Propositional Calculus I: The Formal Language, Rules of Inference (initial), Application to Some Motivating Problems

Selmer Bringsjord

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Department of Computer Science
Lally School of Management & Technology
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Troy, New York 12180 USA

Intro to Logic
2/1/2018
Logistics ...
Logistics ...
Logistics ... 

**Note:** Should now have laptop with you and ready to go with Slate installed — but if not, then certainly on Monday Feb 5!
Logistics ...

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**And** ... HyperGrader will debut in class on Feb 12, led by Rini.
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http://www.logicamodernapproach.com
Logistics ...

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http://www.logicamodernapproach.com

Cannot use without valid, registered code!
Re the CD
Re the CD
Re the CD
Re the CD

Once seal broken, no return. Remember from first class, can opt for “Stanford” paradigm, with its software instead of LAMA paradigm!
Your code for Slate & HyperGrader for the semester:
Your code for Slate & HyperGrader for the semester:
Save sleeve & CD, snapshot sleeve & archive!!
Save sleeve & CD, snapshot sleeve & archive!!
What’s on the CD?

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<th>Size</th>
<th>Kind</th>
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What’s on the CD?

Mac OS
What’s on the CD?

Windows

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What’s on the CD?

- Mac OS: `soft_lic_agree_011818.pdf`
- Windows: `textbook`
What's on the CD?

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Complete, sign, email pdf to Selmer.Bringsjord@gmail.com
Initial Steps
Initial Steps

• Snapshot sleeve with code, and archive.
Initial Steps

- Snapshot sleeve with code, and archive.
- Copy the folder from CD to your laptop.
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• Eject CD and “bank-vault”-save both!
• Depending upon whether you’re Windows or MacOS, expand the relevant zipped file to obtain Slate.
• Open Slate
• & today I’ll explain and show some simple moves in Slate (though of course I’ve already shown some moves in class).
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Intro to Logic
2/1/2018
Note:

skipping to ~ p. 34!
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M. Chi: Self-testers end up being self-made.
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skipping to ~ p. 34!

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M. Chi: Self-testers end up being self-made.

“What category of English sentences does logic focus on?”
# CHAPTER 2. PROPOSITIONAL CALCULUS

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<thead>
<tr>
<th>Syntax</th>
<th>Formula Type</th>
<th>Sample Representation</th>
</tr>
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<tbody>
<tr>
<td>$P, P_1, P_2, Q, Q_1, \ldots$</td>
<td>Atomic Formulas</td>
<td>“Larry is lucky.” as $L_l$</td>
</tr>
<tr>
<td>$\neg \phi$</td>
<td>Negation</td>
<td>“Gary isn’t lucky.” as $\neg L_g$</td>
</tr>
<tr>
<td>$\phi_1 \land \ldots \land \phi_n$</td>
<td>Conjunction</td>
<td>“Both Larry and Carl are lucky.” as $L_l \land L_c$</td>
</tr>
<tr>
<td>$\phi_1 \lor \ldots \lor \phi_n$</td>
<td>Disjunction</td>
<td>“Either Billy is lucky or Alvin is.” as $L_b \lor L_a$</td>
</tr>
<tr>
<td>$\phi \rightarrow \psi$</td>
<td>Conditional (Implication)</td>
<td>“If Ron is lucky, so is Frank.” as $L_r \rightarrow L_f$</td>
</tr>
<tr>
<td>$\phi \leftrightarrow \psi$</td>
<td>Biconditional (Equivalence)</td>
<td>“Tim is lucky if and only if Kim is.” as $L_t \leftrightarrow L_k$</td>
</tr>
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Table 2.1: Syntax of the Propositional Calculus. Note that $\phi$, $\psi$, and $\phi_i$ stand for arbitrary formulas.
The Formal Language

CHAPTER 2. PROPOSITIONAL CALCULUS

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Table 2.1: Syntax of the Propositional Calculus. Note that \( \phi, \psi, \) and \( \phi_i \) stand for arbitrary formulas.

