

Pure Predicate Calculus; Quantification

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Intro to Logic
2/15/2018



Any questions re HyperGrader?

What is Logic?

- The key to becoming rational.
- “The science of reasoning.” — so the not-unreasonable slogan goes.
- The only invincible subject there is.
- The basis for the formal sciences (from mathematics to game theory to decision theory to probability calculi to axiomatic physics) — and hence the basis for disciplines based on the formal sciences (e.g., engineering, computer science).
- The way of escape from shallow content and context to pure, immaterial, and immortal form and structure (which is why the exotic, imaginary, and seemingly non-sensical is so pedagogically useful).
- The most challenging subject there is.
- One of the chief differentiators between dogs and monkeys versus you (let alone bears and you); and mindless machines (like Deep Blue & Watson) versus you.
- A key to riches.
- The key to divining the meaning of life (and other such big questions).
- The better way to program computers; and fundamentally the *only* way to *reliably* program computers.
- One of two fundamental approaches to studying minds, and replicating/simulating minds in machines...
- The thing many creatures of fiction have mastered — have you (as a New Yorker)?
...

What is Logic?

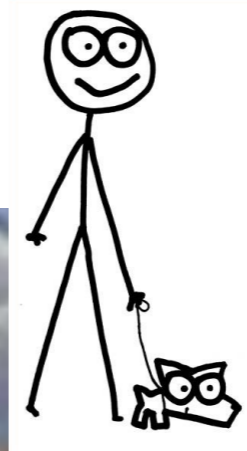
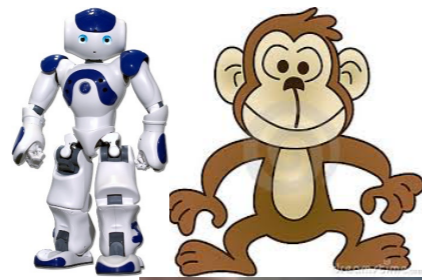
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Watch brainy zoo animals figure out a box puzzle to get at food

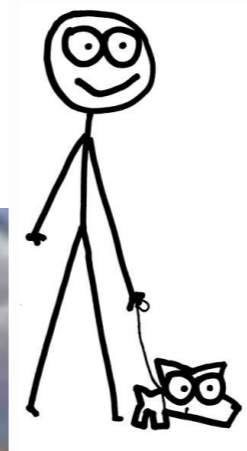
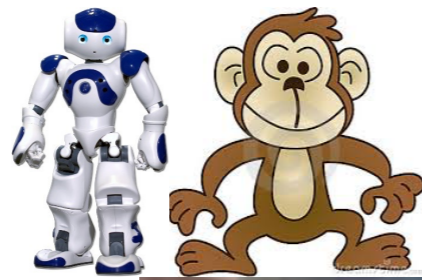


[\(https://www.newscientist.com/article/2075151-watch-brainy-zoo-animals-figure-out-a-box-puzzle-to-get-at-food/\)](https://www.newscientist.com/article/2075151-watch-brainy-zoo-animals-figure-out-a-box-puzzle-to-get-at-food/)

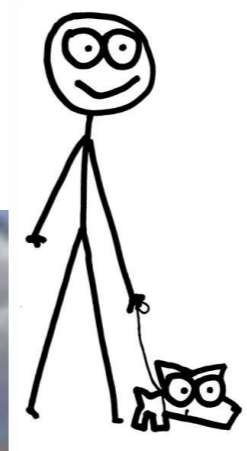
The Canyon of Discontinuity (or Darwin's Dread)



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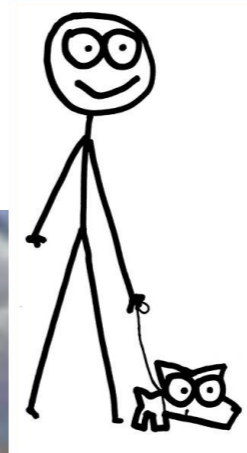
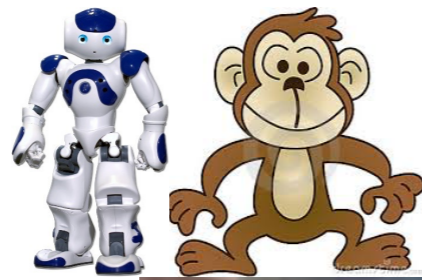


The Canyon of Discontinuity (or Darwin's Dread)



Relations and Functions!

