Introduction to (<u>Formal</u>) Logic

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1 General Orientation

This course is an accelerated, advanced introduction, within the "LAMA" paradigm,[1] to deductive formal logic (with at least some brief but informative pointers to *in*ductive formal logic).[2] The phrase we use to describe what the student is principally introduced to in this class is: *beginning deductive logic, advanced* (BDLA). After this class, the student can proceed to the intermediate level in formal deductive logic, and — with a deeper understanding and better prepared to flourish — to various areas within the *formal sciences*, which are all based on formal logic. The formal sciences include e.g. theoretical computer science (e.g., computability theory, complexity theory, rigorous coverage of programming and programming languages), mathematics in its traditional branches (analysis, topology, algebra, etc), decision theory, game theory, set theory, probability theory, mathematical statistics, etc. (and of course formal logic itself).

What have referred above to "the LAMA paradigm." What is that? This question will be answered in more detail later, but we do say here that while the LAMA paradigm is based upon a number of pedagogical principles, first and foremost among them is what can be labelled the Driving Dictum:

$$\boxed{\text{If you can't prove it, you don't get it.}}$$

Turning back to the nature of formal logic, we it can accuratedly be said that it's the science and engineering of reasoning,[3] but even this supremely general slogan fails to convey the flexibility and enormity of the field. For example, all of classical mathematics can be deductively derived from a small set of formulas (e.g., **ZFC** set theory, which you'll be hearing more about) expressed in the formal logic known as 'first-order logic' (FOL, which you'll *also* be hearing more about), and, as we shall see and discuss in class, computer science emerged from and is in large part based upon logic (for cogent coverage of this emergence, see Glymour 1992). Logic is indeed the foundation for *all* at once rational-and-rigorous intellectual pursuits. (If you can find a counter-example, i.e. such a pursuit that doesn't directly and crucially partake of logic, I would be very interested to see it.)

[1] 'LAMA' is an acronym for 'Logic: A Modern Approach," and is pronounced to rhyme with 'llama' in contemporary English, the name of the exotic and sure-footed camelid whose binomial name is *Lama glama*, and has in fact been referred to in the past by the single-l 'lama.'

[2] Sometimes 'symbolic' is used in place of 'formal,' but that's a bad practice, since — as students in this class will see — formal logic includes the representation of and systematic reasoning over *pictorial* information, and such information is decidedly *not* symbolic. For a discussion of the stark difference between the pictorial vs. the pictorial, and presentation of a formal logic that enables representation of and reasoning over both, see (Arkoudas & Bringsjord 2009).

[3] Warning: Increasingly, the term 'reasoning' is used by some who don't *really* do anything related to reasoning, as traditionally understood, to nonetheless label what they do. Fortunately, it's easy to verify that some reasoning is that which is covered by formal logic: *If the reasoning is explicit, links declarative statements or formulae together via explicit, abstract reasoning schemas or rules of inference (giving rise to at least explicit arguments, and often proofs), is surveyable and inspectable, and ultimately machine-checkable, then the reasoning in question is what formal logic is the science and engineering of.* In order to characterize *in*formal logic, one can remove from the previous sentence the requirements that the links must conform to explicit reasoning schemas or rules of inference, and machine-checkability. It follows that so-called informal logic would revolve around arguments, but not proofs. An excellent overview of informal logic, which will be completely ignored in this class, is provided in "Informal Logic" in the Stanford Encyclopedia of Philosophy. In this article, it's made clear that, yes, informal logic concentrates on the nature and uses of argument.

2 Assistance to Bringsjord

The TA for this course is Rini Palamittam; email address: `palamr@rpi.edu`. Rini will hold office hours on Fridays 230–430 in Carnegie 309 (and by appointment). Some guest lectures may be provided by researchers working in the RAIR Lab, a logic-based AI lab.

3 Prerequisites

There are no formal prerequisites. However, as said above, this course introduces *formal* logic, and does so in an accelerated, advanced way. This implies that — for want of a better phrase — students are expected to have a degree of logico-mathematical maturity. You have this on the assumption that you understood the math you were supposed to learn to make it where you are.[4] For example, to get to where you are now, you were supposed to have learned the technique of *indirect proof* (= proof by contradiction = *reductio ad absurdum*). An example of the list of concepts and techniques you are assumed to be familiar with from high-school geometry can be found in the common-core-connected (Bass & Johnson 2012). An example of the list of concepts and techniques you are assumed to be familiar with from high-school Algebra 2 can be found in the common-core-connected (Bellman, Bragg & Handlin 2012). It's recommended that during the first two weeks of the class, students review their high-school coverage of formal logic, which includes at minimum the rudiments of the propositional calculus.

