Introduction to (Formal) Logic

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Selmer Bringsjord
Rensselaer AI & Reasoning (RAIR) Lab
http://www.rpi.edu/~brings
Department of Cognitive Science
Department of Computer Science
Lally School of Management
Rensselaer Polytechnic Institute (RPI)
Troy NY 12180 USA
Office Hours: M R 2–330 CA312; & by appointment
518.276.6472
Selmer.Bringsjord@gmail.com

version 0125161200NY

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1 General Orientation

This course is an introduction to deductive formal logic (with at least some informative pointers to *in*ductive formal logic). Sometimes 'symbolic' is used in place of 'formal.' The phrase we use to describe what the student is principally introduced to in this class is: *beginning deductive logic* (BDL). After this class, the student can proceed to the intermediate level in formal deductive logic, and — with a deeper understanding and better prepared to flourish — to various areas within the *formal sciences*, which are all based on formal logic. The formal sciences include e.g. theoretical computer science (e.g., computability theory, complexity theory, rigorous coverage of programming and programming languages), mathematics in its traditional branches (analysis, topology, etc), decision theory, game theory, set theory, etc.

In general, formal logic is the science and engineering of reasoning,[1] but even this supremely general description fails to convey the flexibility and enormity of the field. For example, all of classical mathematics can be deductively derived from a small set of formulas (e.g., **ZFC** set theory, which you'll be hearing more about) expressed in the formal logic known as 'first-order logic' (FOL, which you'll *also* be hearing more about), and computer science emerged from and is in large part based upon logic (for cogent coverage of this emergence, see Glymour 1992). Logic is indeed the foundation for *all* at once rational-and-rigorous intellectual pursuits. (If you can find a counter-example, i.e. such a pursuit that doesn't directly and crucially partake of logic, I would be very interested to see it.)

2 Assistance to Bringsjord

The TA for this course is Rini Palamittam; email address: `palamr@rpi.edu`. Some guest lectures will be provided by PhD students working in the RAIR Lab, a logic-based AI lab. Dr Joshua Taylor, whose mastery of the Slate software system is unparalleled, will try to help address any technical problems that might arise with this system, and will be making contributions to a new enhancement to the LAMA paradigm, to be introduced later in this semester (viz., web-based proof-checking).

3 Prerequisites

There are no formal prerequisites. However, as said above, this course covers *formal* logic. This implies that — for want of a better phrase — students are expected to have a degree of logico-mathematical maturity. You have this on the assumption that you understood the math you were

[1] Warning: Increasingly, the term 'reasoning' is used by some who don't really do anything related to reasoning, as traditionally understood, to nonetheless label what they do. Fortunately, it's easy to verify that some reasoning is that which is covered by formal logic: If the reasoning is explicit, links declarative statements or formule together via explicit, abstract reasoning schemas or rules of inference (giving rise to at least explicit arguments, and often proofs), is surveyable and inspectable, and ultimately machine-checkable, then the reasoning in question is what formal logic is the science and engineering of. In order to characterize *in*formal logic, one can remove from the previous sentence the requirements that the links must conform to explicit reasoning schemas or rules of inference, and machine-checkability. It follows that so-called informal logic would revolve around arguments, but not proofs. An excellent overview of informal logic, which will be completely ignored in this class, is provided in "Informal Logic" in the Stanford Encyclopedia of Philosophy. In this article, it's made clear that, yes, informal logic concentrates the nature and uses of argument.

supposed to learn to make it where you are. For example, to get to where you are now, you were supposed to have learned the technique of indirect proof (= proof by contradiction = *reductio ad absurdum*). An example of the list of concepts and techniques you are assumed to be familiar with from high-school geometry can be found in the common-core-connected (Bass & Johnson 2012). An example of the list of concepts and techniques you are assumed to be familiar with from high-school Algebra 2 can be found in the common-core-connected (Bellman, Bragg & Handlin 2012). It's recommended that during the first two weeks of the class, students review their high-school coverage of formal logic.

4 Texts

Students will purchase an inseparable combination of the e-text *Logic: A Modern Approach; Beginning Deductive Logic via Slate* (LAMA-BDL) and the Slate software system; both will be available in the RPI Bookstore. Full logistics of the purchase, and the contents of the CD that holds this pair (and other files), will be explained the first class. Updates to LAMA-BDL, and additional exercises, will be provided by email through the course of the semester. You will need to manage many electronic files in the course of this course, and e-housekeeping and e-orderliness is of paramount importance. You will specifically need to assemble a library of completed and partially completed proofs so that you can use them as building blocks in harder proofs; in other words, building up your own "logical library" will be crucial.

Please note that Slate is copyrighted software: copying and/or distributing this software to others is strictly prohibited. You will need to submit (via hard copy in person, or email) to Bringsjord a signed hard-copy version of a Software License Agreement (a pdf is included on the aforementioned CD). This agreement will also cover the textbook, which is copyrighted as well, and cannot be copied or distributed.

In addition, occasionally papers may be assigned as reading. Two, indeed, are hereby assigned: (Bringsjord, Taylor, Shilliday, Clark & Arkoudas 2008, Bringsjord 2008).