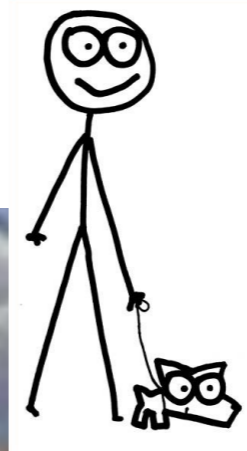
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Relations and Functions!

Quantification!

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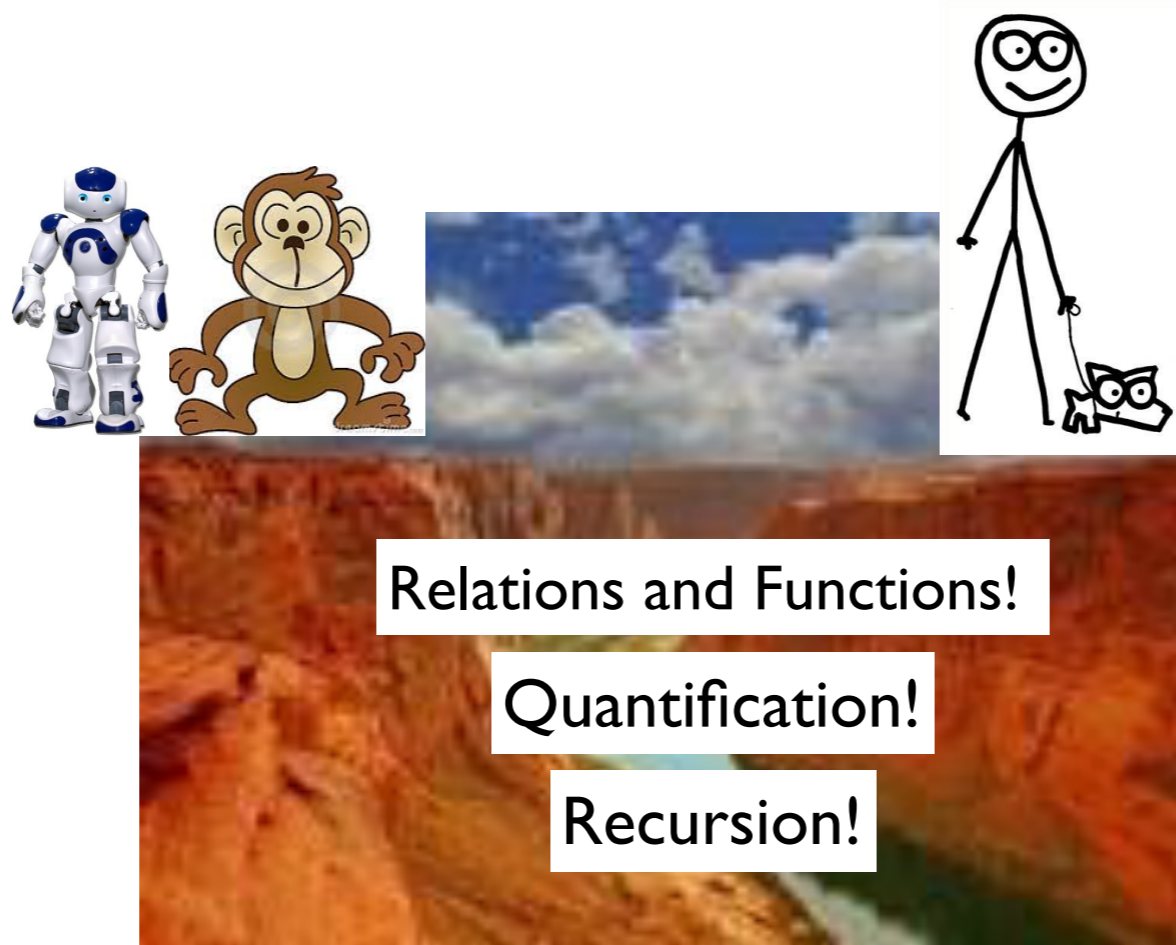


Relations and Functions!

