Immediate Action Items:

Please now, before you do anything else, write down the following details on the Scantron sheets in print: Name, your CD/HyperGrader Code, email address, and RIN.

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1 Multiple Choice Questions

Q1 Suppose that you are presented with a version of the Wason Selection Task exactly the same as the one we considered in class, except for two things: viz., (i) the four cards in front of you have the appearance of what follows the present paragraph; and (ii), the rule from before is supplanted with this new one: “If there’s either a vowel or a consonant on one side, then there is a prime number on the other.” Which card or cards should you turn over?

\[ \begin{align*} &G \quad A \quad 7 \quad 6 \end{align*} \]

a You should flip \[ G \] only.
b You should flip \[ A \] and \[ G \].
c You should flip \[ A \] and \[ G \] and \[ 6 \].
d You should flip over all of the cards.
e You should turn over none of the cards.

Q2 Which of the following inference rules involve the discharging of an assumption?

a universal elimination
b universal introduction
c existential elimination
d existential introduction
e none of the above

Q3 The statement “There exists a heavy object” may be correctly inferred from which of the following sets of statements?

a “There exists a blue object” and “There exists a heavy object if there’s a blue rock”.
b “Rocks are objects” and “There’s a blue rock” and “If a rock is blue, then it’s heavy”
c “There exists a blue object” and “There’s a blue rock” and “All heavy objects are blue”.
d “There exists a heavy object if there’s a blue rock” and “All blue objects are heavy”
e none of the above

Q4 Which one of the following formulae is Russell’s Theorem?

a \( \neg \exists x \forall y (y \in x \leftrightarrow y \notin y) \)
b \( \exists x \forall y (y \in x \leftrightarrow y \notin y) \)
c \( \exists x \forall y (y \in x \leftrightarrow \phi(y)), \text{ where } x \notin \text{free}(\phi). \)
d None of the above.
e Actually, Russell’s Theorem is equivalent to b and c.
Q5 Is it true that \{∀x(Scared(x) ↔ Small(x)), ∃x¬Scared(x)\} ⊢ ∃x¬Small(x)?

a Yes
b No

Q6 Is it true that \{¬∃x(Llama(x) ∧ Small(x)), ∀x(Small(x) ∨ Medium(x) ∨ Large(x))\} ⊢ ∀x(Llama(x) → (Medium(x) ∨ Large(x)))?

a Yes
b No

Q7 Is it true that ⊢ ∃x(Llama(x) → ∀yLlama(y))?

a Yes
b No

Q8 Is it true that \{∃x(Llama(x) ∧ ∀y(Llama(y) → y = x)) ⊢ ∃x∀y(Llama(y) ↔ y = x)\}?

a Yes
b No

Q9 Consider the following three givens:

GIVEN1 Lazy(larry)
GIVEN2 ∀x(Smart(x) → Lazy(x))
GIVEN3 ∀x¬(¬Llama(x) ∨ ∀yLlama(y))

From these givens does the following goal follow deductively?

GOAL Smart(larry)

a Yes
b No

Q10 φₙ entails which of the following?

a φₙ₊₁
b φₙ
ce φₙ₊₁
d φₙ₋₁
e φₙ₋₁
Q11 Assume that we would like to have an inspection robot $R$ verify that a certain zoo has exactly two llamas in it. The robot must be supplied by human overseers with a formal specification, in the form of a single formula $\sigma$ of FOL, which says that there are exactly two llamas. (The machine knows that the context here is the zoo, so we don’t need to bother expressing that context in FOL by using a relation symbol to convey that we’re talking about llamas in the zoo.) Given a correct specification $\sigma$, $R$ will then be able to return an infallibly correct verdict as to whether the zoo “passes” (i.e., has exactly two llamas). Which of the following FOL sentences is a correct specification of $\sigma$?

\[ a \quad \exists x, y \quad [(\text{Llama}(x) \land \text{Llama}(y) \land x \neq y) \land \forall x, y, z [(\text{Llama}(x) \land \text{Llama}(y) \land \text{Llama}(z)) \rightarrow (x = y \lor z)] \]  
\[ b \quad \exists x, y \quad [(\text{Llama}(x) \land \text{Llama}(y) \land x \neq y) \land \forall z (\text{Llama}(z) \rightarrow (z = x \lor z = y))] \]  
\[ c \quad \exists x, y \quad (x \neq y) \land \forall z (\text{Llama}(z) \leftrightarrow (z = x \lor z = y)) \]  
\[ d \quad \exists x, y \quad \forall z (x \neq y) \land (\text{Llama}(x) \leftrightarrow (z = x \lor z = y)) \]  
\[ e \quad \text{All of the above are acceptable as formulae for } \sigma. \]