4 Texts

Students will purchase an inseparable combination of the e-text *Logic: A Modern Approach; Beginning Deductive Logic, Advanced via Slate* (LAMA-BDLA) and the Slate software system; both will be available in the RPI Bookstore in due time. Full logistics of the purchase, and the contents of the CD that holds this pair (and other files), will be explained the second class meeting, Jan 23, and subsequently, as needed. The first use in earnest of Slate will happen in class on Feb 2, so by the start of class on that day students will need have the textbook-software combo, and be able to open it on their laptops in class. Updates to LAMA-BDLA, and additional exercises, will be provided by listing on the course web page (and sometimes by email) through the course of the semester. You will need to manage many electronic files in the course of this course, and e-housekeeping and e-orderliness is of paramount importance. You will specifically need to assemble a library of completed and partially completed proofs/arguments/truth-trees etc. so that you can use them as building blocks in harder proofs; in other words, building up your own "logical library" will be crucial.

Please note that Slate is copyrighted software: copying and/or distributing this software to others is strictly prohibited. You will need to submit (via hard copy in person, or email to `Selmer.Bringsjord@gmail.com`) to S Bringsjord a signed hard-copy version of a Software License Agreement (a pdf is included on the aforementioned CD). This agreement will also cover the

[4] If you happen to be a student reading this as one wanting to be introduced to formal logic, from outside RPI, please examine your own case realistically. If you are not in command of the traditional high-school-level content for algebra, geometry, trigonometry, and at least some (differential and integral) calculus, you will need to go with a standard, non-advanced introduction to logic in the LAMA paradigm. Specifically, you will need the LAMA-BDL textbook, *not* LAMA-BDLA. The 'A' in 'LAMA-BDLA' is for 'Advanced.' Check which textbook you have!

textbook, which is copyrighted as well, and as an ebook cannot be copied or distributed.

In addition, occasionally papers may be assigned as reading. Two background ones, indeed, are hereby assigned: (Bringsjord, Taylor, Shilliday, Clark & Arkoudas 2008, Bringsjord 2008).

Finally, slide decks used in class will contain crucial additional content above and beyond LAMA-BDLA and Slate, and will be available on the web site for course for study.

5 Schedule

The course is basically divided into first a motivating stretch (during which we show that the logically untrained have great trouble reasoning well), and then five additional parts. In the first threee of these remaining parts we'll focus on the **propositional calculus**; in the second on **first-order logic** (= FOL), with a brief look at **second-order logic** and beyond; and in the third, we'll cover **modal logic** (in the form, specifically, of four closely related modal logics: **T**, **S4**, **D** (= **SDL**), and **S5**). Emphasis will be on learning how to construct proofs in each system. The fourth part of the course looks at formal axiom systems, or as they are often called in mathematical logic, **theories**. The fifth part of the course looks at formal *in*ductive logic, and to a degree at logics for reasoning over visual content (e.g., diagrams). The sixth and final part of the course is a synoptic look at some of the astonishing work of perhaps the greatest logician: Kurt Gödel.

A more fine-grained schedule now follows.[5]

[5]Note that the Rensselaer Academic Calendar is available here.

5.1 Why Study Logic?; Its History

- **Jan 19**: General Orientation, Logistics, Mechanics. The syllabus is reviewed in detail. It's made clear to the students that, in this class, there is a very definite theoretical position on formal logic and the teaching thereof, and that in lockstep with this position the LAMA-BDLA textbook, Slate software system, and — in a major innovation over and above prior editions of this course — Hyper-Grader, are used. Students wishing to learn under the "Stanford" paradigm are encouraged to take PHIL 2140 in its other alternating spot (e.g., Fall semester, annually). Students may also find it helpful to consult the content available for another Bringsjord course: *Are Humans Rational?*.
- **Jan 23**: Motivating Puzzles, Problems, Paradoxes, and $\mathcal{R}$, Part I.
- **Jan 26**: Motivatng Puzzles, Problems, Paradoxes, and $\mathcal{R}$, Part II.
- **Jan 30**: Whirlwind History and Overview of Formal Logic (in intimate connection with Computer Science), From Euclid to today's Cutting-Edge Computational Logic

5.2 Propositional Calculus & "Pure" Predicate Calculus

- **Feb 2**: Review from High School: Variables & Connectives. This meeting will tie up any loose ends on the history side of things, and will also be the first time that Slate is used substantively in class, along with Hyper-Grader. Students by this point must have the system running on their laptops, have their codes registered, and have submitted their signed SLA.
- **Feb 6**: Propositional Calculus I: The Formal Language, & Rules of Inference. Application to some of the original problems.
- **Feb 13**: Propositional Calculus II
- **Feb 20**: No class (President's Day Holiday)
- **Feb 21** (a Tuesday)*: Propositional Calculus III. Problems presented by Atriya Sen.
- **Feb 23**: The Pure Predicate Calculus

5.3 First-Order Logic (FOL), and a Glimpse of SOL, TOL

- **Feb 27**: The Need for Quantification
- **Mar 2**: **Test #1**
- **Mar 6**: New Inference Schemata in FOL
- **Mar 9***: Proofs/Problems in FOl, I
- **Spring Break**: Mar 13–16
- **Mar 20**: Measuring Intelligence. Here we provide a glimpse of the Arithmetic Hierarchy, and highlight numerical quantification.
- **Mar 23**: The Liar; Russell's Paradox
- **Mar 27**: ZFC; Second-Order Logic (SOL), Third-Order Logic (TOL), and Beyond (e.g. Type Theory)