Quantification!

Recursion!

The Canyon of Discontinuity (or Darwin's Dread)



(Interesting paper:

http://idiom.ucsd.edu/~ivano/SemBabble_old/LogicSeminar_15W/Material/Partee_2013_History-of-Quantifiers.pdf.)

Karkooking Problem ...

Everyone karkooks anyone who karkooks someone.

Alvin karkooks Bill.

Can you infer that everyone karkooks Bill?

ANSWER:

JUSTIFICATION:

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Everyone karkooks anyone who karkooks someone.

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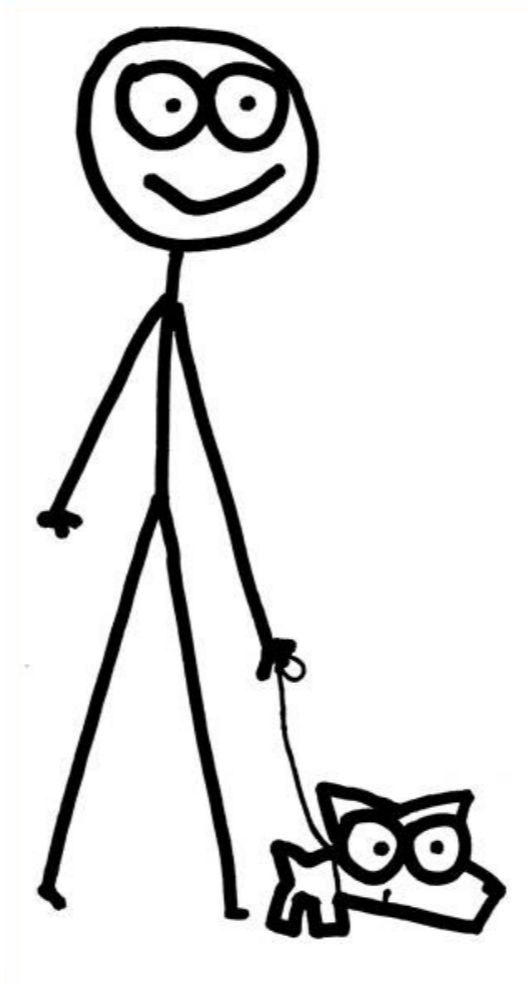
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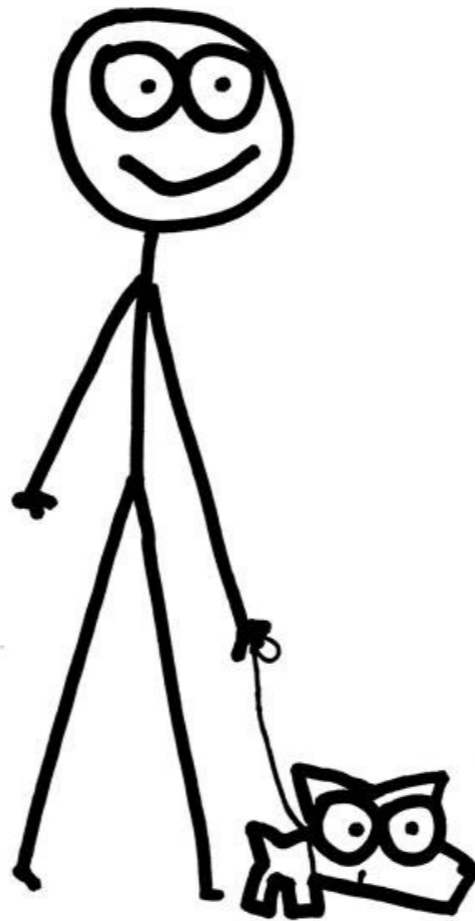
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Recursion!

