Propositional Calculus II:
More Rules of Inference, Application to Additional Motivating Problems

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Troy, New York 12180 USA

Intro to Logic
2/5/2018
Logistics ...
Any takers?

Logistics ...
Logistics ...
Any takers? NO

Logistics ...
Any takers? NO

Logistics ...

Note: Should now have laptop with you and ready to go with Slate installed.
Any takers?  NO

Logistics . . .

Note: Should now have laptop with you and ready to go with Slate installed.
Any takers? NO

Logistics ...

Note: Should now have laptop with you and ready to go with Slate installed.

And … HyperGrader will debut in class on Feb 12, led by Rini.
Any takers? NO

Logistics . . .

Note: Should now have laptop with you and ready to go with Slate installed.

And . . . HyperGrader will debut in class on Feb 12, led by Rini.

http://www.logicamodernapproach.com
Any takers?  NO

Logistics ...

Note: Should now have laptop with you and ready to go with Slate installed.

And ... HyperGrader will debut in class on Feb 12, led by Rini.

http://www.logicamodernapproach.com

Plenty of problems there to learn/practice on!
Your code for Slate & HyperGrader for the semester:
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18450
Save sleeve & CD, snapshot sleeve & archive!!
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What’s on the CD?

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Windows
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- **textbook**
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Textbook: LAMA-BDLA011818.pdf

Windows: slate_20160105.exe.zip, Slate_20160125.app.zip

Mac OS: soft_lnc_agree_011818.pdf
What’s on the CD?

- **Mac OS**
  - textbook

- **Windows**
  - Mac OS

Complete, sign, email pdf to Selmer.Bringsjord@gmail.com
Again: Initial Steps
Again: Initial Steps

- Snapshot sleeve with code, and archive.
Again: Initial Steps

• Snapshot sleeve with code, and archive.
• Copy the folder from CD to your laptop.
Again: Initial Steps

- Snapshot sleeve with code, and archive.
- Copy the folder from CD to your laptop.
- Eject CD and “bank-vault”-save both!
Again: Initial Steps

• Snapshot sleeve with code, and archive.
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• Depending upon whether you’re Windows or MacOS, expand the relevant zipped file to obtain Slate.
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• Snapshot sleeve with code, and archive.
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• Depending upon whether you’re Windows or MacOS, expand the relevant zipped file to obtain Slate.
• Open Slate
Again: Initial Steps

• Snapshot sleeve with code, and archive.
• Copy the folder from CD to your laptop.
• Eject CD and “bank-vault”-save both!
• Depending upon whether you’re Windows or MacOS, expand the relevant zipped file to obtain Slate.
• Open Slate
• Today I’ll continue to explain and show inference rules in Slate.
Propositional Calculus II: More Rules of Inference, Application to Additional Motivating Problems

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Intro to Logic
2/5/2018
Where we are in the schedule ...
Last time we introduced and and lauded the power of oracles, and now … picking up where we left off …
“NYS 3” Revisited

Given the statements

\[ \neg \neg c \]
\[ c \rightarrow a \]
\[ \neg a \lor b \]
\[ b \rightarrow d \]
\[ \neg (d \lor e) \]

which one of the following statements must also be true?

\[ \neg c \]
\[ e \]
\[ h \]
\[ \neg a \]

all of the above
“NYS 3” Revisited

Given the statements
\[
\neg\neg c \\
c \rightarrow a \\
\neg a \lor b \\
b \rightarrow d \\
\neg(d \lor e)
\]

which one of the following statements must also be true?

\[
\neg c \\
e \\
h \\
\neg a \\
all of the above
\]
“NYS 3” Revisited

Given the statements

\( \neg \neg c \)
\( c \to a \)
\( \neg a \lor b \)
\( b \to d \)
\( \neg (d \lor e) \)

which one of the following statements must also be true?

\( \neg c \)
\( e \)
\( h \)
\( \neg a \)
all of the above

After last class, should have done …

Exercise: Show in Slate that each of the first four options can be proved using the PC entailment oracle.
(and (not C) E H (not A))
(and (not C) E H (not A))
(and (not C) E H (not A))
(and (not C) E H (not A))
(and (not C) E H (not A))
(and (not C) E H (not A))
\((\text{and } \neg C) \land E \land H \land \neg A)\)
Proof Plan …
Proof Plan …
Proof Plan ...
Proof Plan ...