Exercise: Is this language Roger-decidable? Prove it!
Given the statements

\neg a \lor \neg b
b
c \rightarrow a

which one of the following statements must also be true?

c
\neg b
\neg c
h
a
none of the above
“NYS I” Revisited

Given the statements

¬a ∨ ¬b
b
c → a

which one of the following statements must also be true?

c
¬b
¬c
h
a
none of the above
Our First Rule of Inference: PC (Entailment) Oracle
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Our First Rule of Inference: PC (Entailment) Oracle
The Rules of Inference: PC (Consequence) Oracle

Premise 1. \( \neg A \lor \neg B \)
   (Premise 1) Assume \( \checkmark \)

Premise 2. \( B \)
   (Premise 2) Assume \( \checkmark \)

Premise 3. \( C \rightarrow A \)
   (Premise 3) Assume \( \checkmark \)

Intermediary Conclusion. \( \neg A \)
   PC \( \not\vdash x \)

Option 1. \( C \)
   (Premise 1, Premise 2, Premise 3)

Counterexample

The following truth assignment makes all of the premises true and the conclusion false.

- Premises
  - \( \neg A \lor \neg B \)
  - \( B \)
  - \( C \rightarrow A \)

- True Literals
  - \( B \)

- False Literals
  - \( A \)
  - \( C \)

Conclusion

- Option 2. \( \neg B \)
  PC \( \not\vdash x \)

- Option 4. \( H \)
  PC \( \not\vdash x \)

- Option 5. \( A \)
  PC \( \not\vdash x \)

Conclusion (Option 3). \( \neg C \)
   PC \( \not\vdash x \)
The Rules of Inference:
PC (Consequence) Oracle

Premise 1. \( \neg A \lor \neg B \) (Premise 1) Assume
Premise 2. \( B \) (Premise 2) Assume
Premise 3. \( C \rightarrow A \) (Premise 3) Assume

Intermediate Conclusion. \( \neg A \)
\( PC \vdash x \)

Option 1. \( C \) (Premise 1, Premise 2, Premise 3)

Counterexample
The following truth assignment makes all of the premises true and the conclusion false.

Premises
\( \neg A \lor \neg B \)
\( B \)
\( C \rightarrow A \)
Conclusion
\( C \)

True Literals
\( B \)
False Literals
\( A \)

Conclusion (Option 3). \( \neg C \)
\( PC \vdash x \)

Option 2. \( \neg B \)
\( PC \vdash x \)

Option 4. \( H \)
\( PC \vdash x \)

Option 5. \( A \)
\( PC \vdash x \)
The Problem in Slate

Premise 1. \( \neg A \lor \neg B \)
Premise 2. \( B \)
Premise 3. \( C \rightarrow A \)

Conclusion (Option 3). \( \neg C \)

Option 1. \( C \)
\( PC = x \)

Option 2. \( \neg B \)
\( PC = x \)

Option 4. \( H \)
\( PC = x \)

Option 5. \( A \)
\( PC = x \)
The Problem in Slate

Premise 1. \( \neg A \lor \neg B \)
(Premise 1) Assume

Premise 2. \( B \)
(Premise 2) Assume

Premise 3. \( C \rightarrow A \)
(Premise 3) Assume

Conclusion (Option 3). \( \neg C \)
\( PC \models X \)

Option 1. \( C \)
\( PC \models X \)

Option 2. \( \neg B \)
\( PC \models X \)

Option 4. \( H \)
\( PC \models X \)

Option 5. \( A \)
\( PC \models X \)
“NYS 3” Revisited

Given the statements
\[-\neg c\]
c \implies a
\[-a \vee b\]
b \implies d
\[-(d \vee e)\]

which one of the following statements must also be true?

\[-c\]  
\[e\]  
\[h\]  
\[-a\]  
all of the above
“NYS 3” Revisited

Given the statements
\( \neg \neg c \)
\( c \rightarrow a \)
\( \neg a \lor b \)
\( b \rightarrow d \)
\( \neg(d \lor e) \)

which one of the following statements must also be true?

\( \neg c \)
e
h
\( \neg a \)
all of the above
Given the statements
\neg \neg c
\neg a \lor b
b \rightarrow d
\neg (d \lor e)

which one of the following statements must also be true?

\neg c
e
h
\neg a
all of the above

Exercise: Show in Slate that each of the first four options can be proved using the PC entailment oracle.