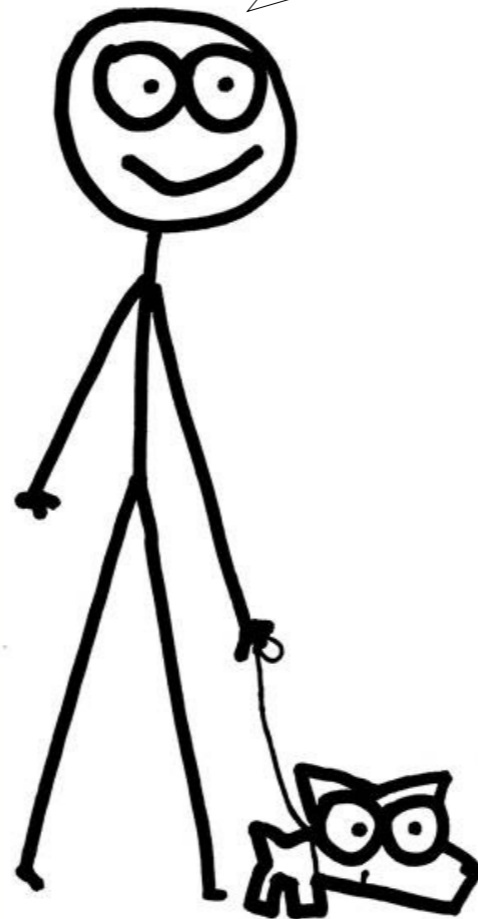
ANSWER:

JUSTIFICATION:





Does everyone karkook Bill?



Yup! Want me to prove it?

Does everyone karkook Bill?

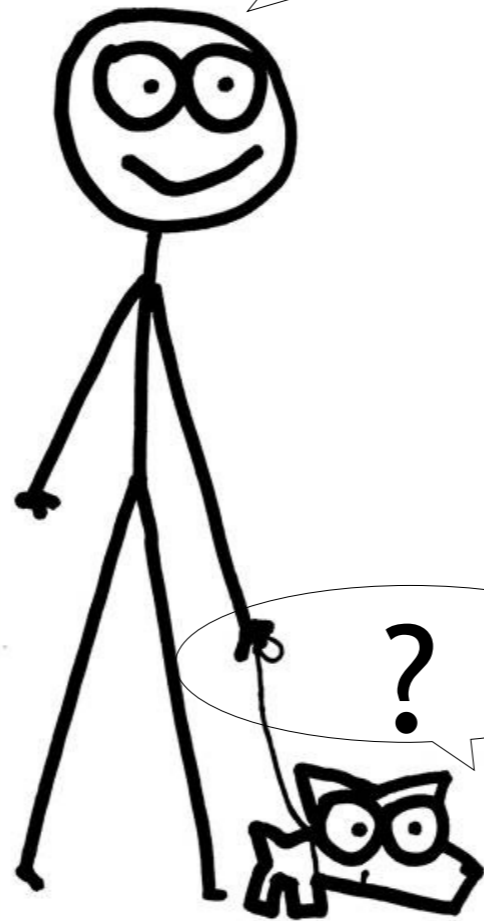
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?