5.4 Theories (= Axiom Systems) of Arithmetic

- **Mar 30**: **Test #2**
- **Apr 3**: Theories of Arithmetic I (e.g., **EA**)
- **Apr 6**: Theories of Arithmetic II (e.g., **PA**)

5.5 Deontic Logic and Killer Robots

- **Apr 10**: The System **D** = **SDL**
- **Apr 13**: The Threat of "Killer" Robots
- **Apr 17**: Logic Can Save Us; Here's How

5.6 Beginning Heterogenous Logic & Beginning *In*ductive Logic (BIL): Glimpses

- **Apr 20**: Heterogeneous Logic; Whirlwind History & Overview of the Modern Approach to Beginning (Formal) Inductive Logic (LAMA-BIL).
- **Apr 24**: The Lottery Paradox, Solved. Recent work in the RAIR Lab devoted to solving the St Petersburg Paradox will also be covered. Kevin O'Neill will make a guest appearance.

5.7 Gödel

- **Apr 27**: Gödel's Completeness Theorem. We seek here to understand the brilliant core of Gödel's CT, from his doctoral dissertation. In addition, we provide the class with a glimpse of Gödel's stunning *in*completeness theorems, and briefly take up the question: Could an AI ever match Gödel?
- **May 1**: **Test #3**

6 Grading

Grades are based on three in-class tests,[6] weighted 10%, 20%, and 30%, resp; and on a series of problems to be done in the Slate system, and verified by Hyper-Grader. Every problem in the series must be certified 100% correct by Hyper-Grader in order to pass the course, and a grade of 'A' is earned for the series, which is 40% of the final grade. All assignments must be completed and submitted in order to receive a final grade. In addition, please note that class attendance is mandatory. Any more than two unexcused absences will result in a failing grade.

7 Some Learning Outcomes

There are four desired outcomes. *One*: Students will be able to carry out formal proofs and disproofs, within the Slate system and its workspaces, at the level of the propositional and predicate calculi, and propositional modal logic (the aforementioned systems **T**, **S4**, **D**, and **S5**). *Two*: Students will be able to translate suitable reasoning in English into interconnected formulae in the languages of these four calculi, and assess this reasoning by determining if the desired structures are present in the formulae and relationships between them. *Three*, students will be able to carry out informal proofs. *Four*, students will demonstrate significant understanding of the advanced topics covered.

8 Academic Honesty

Student-teacher relationships are built on mutual respect and trust. Students must be able to trust that their teachers have made responsible decisions about the structure and content of the course, and that they're conscientiously making their best effort to help students learn. Teachers must be able to trust that students do their work conscientiously and honestly, making their best effort to learn. Acts that violate this mutual respect and trust undermine the educational process; they counteract and contradict our very reason for being at Rensselaer and will not be tolerated. Any student who engages in any form of academic dishonesty will receive an F in this course and will be reported to the Dean of Students for further disciplinary action. (The *Rensselaer Handbook* defines various forms of Academic Dishonesty and procedures for responding to them. All of these forms are violations of trust between students and teachers. Please familiarize yourself with this portion of the handbook.)

References

Arkoudas, K. & Bringsjord, S. (2009), 'Vivid: An AI Framework for Heterogeneous Problem Solving', *Artificial Intelligence* **173**(15), 1367–1405. The url http://kryten.mm.rpi.edu/vivid/vivid.pdf provides a preprint of the penultimate draft only. If for some reason it is not working, please contact either author directly by email.
URL: *http://kryten.mm.rpi.edu/vivid_030205.pdf*

Bass, L. & Johnson, A. (2012), *Geometry: Common Core*, Pearson, Upper Saddle River, NJ. Series Authors: Charles, R., Kennedy, D. & Hall, B. Consulting Authors: Murphy, S. & Wiggins, G.

[6] Each of these tests will have approximately 50 multiple-choice questions, and one to three problems that each call for an informal proof.

Bellman, A., Bragg, S. & Handlin, W. (2012), *Algebra 2: Common Core*, Pearson, Upper Saddle River, NJ. Series Authors: Charles, R., Kennedy, D. & Hall, B. Consulting Authors: Murphy, S. & Wiggins, G.

Bringsjord, S. (2008), Declarative/Logic-Based Cognitive Modeling, *in* R. Sun, ed., 'The Handbook of Computational Psychology', Cambridge University Press, Cambridge, UK, pp. 127–169.
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Bringsjord, S., Taylor, J., Shilliday, A., Clark, M. & Arkoudas, K. (2008), Slate: An Argument-Centered Intelligent Assistant to Human Reasoners, *in* F. Grasso, N. Green, R. Kibble & C. Reed, eds, 'Proceedings of the 8th International Workshop on Computational Models of Natural Argument (CMNA 8)', University of Patras, Patras, Greece, pp. 1–10.
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