Premise1. \( \neg \neg C \)
    \{Premise1\} Assume \( \checkmark \)

1. C
    \{Premise1\}

Premise2. C \( \rightarrow \) A
    \{Premise2\} Assume \( \checkmark \)

Premise3. \( \neg A \lor B \)
    \{Premise3\} Assume \( \checkmark \)

2. A
    \{Premise1,Premise2\}

Premise4. B \( \rightarrow \) D
    \{Premise4\} Assume \( \checkmark \)

3. B
    \{Premise1,Premise2,Premise3\}

Premise5. \( \neg (D \lor E) \)
    \{Premise5\} Assume \( \checkmark \)

5. \( \neg D \land \neg E \)
    \{Premise5\}

6. \( \neg D \)
    \{Premise5\}

Option1. \( \neg C \)
    \{Premise1\} \( PC \leftarrow \times \)

Option2. E
    \{Premise1\} \( PC \leftarrow \times \)

Option3. H
    \{Premise1\} \( PC \leftarrow \times \)

Option4. \( \neg A \)
    \{Premise1\} \( PC \leftarrow \times \)

Option5. \( \neg C \land E \land H \land \neg A \)
    \{Premise1,Premise2,Premise3,Premise4,Premise5\}

PC \( \leftarrow \checkmark \)
Proof Plan …

Premise 1. \( \neg C \)  
\{Premise 1\} Assume \( \checkmark \)

PC \( \leftarrow \checkmark \)

1. C  
\{Premise 1\}

PC \( \leftarrow \checkmark \)

2. A  
\{Premise 1, Premise 2\}

PC \( \leftarrow \checkmark \)

3. B  
\{Premise 1, Premise 2, Premise 3\}

PC \( \leftarrow \checkmark \)

4. D  
\{Premise 1, Premise 2, Premise 3, Premise 4\}

PC \( \leftarrow \checkmark \)

Option 5. \( \neg C \wedge E \wedge H \wedge \neg A \)  
\{Premise 1, Premise 2, Premise 3, Premise 4, Premise 5\}

Sub-Proof

Premise 5. \( \neg (D \vee E) \)  
\{Premise 5\} Assume \( \checkmark \)

PC \( \leftarrow \checkmark \)

5. \( \neg D \wedge \neg E \)  
\{Premise 5\}

PC \( \leftarrow \checkmark \)

6. \( \neg D \)  
\{Premise 5\}

Option 1. \( \neg C \)  
PC \( \leftarrow \times \)

Option 2. \( E \)  
PC \( \leftarrow \times \)

Option 3. \( H \)  
PC \( \leftarrow \times \)

Option 4. \( \neg A \)  
PC \( \leftarrow \times \)
Proof Plan ...

New Inference Rule:

Premise1. \( \neg C \)
{Premise1) Assume \( \checkmark \)

Premise2. \( C \rightarrow A \)
{Premise2) Assume \( \checkmark \)

Premise3. \( \neg A \lor B \)
{Premise3) Assume \( \checkmark \)

Premise4. \( B \rightarrow D \)
{Premise4) Assume \( \checkmark \)

Premise5. \( \neg (D \lor E) \)
{Premise5) Assume \( \checkmark \)

Option1. \( \neg C \)
PC \( \leftarrow X \)

Option2. \( E \)
PC \( \leftarrow X \)

Option3. \( H \)
PC \( \leftarrow X \)

Option4. \( \neg A \)
PC \( \leftarrow X \)

Option5. \( \neg C \land E \land H \land \neg A \)
{Premise1,Premise2,Premise3,Premise4,Premise5}
Proof Plan ...

New Inference Rule:

Sub-Proof

Premise5. \(\neg(D \lor E)\)
   (Premise5) Assume \(\checkmark\)

5. \(\neg D \land \neg E\)
   (Premise5) Assume \(\checkmark\)

6. \(\neg D\)
   (Premise5) Assume \(\checkmark\)

Option1. \(\neg C\)
   PC \(\leftarrow \times\)

Option2. E
   PC \(\leftarrow \times\)

Option3. H
   PC \(\leftarrow \times\)

Option4. \(\neg A\)
   PC \(\leftarrow \times\)
Proof Plan …

New Inference Rule: conditional elim a.k.a., modus ponens
Proof Plan …

New Inference Rule: conditional elim
a.k.a., modus ponens
The Original King-Ace

Suppose that the following premise is true:

If there is a king in the hand, then there is an ace in the hand, or else if there isn’t a king in the hand, then there is an ace.

What can you infer from this premise?
The Original King-Ace

Suppose that the following premise is true:

If there is a king in the hand, then there is an ace in the hand, or else if there isn’t a king in the hand, then there is an ace.

What can you infer from this premise?

There is an ace in the hand.
The Original King-Ace

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What can you infer from this premise?

There is an ace in the hand.
The Original King-Ace

Suppose that the following premise is true:

If there is a king in the hand, then there is an ace in the hand, or else if there isn’t a king in the hand, then there is an ace.

What can you infer from this premise?

NO! There is an ace in the hand.
The Original King-Ace

Suppose that the following premise is true:

If there is a king in the hand, then there is an ace in the hand, or else if there isn’t a king in the hand, then there is an ace.

What can you infer from this premise?

NO! There is an ace in the hand. NO!
The Original King-Ace

Suppose that the following premise is true:

If there is a king in the hand, then there is an ace in the hand, or else if there isn’t a king in the hand, then there is an ace.

What can you infer from this premise?

NO! There is an ace in the hand. NO!