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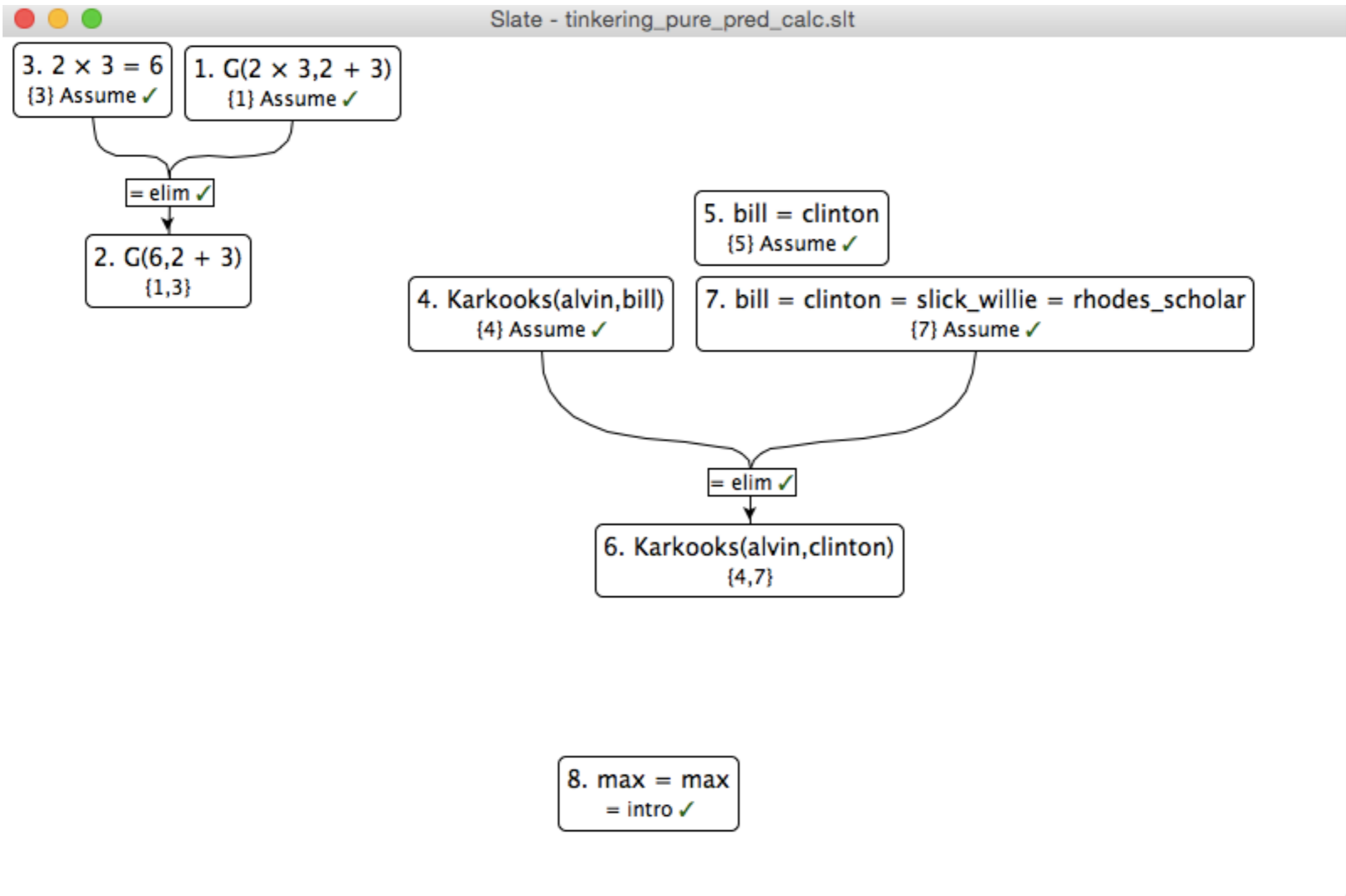
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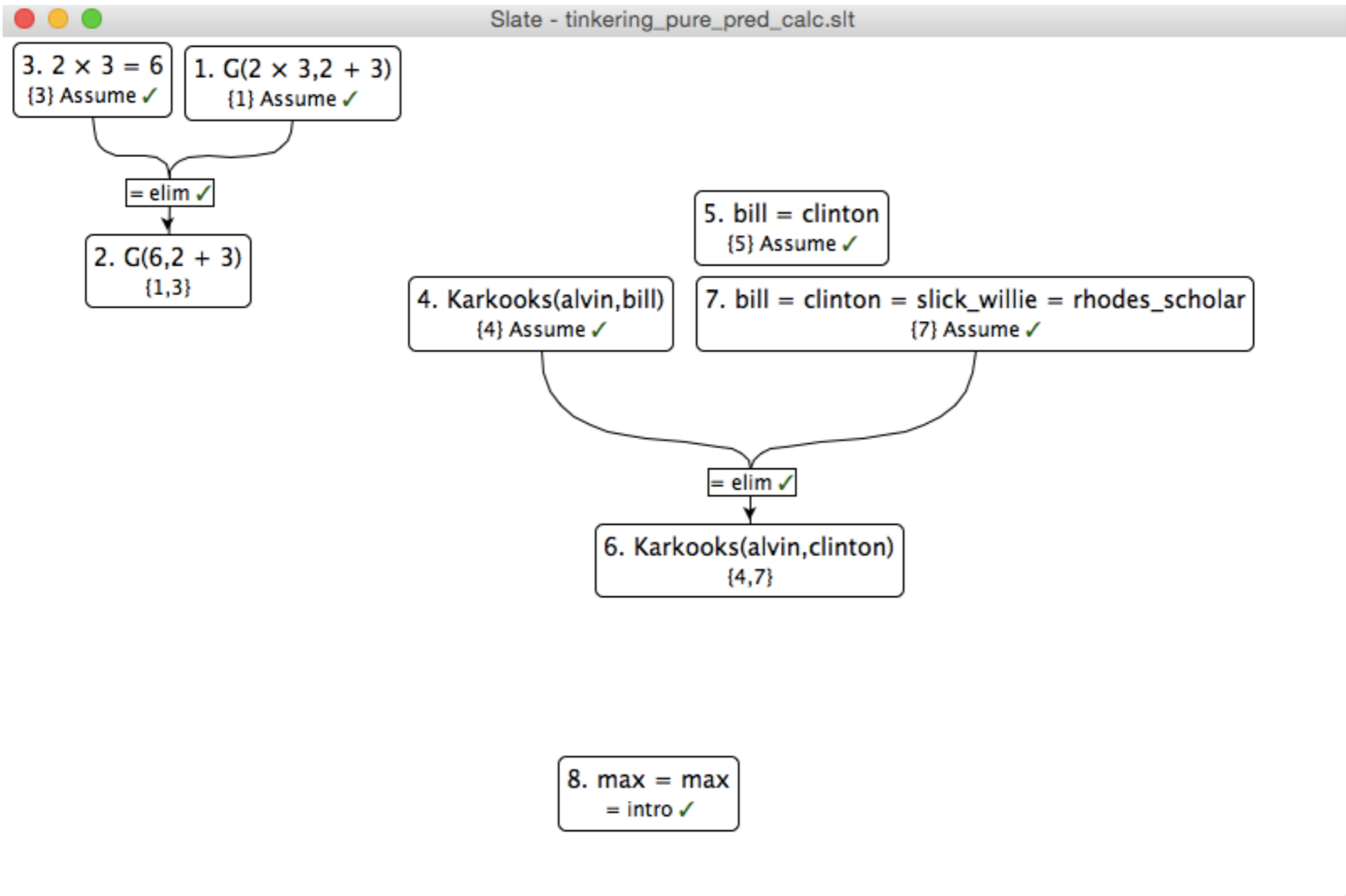
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Recursion!