Q12 Consider again the situation that sets the stage for the previous question. Two zookeepers, Zach and Zelda, are tasked with making sure that the robot $R$ that patrols the zoo verifies that at all times there are at least 6 zebras. In order to do this, they must supply the robot with a specification $\sigma'$ which expresses in FOL the proposition that there exist at least 6 zebras. (Once again, we leave aside needing to capture that the zebras must exist in the zoo in question.) Unfortunately, the system that Zach and Zelda must work with in order to type in $\sigma'$ provides only five variables to use: $x, y, z, u, v$. Zach says that despite this constraint, since they are free to use as many quantifiers as they wish, they can build a formula that gets the job done. Zelda says: “No, there’s no way to do it. We have an impossible task.” Who is right?

\[ a \quad \text{Zach} \]  
\[ b \quad \text{Zelda} \]

Q13 Consider the following sentence: ‘This sentence is false.’ Let’s label this sentence ‘L.’ As you know, this self-referential sentence leads directly to a contradiction. Can we solve this problem by simply stipulating that we don’t allow self-referential sentences?

\[ a \quad \text{Yes} \]  
\[ b \quad \text{No} \]

Q14 Frege and Russell were contemporaries, as you know. You also know that Russell sent a short proof to Frege that obliterated Frege’s Axiom V, which was part of Frege’s foundation for all of mathematics. Now, did Russell first discover this proof upon studying Frege’s work, or did he discover it in thinking separate from and prior to studying Frege’s work?

\[ a \quad \text{Given only what we examined in class, and taking Russell’s letter at face value, upon studying Frege’s work.} \]  
\[ b \quad \text{Given only what we examined in class, and taking Russell’s letter at face value, in thinking separate from and prior to studying Frege’s work.} \]
c Given only what we examined in class, and taking Russell’s letter at face value, there’s no clear answer to this question.

Q15 As you know, we have introduced the following numerical quantifiers: \( \exists = k, \exists \leq k, \exists \geq k \), where of course \( k \in \mathbb{Z}^+ \). This allows us for instance to economically represent such statements as “There are exactly 4 dim llamas.”\footnote{This statement would be represented by \( \exists^4 x (\text{Dim}(x) \land \text{Llama}(x)) \).} The general form of such a formula would be \( \exists^k x \phi(x) \), where \( \phi(x) \),” as we have said before, simply says that \( x \) is free in \( \phi \). Now, given this context, which of the following is true?

\[
\begin{align*}
&\text{a} \quad \{\exists = k x \phi(x)\} \vdash \exists \leq k x \phi(x) \\
&\text{b} \quad \{\exists \geq k x \phi(x), \exists \leq k x \phi(x)\} \vdash \exists = k x \phi(x) \\
&\text{c} \quad \{\exists = k x \phi(x)\} \vdash \exists \geq k x \phi(x) \\
&\text{d} \quad \text{All of the above.} \\
&\text{e} \quad \text{None of the above.}
\end{align*}
\]

Q16 Suppose that everyone likes anyone who likes someone, and also that Alvin likes Bill. Does it follow deductively from this pair of assumptions that Xavier likes Yolanda?

\[
\begin{align*}
&\text{a} \quad \text{Yes} \\
&\text{b} \quad \text{No}
\end{align*}
\]

Q17 Suppose that no one is liked by anyone who likes everyone, and also that Alvin likes everyone. Does it follow deductively from this pair of assumptions that Xavier likes Yolanda?

\[
\begin{align*}
&\text{a} \quad \text{Yes} \\
&\text{b} \quad \text{No}
\end{align*}
\]

Q18 Suppose that \( \forall x \text{LlamaCute}(x) \). (There is no typographical error in the previous sentence.) Suppose, secondly, that \( \text{Llama}(\text{lonnie}) \). Does it follow deductively from these two suppositions that \( \text{Cute}(\text{lonnie}) \)?

\[
\begin{align*}
&\text{a} \quad \text{Yes} \\
&\text{b} \quad \text{No}
\end{align*}
\]

Q19 Suppose that \( \exists x \text{LlamaCute}(x) \). (There is no typographical error in the previous sentence.) Suppose, secondly, that \( \text{Llama}(\text{lonnie}) \). Does it follow deductively from these two suppositions that \( \text{Cute}(\text{lonnie}) \)?

\[
\begin{align*}
&\text{a} \quad \text{Yes} \\
&\text{b} \quad \text{No}
\end{align*}
\]
Two Problems on HyperGrader: OnlyMediumOrLargeLlamas and Chimerical Barber

Two new problems for solving through HyperGrader have just been posted, or will momentarily be posted; they’re called ‘OnlyMediumOrLargeLlamas’ and ‘ChimericalBarber.’ These two problems are part of the present test. To earn the relevant points on the test, go to

http://www.logicamodernapproach.com/allProblems

and obtain the underlying Slate files, create in each case a proof that obtains the GOAL from the GIVEN/S (you are allowed to use the PC provability oracle, but only that oracle), submit your solution file, and earn a trophy (which signifies that you have earned the points at stake on Test 2). Only if you earn a trophy can you be sure that your points have been earned.