In fact, what you can infer is that there isn’t an ace in the hand!
King-Ace Solved

**Proposition:** There is *not* an ace in the hand.

**Proof:** We know that at least one of the if-thens (i.e., at least one of the **conditionals**) is false. So we have two cases to consider, viz., that $K \Rightarrow A$ is false, and that $\neg K \Rightarrow A$ is false. Take first the first case; accordingly, suppose that $K \Rightarrow A$ is false. Then it follows that $K$ is true (since when a conditional is false, its antecedent holds but its consequent doesn’t), and $A$ is false. Now consider the second case, which consists in $\neg K \Rightarrow A$ being false. Here, in a direct parallel, we know $\neg K$ and, once again, $\neg A$. In both of our two cases, which are exhaustive, there is no ace in the hand. The proposition is established. **QED**
Suppose that the following premise is true:

*If there is a king in the hand, then there is an ace in the hand; or if there isn’t a king in the hand, then there is an ace; but not both of these if-then statements are true.*

What can you infer from this premise?
Suppose that the following premise is true:

*If there is a king in the hand, then there is an ace in the hand; or if there isn’t a king in the hand, then there is an ace; but not both of these if-then statements are true.*

What can you infer from this premise?

There is an ace in the hand.
King-Ace 2

Suppose that the following premise is true:

*If there is a king in the hand, then there is an ace in the hand; or if there isn’t a king in the hand, then there is an ace; but not both of these if-then statements are true.*

What can you infer from this premise?

There is an ace in the hand.
King-Ace 2

Suppose that the following premise is true:

*If there is a king in the hand, then there is an ace in the hand; or if there isn’t a king in the hand, then there is an ace; but not both of these if-then statements are true.*

What can you infer from this premise?

NO! There is an ace in the hand.
King-Ace 2

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Study the S-expression
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(\text{Formula:})
(\text{and} \ (\text{or} \ (\text{if} \ K \ A) \ (\text{if} \ (\text{not} \ K) \ A)) \ (\text{not} \ (\text{and} \ (\text{if} \ K \ A) \ (\text{if} \ (\text{not} \ K) \ A))))

(\text{Justification:})
(\text{Name (optional):})
Assume Premise

(\text{Premise.})
((K \rightarrow A) \lor (\neg K \rightarrow A)) \land \neg((K \rightarrow A) \land (\neg K \rightarrow A)) \{\text{Premise} \} \ Assume \checkmark

(\text{Reality.})
\neg A \{\text{Premise}\}

(\text{Illusion.})
A \{\text{Premise}\}
We need another rule of inference to crack this problem … …
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disjunction elimination
from each \( \phi_i \), then we may conclude \( \psi \). That is, if we can, for each \( \phi_i \), assume \( \phi_i \) and show that \( \psi \) follows, then we may conclude \( \psi \) from the disjunction \( \phi_1 \lor \ldots \lor \phi_n \) and the derivations of \( \psi \). There is one more subtle point, however. In the days-of-the-week example above, the conclusion that Susan has class on a weekday should not be in the scope of both the assumptions that she has class on Monday and that she has class on Tuesday; these assumptions are discharged. Disjunction elimination discharges each assumption \( \phi_i \) from the line of reasoning that corresponds to that case.

\[
\begin{align*}
\phi_1 \lor \ldots \lor \phi_n & \quad \psi \{\phi_1\} \in \Gamma_1 & \quad \ldots & \quad \psi \{\phi_n\} \in \Gamma_n \\
\Gamma_0 & \quad \lor \text{elim} & \quad \vdash \psi & \quad \Gamma_0 \cup \ldots \cup \Gamma_n
\end{align*}
\]

(2.25)

The various \( \Gamma_i \) on the premises of disjunction elimination might make this rule seem more complicated than it really is. Their presence makes it clear that the only assumptions discharged from each line of reasoning is the assumption corresponding to that particular case.
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\begin{equation}
\phi_1 \lor \ldots \lor \phi_n \\
\Gamma_0
\end{equation}

\begin{equation}
\psi \\
\{\phi_i\} \cup \Gamma_1
\end{equation}

\begin{equation}
\vdots
\end{equation}

\begin{equation}
\psi \\
\{\phi_n\} \cup \Gamma_n
\end{equation}

\begin{equation}
\lor \text{elim} \\
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\end{equation}

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\begin{align*}
\phi_1 \lor \ldots \lor \phi_n & \quad \text{\( \Gamma_0 \)} \\
\psi & \quad \text{\( \{\phi_1\} \cup \Gamma_1 \)} \\
\ldots & \\
\psi & \quad \text{\( \{\phi_n\} \cup \Gamma_n \)} \\
\lor \text{\( \text{ elim } \)} & \\
\psi & \quad \text{\( \Gamma_0 \cup \ldots \cup \Gamma_n \)}
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\[
\begin{array}{ccc}
\phi_1 \lor \ldots \lor \phi_n & \Rightarrow & \psi \\
\Gamma_0 & \Rightarrow & \{\phi_1\} \cup \Gamma_1 \\
& \Rightarrow & \{\phi_n\} \cup \Gamma_n \\
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\psi & \\
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\psi & \\
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\vee \text{elim} & \\
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