ANSWER:

JUSTIFICATION:





Larry

Larry attended Hotchkiss before entering Harvard, from which he graduated with a degree in Scandinavian Studies. He aspires to be a Diplomat representing the United States to Sweden. Larry is from a rather wealthy family: his trust fund is valued at \$7 billion; his father collects exotic islands, his mother precursor-to-Impressionism masterpieces, and his three bachelor brothers, fast, classic European luxury sedans capable of heart-pounding top speeds. Larry's command of math never exceeded what is covered on the SAT, and he has long forgotten even this material. He does not understand what a computer or computer program is, but nonetheless makes continuous use of social networking technology, including specifically **facebook**, on which he is liked by four people, all in his nuclear family, save for one, and that one is a brilliant female with a penchant for driving fast European sedans *really* fast. Say what you will about Larry, he is arrestingly eloquent without notes of any kind when speaking about geopolitics, in any venue; knows perhaps more about the history of Northern Europe (including its mythology) than any man alive; and while in keeping with his upbringing is a Hayekian capitalist, is rumored to generously donate millions each year to Big Brothers Big Sisters, AA, and Samaritan's Purse.



Lucy

Lucy is a brilliant but poor hacker from a broken, impoverished home in Buffalo NY. A motherless only child raised by a single, devoted father who made ends meet (between binges on the bottle) as a brilliant but itinerant Daimler mechanic, she saw more heartache in her youth than that catalogued in a thousand country-song sagas. As a sophomore at MIT, she (successfully) petitioned to move directly to the PhD program in computer science without having to suffer the — to use her words — “torturous tedium” of the junior and senior years. This rapid “ascension” was all the more remarkable because her first year in college was not spent at MIT, but at Erie Community College, where on day one of *Java 101*, the professor insisted she come to her office after class, whereupon was launched a tutor-student relationship that initially centered not around not Java, but the language for which Professor Kuth has a secret passion: Prolog. Lucy has consistently rebuffed the overtures of all males at MIT, a group she disdains for their universally poor command of matters computational. Lucy stays in touch with her father by email (and as of this writing has managed to maintain her atheism despite his conversion and testimony), and with but five friends on **facebook**, one of whom has attended an Ivy-League institution, and one of whom, an entrepreneur co-running a startup company in the mobile computing space, attends a likewise techie university 2.5-hrs-drive to-the-west-of MIT.



