How’d We Arrive Here?
(Selmer’s Leibnizian Whirlwind History of Logic)

Selmer Bringsjord

Rensselaer AI & Reasoning (RAIR) Lab
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Troy, New York 12180 USA

Intro to Logic
1/24/2019
The Starting Code to Purchase in Bookstore

Your code for starting the registration process is:

To access HyperGrader, HyperSlate, the license agreement, and to obtain the textbook LAMA-BDLA, go to:

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Once seal broken on envelope, no return. Remember from first class, any reservations, opt for “Stanford” paradigm, with its software instead of LAMA™ paradigm!
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Intro to Logic
1/29/2018
Caveat; Apology; Rain Check
Caveat; Apology; Rain Check

LAMA–BDLA
Caveat; Apology; Rain Check

LAMA–BDLA
Caveat; Apology; Rain Check

LAMA–BDLA
Caveat; Apology; Rain Check

LAMA–BDLA

LAMA–BIL, a bit.
The Monty Hall Problem

$1M
The Monty Hall Problem

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The Monty Hall Problem

$1M
Jones has come to a game show, and finds himself thereon selected to play a game on national TV with the show’s suave host, Full Monty. Jones is told correctly by Full that hidden behind one of three closed, opaque doors facing the two of them is $1,000,000, while behind each of the other two is a feculent, obstreperous llama whose value on the open market is charitably pegged at $1. Full reminds Jones that this is a game, and a fair one, and that if Jones ends up selecting the door with $1M behind it, all that money will indeed be his. (Jones' net worth has nearly been exhausted by his expenditures in traveling to the show.) Full also reminds Jones that he (= Full) knows what’s behind each door, fixed in place until the game ends.

Full asks Jones to select which door he wants the contents of. Jones says, "Door 1." Full then says: "Hm. Okay. Part of this game is my revealing at this point what's behind one of the doors you didn't choose. So ... let me show you what's behind Door 3." Door 3 opens to reveal a very unsavory llama. Full now to Jones: "Do you want to switch to Door 2, or stay with Door 1? You'll get what's behind the door of your choice, and our game will end." Full looks briefly into the camera, directly.

(P1.1) What should Jones do if he's rational?

(P1.2) Prove that your answer is correct. (Diagrammatic proofs are allowed.)

(P1.3) A quantitative hedge fund manager with a PhD in finance from Harvard zipped this email off to Full before Jones made his decision re. switching or not: "Switching would be a royal waste of time (and time is money!). Jones hasn't a doggone clue what's behind Door 1 or Door 2, and it's obviously a 50/50 chance to win whether he stands firm or switches. So the chap shouldn't switch!" Is the fund manager right? Prove that your diagnosis is correct.

(P1.4) Can these answers and proofs be exclusively Bayesian in nature?
Elements of the branch of logic known as conditional logic are analyzed informally but systematically, as indicated e.g. by Mikhail (2011), programming languages is nothing more than reliance upon the material formal logics that include conditionals much more expressive and nuanced. A module is, if we so wished we could always talk about modules.

**Intro to (Formal) Logic @ RPI**
Intro to (Formal) Logic @ RPI
Intro to (Formal) Logic @ RPI
Euclidean “Magic”

**Theorem:** There are infinitely many primes.

**Proof:** We take an indirect route. Let $\Pi = p_1 = 2, p_2 = 3, p_3 = 5, \ldots, p_k$ be a finite, exhaustive consecutive sequence of prime numbers. Next, let $M_{\Pi}$ be $p_1 \times p_2 \times \cdots \times p_k$, and set $M'_{\Pi}$ to $M_{\Pi} + 1$. Either $M'_{\Pi}$ is prime, or not; we thus have two (exhaustive) cases to consider.

C1 Suppose $M'_{\Pi}$ is prime. In this case we immediately have a prime number beyond any in $\Pi$ — contradiction!

