here to understand the brilliant core of Gödel's CT, from his doctoral dissertation. In addition, we provide the class with a glimpse of Gödel's stunning *incompleteness* theorems, and briefly take up the question: Could an AI ever match Gödel?
- **Apr 30: Test #3**

6 Grading

Grades are based on three in-class tests,⁷ weighted 10%, 20%, and 30%, resp; and on a series of problems to be done in the Slate system, and verified by HyperGrader. Every problem in the series must be certified 100% correct by Hyper-Grader in order to pass the course, and a grade of ‘A’ is earned for the series, which is 40% of the final grade. All assignments must be completed and submitted in order to receive a final grade. In addition, please note that class attendance is mandatory. Any more than two unexcused absences will result in a failing grade.

7 Some Learning Outcomes

There are four desired outcomes. *One*: Students will be able to carry out formal proofs and disproofs, within the Slate system and its workspaces, at the level of the propositional and predicate calculi, and propositional modal logic (the aforementioned systems **T**, **S4**, **D**, and **S5**). *Two*: Students will be able to translate suitable reasoning in English into interconnected formulae in the languages of these four calculi, and assess this reasoning by determining if the desired structures are present in the formulae and relationships between them. *Three*, students will be able to carry out informal proofs. *Four*, students will demonstrate significant understanding of the advanced topics covered.

8 Academic Honesty

Student-teacher relationships are built on mutual respect and trust. Students must be able to trust that their teachers have made responsible decisions about the structure and content of the course, and that they’re conscientiously making their best effort to help students learn. Teachers must be able to trust that students do their work conscientiously and honestly, making their best effort to learn. Acts that violate this mutual respect and trust undermine the educational process; they counteract and contradict our very reason for being at Rensselaer and will not be tolerated. Any student who engages in any form of academic dishonesty will receive an F in this course and will be reported to the Dean of Students for further disciplinary action. (The *Rensselaer Handbook* defines various forms of Academic Dishonesty and procedures for responding to them. All of these forms are violations of trust between students and teachers. Please familiarize yourself with this portion of the handbook.)

References

Arkoudas, K. & Bringsjord, S. (2009), ‘Vivid: An AI Framework for Heterogeneous Problem Solving’, *Artificial Intelligence* **173**(15), 1367–1405. The url <http://kryten.mm.rpi.edu/vivid/vivid.pdf> provides a preprint of the penultimate draft only. If for some reason it is not working, please contact either author directly by email.

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Bass, L. & Johnson, A. (2012), *Geometry: Common Core*, Pearson, Upper Saddle River, NJ. Series Authors: Charles, R., Kennedy, D. & Hall, B. Consulting Authors: Murphy, S. & Wiggins, G.

⁷Each of these tests will have approximately 20 multiple-choice questions, and one to three problems that each call for in-class use of Slate in conjunction with HyperGrader.

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