Key

'Larry' :: larry
'Lucy' :: lucy
'Virginia' :: virginia
'Prescott' :: prescott
'Hank' :: hank
'Abe' :: abe
'Ben' :: ben
'Charles' :: charles
'Christian' :: christian
'Harvard' :: harvard
'MIT' :: mit
'RPI' :: rpi
'Rensselaer' :: renselaer

father-of is a function; eg we can say: (father-of lucy)

brother-of is a function; eg we can say: (brother-of larry)

x is rich iff (Rich x)

x is employed at y iff (EmployedAt x y)

x is west of y iff (WestOf x y)

x attends y iff (Attends x y)

x had i-contact with y iff (IContact x y)

x facebook-likes y iff (F-Likes x y)

x truly likes y iff (T-Likes x y)

x is an alum of y iff (Alum x y)

x is a hacker iff (Hacker x)

x is computationally sophisticated iff (Comp-Soph x)

x is a female iff (Female x)

x is a generous iff (Generous x)

x is eloquent iff (Eloquent x)

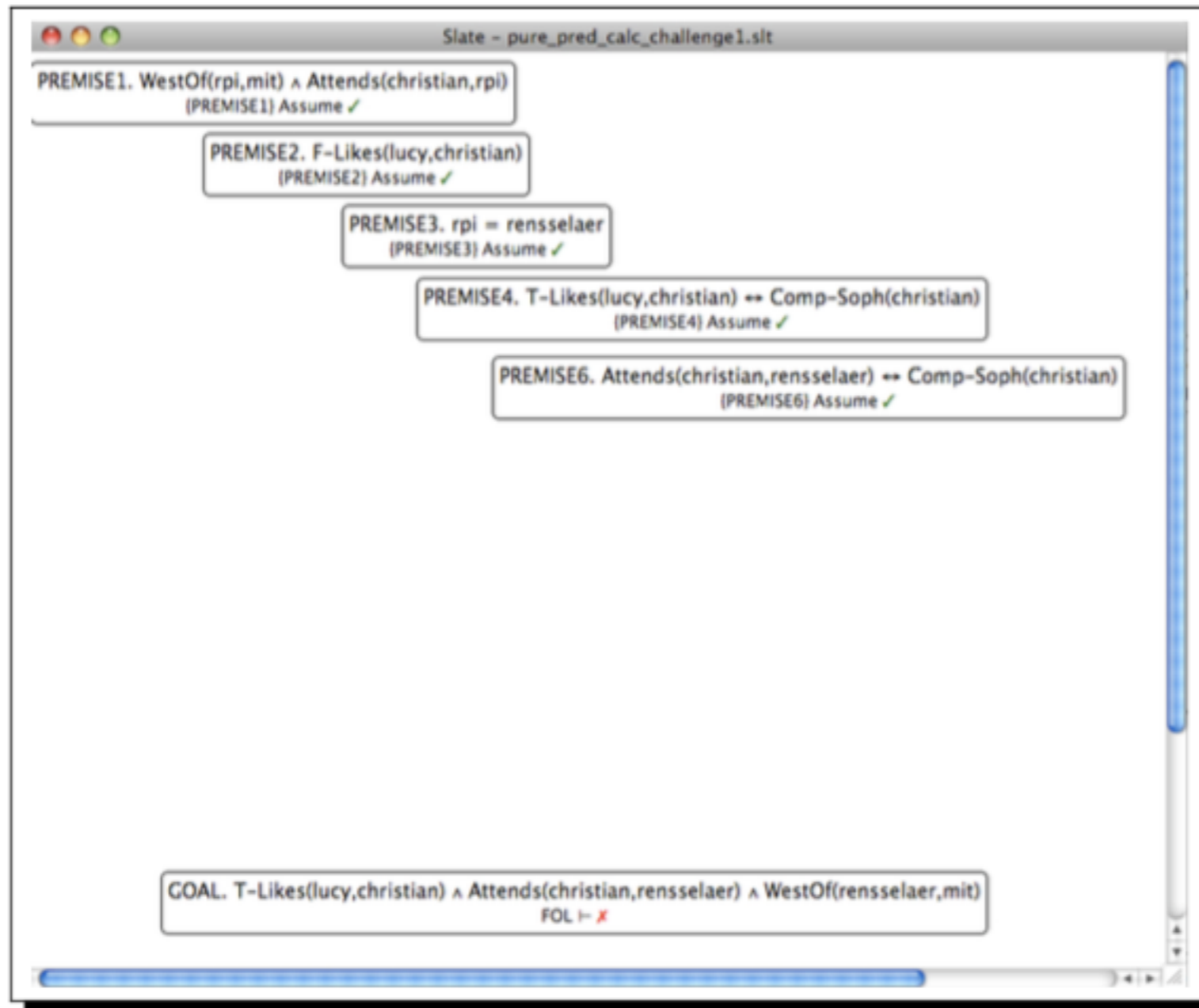
x is a brother of y iff (Brother x y)