Finally, slide decks used in class will contain crucial additional content above and beyond LAMA-BDL and Slate, and will be available on the web site for course.

5 Schedule

The course is basically divided into first a motivating stretch (during which we show that the logically untrained have great trouble reasoning well), and then five additional parts. In the first threee of these remaining parts we'll focus on the **propositional calculus**; in the second on FOL; and in the third, we'll cover **modal logic** (in the form, specifically, of four closely related modal logics: **T**, **S4**, **D** (= **SDL**), and **S5**). Emphasis will be on learning how to construct proofs in each system. The fifth part of the course looks at formal axiom systems, or as they are often called in mathematical logic, **theories**. The last part of the course is a synoptic look at some of the astonishing work of perhaps the greatest logician: Kurt Gödel.

A more fine-grained schedule now follows.[2]

[2]Note that the Rensselaer Academic Calendar is available here.

5.1 Why Study Logic?; Its History

- **Jan 25**: General Orientation, Logistics, Mechanics. The syllabus is reviewed in detail. It's made clear to the students that, in this class, there is a very definite theoretical position on formal logic and the teaching thereof, and that in lockstep with this position the LAMA-BDL textbook and Slate software system are used. Students wishing to learn under the "Stanford" paradigm are encouraged to take PHIL 2140 in its other alternating spot (e.g., Fall semester, annually). Relevant reading includes "Preface" in the LAMA-BDL text. Students may also find it helpful to consult the content available for another Bringsjord course: *Are Humans Rational?*.
- **Jan 28**: Motivating Puzzles, Problems, Paradoxes, and $\mathcal{R}$I.
- **Feb 1**: Motivatng Puzzles, Problems, and Paradoxes, and $\mathcal{R}$, II.
- **Feb 4**: Whirlwind History and Overview of Formal Logic, From Euclid to today's Cutting-Edge Computational Logic

5.2 Propositional Calculus & "Pure" Predicate Calculus

- **Feb 8**: Review from High School: Variables & Connectives
- **Feb 11**: Propositional Calculus I
- **Feb 15**: No class (President's Day Holiday)
- **Feb 18**: Propositional Calculus II
- **Feb 22***: Propositional Calculus III
- **Feb 25**: **Test #1**

5.3 First-Order Logic (FOL)

- **Feb 29**: The Need for Quantification
- **Mar 3**: New Inference Schemata in FOL, I
- **Mar 7**: New Inference Schemata in FOL, II
- **Mar 10**: Proofs/Problems in FOl, I
- **Spring Break**: Mar 14–18
- **Mar 21**: Proofs/Problems in FOL, II
- **Mar 24**: Review, Q&A, Help for FOL
- **Mar 28**: **Test #2**

5.4 Theories (= Axiom Systems)

- **Apr 4**: Russell Paradox, Richard's Paradox
- **Apr 7**: ZFC
- **Apr 11**: Theories of Arithmetic (e.g., **PA**)

5.5 Deontic Logic and Killer Robots

- **Apr 14**: The System **D** = **SDL**
- **Apr 18**: The Threat of "Killer" Robots
- **Apr 21**: Logic Can Save Us; Here's How

5.6 Beginning *In*ductive Logic (BIL): A Glimpse

- **Apr 25**: Whirlwind History & Overview of LAMA-BIL
- **Apr 28**: The Lottery Paradox, Solved

5.7 Gödel

- **May 2**: Gödel's Incompleteness Theorems
- **May 5**: Could an AI ever match Gödel?

- **May 9**: **Test #3**

6 Grading

Grades are based on three in-class tests, weighted 10%, 20%, and 30%, resp; and on four homeworks, each worth 10%. All assignments must be completed and submitted in order to receive a final grade.

7 Some Learning Outcomes

There are four desired outcomes. *One*: Students will be able to carry out formal proofs and disproofs, within the Slate system and its workspaces, at the level of the propositional and predicate calculi, and propositional modal logic (the aforementioned systems **T**, **S4**, **D**, and **S5**). *Two*: Students will be able to translate suitable reasoning in English into interconnected formulae in the languages of these four calculi, and assess this reasoning by determining if the desired structures are present in the formulae and relationships between them. *Three*, students will be able to carry out informal proofs. *Four*, students will demonstrate significant understanding of the advanced topics covered.

8 Academic Honesty

Student-teacher relationships are built on mutual respect and trust. Students must be able to trust that their teachers have made responsible decisions about the structure and content of the course, and that they're conscientiously making their best effort to help students learn. Teachers must be able to trust that students do their work conscientiously and honestly, making their best effort to learn. Acts that violate this mutual respect and trust undermine the educational process; they counteract and contradict our very reason for being at Rensselaer and will not be tolerated. Any student who engages in any form of academic dishonesty will receive an F in this course and will be reported to the Dean of Students for further disciplinary action. (The *Rensselaer Handbook* defines various forms of Academic Dishonesty and procedures for responding to them. All of these forms are violations of trust between students and teachers. Please familiarize yourself with this portion of the handbook.)

References

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