Q1 This problem concerns what can be called *Chisholm’s Paradox Paradox*. The repetition is intentional. Chisholm’s Paradox (CP) is that from a particular representation in Standard Deontic Logic (SDL) of four seemingly innocuous givens, a contradiction $\zeta \land \neg \zeta$ can be deduced. (We covered this in class, of course.) The four givens are based on the story of a character Jones, who is obligated to go to assist his neighbors (move to a different domicile, e.g.). It would be wrong of him to show up unannounced, though; so if he goes to assist them, it ought to be that he tells them he is coming. In addition, if it’s not the case that Jones goes to assist them, then it ought to be that it not be the case that he tells them he’s coming. Finally, as a matter of fact, it’s not case the Jones goes to assist (because on the way he comes across a car accident, and has an opportunity to save one of the victims).

Here are the four givens, in English:

A. It ought to be that Jones goes to the assistance of his neighbors.
B. It ought to be that if Jones goes, then he tells them he’s coming.
C. If Jones doesn’t go, then it ought to be that it’s not the case that he tells them he’s coming.
D. Jones doesn’t go.

Represent each of A. to D. in Slate, and prove (in system D) that from your four formulae you can deduce $\zeta \land \neg \zeta$. Your proof constitutes Chisholm’s Paradox. Two more proofs are needed in your Slate file in order to create Chisholm’s Paradox Paradox. We now explain . . .

Q2 The Continuum Hypothesis (CH), exactly as we formalized it (and discussed in connection with the “detective work” of Gödel) would be what as an s-expression in Slate? Next, does it follow deductively in FOL from the negation of CH that there exists an infinite subset $x$ of the reals such that no bijective function mapping $x$ to the natural numbers exists? If so, show this by way of a Slate proof.

Q3 As you know, $\Diamond$ denotes permissibility in D. The formula $\Diamond \phi$ says that it’s permissible that $\phi$ be the case. Is it a theorem in D that $\phi \rightarrow \Diamond \Diamond \phi$? Give a Slate proof that your answer is correct. Now, next, we write $\Diamond^n$, where $n$ is a positive integer, to indicate that there are $n$ successive permissibility operators in sequence. Hence, the formula $\Diamond^5 \psi$, when unpacked, becomes, $\Diamond \Diamond \Diamond \Diamond \Diamond \psi$. Where ‘1T’ is short for 1 trillion, Betty claims that the following formula is a theorem in D:

$$\phi \rightarrow \Diamond^{1T} \phi$$

Is she right? Prove it.

Q4 Austin Hardy, as we noted, proved one side of Gödel’s first incompleteness theorem (and we reviewed his reasoning). You were given the challenge of proving the other half, viz. that the negation of $g^*$, $\neg g^*$, isn’t provable from PA, in the specific context laid out in our slides. Give here your proof that meets this challenge.

Q5 Extra credit problem: As Selmer types this sentence (at 850pm NY time, May 7), his wife is asking him to buy a Powerball ticket. Selmer replies: “That would be a waste of time. You won’t win. Not only do I believe you won’t win, but I know you won’t win. It’s no different that believing (and knowing) that if I go outside to run down to Stewart’s for a ticket, I won’t be mashed by a meteorite.” Would Kevin O’Neill agree with Selmer? Why?