x knows Norse mythology iff (K-Norse-Myth x)

x knows who Huginn and Muninn are iff (K-H-M x)

Work through this example from the book!

Figure 3.1: A Proof Challenge in the Pure Predicate Calculus



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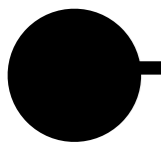
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ANSWER:

all, any, some, most, none, several, ...

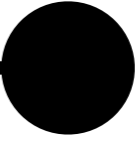
JUSTIFICATION:



350 BC

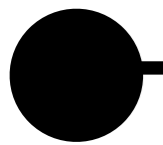


Euclid



2016

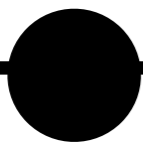
Intro to Logic @ RPI



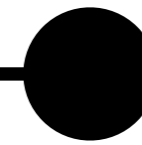
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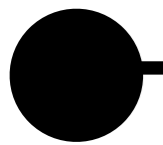


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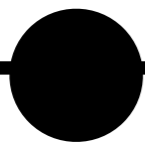


2016

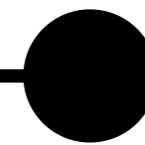
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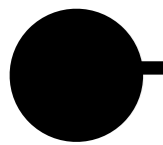
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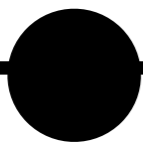
I don't believe in magic! Why exactly is that so convincing? What the heck is he doing??



350 BC



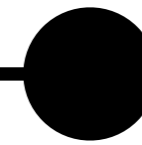
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Organon



2016

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Intro to Logic @ RPI

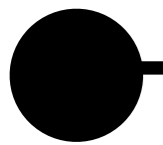
He's using syllogisms!

E.g.,

All As are Bs.

All Bs are Cs.

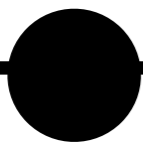
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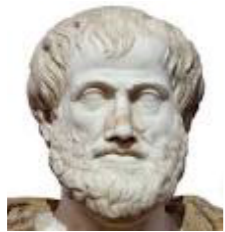
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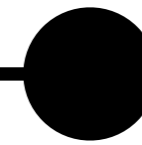
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Two Proposed Arguments; Valid?

- All mammals walk.
- Whales are mammals.
- Therefore:
- Whales walk.
- All of the Frenchmen in the room are wine-drinkers.
- Some of the wine-drinkers in the room are gourmets.
- Therefore:
- Some of the Frenchmen in the room are gourmets.

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ENGLISH_SOURCE. "None of the candidates are rich."
{ENGLISH_SOURCE} Assume ✓

ENGLISH_SOURCE. "All mammals walk."
{ENGLISH_SOURCE} Assume ✓

ENGLISH_SOURCE. "Some of the wine-drinkers are gourmets."
{ENGLISH_SOURCE} Assume ✓

1. $\neg\exists x \text{ Rich}(x)$
{1} Assume ✓

S1.1. $\forall x (\text{Mammal}(x) \rightarrow \text{Walk}(x))$
{S1.1} Assume ✓

S2.2. $\exists x (W(x) \wedge G(x))$
{S2.2} Assume ✓

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Exercise: Symbolize and settle the matter in Slate.