C2 Suppose on the other hand that $M'_{\Pi}$ is *not* prime. Then some prime $p$ divides $M'_{\Pi}$. (Why?) Now, $p$ itself is either in $\Pi$, or not; we hence have two sub-cases. Supposing that $p$ is in $\Pi$ entails that $p$ divides $M_{\Pi}$. But we are operating under the supposition that $p$ divides $M'_{\Pi}$ as well. This implies that $p$ divides 1, which is absurd (a contradiction). Hence the prime $p$ is outside $\Pi$.

Hence for *any* such list $\Pi$, there is a prime outside the list. That is, there are infinitely many primes. \( \text{QED} \)
Euclidean “Magic”

Theorem: There are infinitely many primes.

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Intro to (Formal) Logic @ RPI

350 BC

Euclid
Intro to (Formal) Logic @ RPI

350 BC
300 BC
2019

Euclid
I don’t believe in magic! Why exactly is that so convincing? What exactly is he doing?!

Euclid

Intro to (Formal) Logic @ RPI
I don’t believe in magic! Why exactly is that so convincing? What exactly is he doing?!!

Intro to (Formal) Logic @ RPI
He’s using syllogisms!

E.g.,

All As are Bs.
All Bs are Cs.

All As are Cs.

I don’t believe in magic! Why exactly is that so convincing? What exactly is he doing?!!

Intro to (Formal) Logic @ RPI
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E.g.,

All As are Bs.
All Bs are Cs.

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Balderdash!
I don’t believe in magic! Why exactly is that so convincing? What exactly is he doing?!?

Balderdash!
Intro to (Formal) Logic @ RPI

350 BC

Euclid

Organon
Euclid 350 BC

Organon 300 BC

1666

Intro to (Formal) Logic @ RPI
Intro to (Formal) Logic @ RPI
1666

Leibniz

“Universal Computational Logic”

350 BC

Euclid

300 BC

300 BC

Organon

1666

Leibniz

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“Universal Computational Logic”

350 BC  300 BC  1666  1854

Euclid  Organon  Leibniz

Intro to (Formal) Logic @ RPI
“Universal Computational Logic”

350 BC
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1666
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1854

Intro to (Formal) Logic @ RPI
1666
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1854
Logic Theorist
(birth of modern logicist AI)

1854
Intro to (Formal) Logic @ RPI

1956
Simon

1956
Logic Theorist
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“Astonishing” Logic Theorist Proof @ Dawn of AI
“Astonishing” Logic Theorist
Proof @ Dawn of AI

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<table>
<thead>
<tr>
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<tbody>
<tr>
<td>1</td>
<td>((\phi \lor \phi) \rightarrow \phi)</td>
<td>axiom</td>
</tr>
<tr>
<td>2</td>
<td>((\neg \phi \lor \neg \phi) \rightarrow \neg \phi)</td>
<td>substitution</td>
</tr>
<tr>
<td>3</td>
<td>((\phi \rightarrow \neg \phi) \rightarrow \neg \phi)</td>
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<tr>
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“Astonishing” Logic Theorist
Proof @ Dawn of AI

1. \((\phi \lor \phi) \rightarrow \phi\)  \hspace{1cm} \text{axiom}
2. \((\neg \phi \lor \neg \phi) \rightarrow \neg \phi\)  \hspace{1cm} \text{substitution}
3. \((\phi \rightarrow \neg \phi) \rightarrow \neg \phi\)  \hspace{1cm} \text{a “replacement rule”}
4. \((A \rightarrow \neg A) \rightarrow \neg A\)  \hspace{1cm} \text{substitution}

At dawn of AI: 10 seconds.
“Astonishing” Logic Theorist
Proof @ Dawn of AI

1. \((\phi \lor \phi) \rightarrow \phi\)  
   | axiom
2. \((\neg \phi \lor \neg \phi) \rightarrow \neg \phi\)  
   | substitution
3. \((\phi \rightarrow \neg \phi) \rightarrow \neg \phi\)  
   | a “replacement rule”
4. \((A \rightarrow \neg A) \rightarrow \neg A\)  
   | substitution

At dawn of AI: 10 seconds.

AI of today: vanishingly small amount of time.
“Universal Computational Logic”

350 BC
Euclid

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“Universal Computational Logic”

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Intro to (Formal) Logic @ RPI

2019

2020
350 BC  300 BC  1666  1854  1956  2019  2020

Euclid  Organon  Leibniz

"Universal Computational Logic"

Logic Theorist
(birth of modern logicist AI)

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Intro to (Formal) Logic @ RPI

The Singularity?
Entscheidungsproblem

Logic Theorist
(birth of modern logicist AI)

350 BC 300 BC 1666 1854 1956 2019 2020

Euclid Organon Leibniz Simon

1956 Logic Theorist

Intro to Logic @ RPI

The Singularity?

"Universal Computational Logic"
Entscheidungsproblem

“Universal Computational Logic”

350 BC 300 BC 1666 1854 1956 2019 2020

Euclid

Organon

Leibniz

An Investigation of the Laws of Thought

Logic Theorist
(birth of modern logicist AI)

Simon

Intro to Logic @ RPI
Exceeds Leibniz & de-mystifies Euclid: the “compellingness” of these proofs consists in their being, at bottom, formal proofs in first-order logic (FOL).
Entscheidungsproblem

"Universal Computational Logic"

350 BC 300 BC 1666 1854 1956 2019 2020

Euclid

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Leibniz

Simon

Logic Theorist (birth of modern logicist AI)

Intro to Logic @ RPI

Frege

Church

The Singularity?
Exceeds Leibniz & de-mystifies
Euclid: the “compellingness” of these proofs consists in their being, at bottom, formal proofs in first-order logic (FOL).
Exceeds Leibniz & de-mystifies Euclid: the “compellingness” of these proofs consists in their being, at bottom, formal proofs in first-order logic (FOL).
Here’s what a computer is, and given that, sorry, the *Entscheidungsproblem* can’t be solved by such a machine!
First, the Theoremhood Decision Problem (THEOREM_{PC}) for the Propositional Calculus
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\[(K \land A) \rightarrow A\]
First, the Theoremhood Decision Problem (THEOREM$_{PC}$) for the Propositional Calculus

\[(K \land A) \rightarrow A\]

input \hspace{1cm} output
First, the Theoremhood Decision Problem (THEOREM_{PC}) for the Propositional Calculus

\[(K \land A) \rightarrow A\]
First, the Theoremhood Decision Problem \((\text{THEOREM}_{PC})\) for the Propositional Calculus

\[(K \land A) \rightarrow A\]

**input** \(\rightarrow\) **output**

Yes, proof

Hard!! — for apparently no polynomial-time algorithm for this!
First, the Theoremhood Decision Problem (THEOREM\textsubscript{PC}) for the Propositional Calculus

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\[(K \rightarrow A) \land \neg A \rightarrow \neg K\]

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First, the Theoremhood Decision Problem (THEOREM\(_{\text{PC}}\)) for the Propositional Calculus

\[(\neg K \rightarrow (A \rightarrow \neg A)) \rightarrow \neg K\]

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input \[\rightarrow\] Yes, proof output

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And now, the Theoremhood Decision Problem, i.e., the *Entscheidungsproblem*, (THEOREM\textsubscript{FOL}) for First-Order Logic (FOL)
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**Not just hard: impossible** for a (and this needed to be *invented* in the course of clarifying and solving the problem) standard computing machine.
Applying this to …

The Singularity Question
Applying this to …

The Singularity Question

A:
Premise 1  There will be AI (created by HI and such that $AI = HI$).
Premise 2  If there is AI, there will be $AI^+$ (created by AI).
Premise 3  If there is $AI^+$, there will be $AI^{++}$ (created by $AI^+$).

\[ \therefore S \]
There will be $AI^{++}$ ($= S$ will occur).

(Good-Chalmers Argument)

(Kurzweil is an “extrapolationist.”)
Applying this to …

The Singularity Question

So, these super-smart machines that will be built by human-level-smart machines, they can’t possibly be smart enough to solve the *Entscheidungsproblem*. Hence they’ll be just faster at solving problems we can routinely solve? What’s so super-smart about *that*?