Illogical Thought of the Day; Exhortation; Truth Trees; Measuring Intelligence & FOL IV; Letter Grades

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Intro to (Formal) Logic
3/8/2018
Illogical Thought of the Day …
Just like we made up the game of Poker, we make it up!
Just like we made up the game of Poker, we make it up!
Frege
Don’t be silly. It comes from using logic for discovery, specifically from using …

FOL (which prominently includes quantification) + finding arithmetic (which models reality that any aliens —indeed, any sentient-and-truly-intelligent minds — must grasp) + …
Don’t be silly. It comes from using logic for discovery, specifically from using …

FOL (which prominently includes quantification) + finding arithmetic (which models reality that any aliens — indeed, any sentient-and-truly-intelligent minds — must grasp) + …
Exhortation ...
Too many students are behind, HyperGrader metrics say!
Truth Trees vs. Truth Tables
Truth Trees vs. Truth Tables
Truth Trees vs. Truth Tables

First very simple: truth-tree for *modus ponens* ...
Truth Trees vs. Truth Tables

First very simple: truth-tree for *modus ponens* ...
\{P \rightarrow Q, P\} \vdash Q
\{P \rightarrow Q, P\} \vdash Q
\{P \rightarrow Q, P\} \vdash Q
Truth Trees
Truth Trees

\[ \vdash (P \rightarrow (Q \rightarrow R)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R)) \]
Truth Trees

\[ \vdash (P \rightarrow (Q \rightarrow R)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R)) \]

(This is the axiom THEN-2 in Frege’s (brutal) axiomatization of the propositional calculus.)

Frege
Truth Trees

\[ \vdash (P \rightarrow (Q \rightarrow R)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R)) \]

(This is the axiom THEN-2 in Frege’s (brutal) axiomatization of the propositional calculus.)

Frege

https://en.wikipedia.org/wiki/Frege%27s_propositional_calculus
Truth Trees

⊢ (P → (Q → R)) → ((P → Q) → (P → R))

(This is the axiom THEN-2 in Frege’s (brutal) axiomatization of the propositional calculus.)

Frege
https://en.wikipedia.org/wiki/Frege%27s_propositional_calculus
The Rules of the Game!
The Rules of the Game!

B1. $\varphi \rightarrow \psi$

B1.1. $\neg \varphi$
  (B1.1) Assume ✓

B1.2. $\psi$
  (B1.2) Assume ✓

B2. $\varphi \lor \psi$

B2.1. $\varphi$
  (B2.1) Assume ✓

B2.2. $\psi$
  (B2.2) Assume ✓

B3. $\neg (\varphi \land \psi)$

B3.1. $\neg \varphi$
  (B3.1) Assume ✓

B3.2. $\neg \psi$
  (B3.2) Assume ✓

B4. $\varphi \leftrightarrow \psi$

B4.1a. $\varphi$
  (B4.1a) Assume ✓

B4.1b. $\psi$
  (B4.1b) Assume ✓

B4.2a. $\neg \varphi$
  (B4.2a) Assume ✓

B4.2b. $\neg \psi$
  (B4.2b) Assume ✓

B5. $\neg (\varphi \leftrightarrow \psi)$

B5.1a. $\varphi$
  (B5.1a) Assume ✓

B5.1b. $\neg \psi$
  (B5.1b) Assume ✓

B5.2a. $\neg \varphi$
  (B5.2a) Assume ✓

B5.2b. $\psi$
  (B5.2b) Assume ✓
The Rules of the Game!
Measuring Intelligence, Period
Measuring Intelligence, Period

The Singularity (superhuman machine intelligence) is near!!
Measuring Intelligence, Period

The Singularity (superhuman machine intelligence) is near!!

Is that so? And how are you measuring intelligence, pray tell?
Measuring Intelligence, Period

Is that so? And how are you measuring intelligence, pray tell?
Measuring Intelligence, Period
Measuring Intelligence, Period

Polynomial Hierarchy
Measuring Intelligence, Period

Polynomial Hierarchy

\[ P \subseteq NP \subseteq PSPACE = NPSPACE \subseteq \text{EXPTIME} \subseteq \text{NEXPTIME} \subseteq \text{EXPSPACE} \]
Measuring Intelligence, Period

Checkers: Chinook

Polynomial Hierarchy

$P \subseteq NP \subseteq PSPACE = NPSPACE \subseteq \text{EXPTIME} \subseteq \text{NEXPTIME} \subseteq \text{EXPSPACE}$
Polynomial Hierarchy

Checkers: Chinook

$P \subseteq NP \subseteq PSPACE = NPSPACE \subseteq EXPTIME \subseteq NEXPTIME \subseteq EXPSPACE$
Measuring Intelligence, Period

Polynomial Hierarchy

P ⊆ NP ⊆ PSPACE = NPSPACE ⊆ EXPTIME ⊆ NEXPTIME ⊆ EXPSPACE

Go: AlphaGo

Checkers: Chinook
Measuring Intelligence, Period

Polynomial Hierarchy

Go: AlphaGo
Checkers: Chinook

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Measuring Intelligence, Period

Polynomial Hierarchy

P ⊆ NP ⊆ PSPACE = NPSPACE ⊆ EXPTIME ⊆ NEXPTIME ⊆ EXPSPACE

Go: AlphaGo
Checkers: Chinook

Jeopardy!
Measuring Intelligence, Period

Polynomial Hierarchy

\[ P \subseteq \text{NP} \subseteq \text{PSPACE} = \text{NPSPACE} \subseteq \text{EXPTIME} \subseteq \text{NEXPTIME} \subseteq \text{EXPSPACE} \]
Measuring Intelligence, Period

Polynomial Hierarchy

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- Jeopardy!
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Polynomial Hierarchy

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Measuring Intelligence, Period

- Checkers: Chinook
- Go: AlphaGo
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Measuring Intelligence, Period

Polynomial Hierarchy

\[
P \subseteq \text{NP} \subseteq \text{PSPACE} = \text{NPSPACE} \subseteq \text{EXPTIME} \subseteq \text{NEXPTIME} \subseteq \text{EXPSPACE}
\]
### Measuring Intelligence, Period

<table>
<thead>
<tr>
<th>Arithmetical Hierarchy</th>
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<tbody>
<tr>
<td>Checkers: Chinook</td>
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- $P \subseteq NP \subseteq PSPACE = NPSPACE \subseteq \text{EXPTIME} \subseteq \text{NEXPTIME} \subseteq \text{EXPSPACE}$
Measuring Intelligence, Period

Arithmetical Hierarchy

Polynomial Hierarchy

Go: AlphaGo

Checkers: Chinook

P \subseteq NP \subseteq PSPACE = NPSPACE \subseteq \text{EXPTIME} \subseteq \text{NEXPTIME} \subseteq \text{EXPSPACE}
Measuring Intelligence, Period

Arithmetical Hierarchy

“Hey, do these two Java programs compute the very same function?”

Polynomial Hierarchy

\[ P \subseteq NP \subseteq \text{PSPACE} = \text{NPSPACE} \subseteq \text{EXPTIME} \subseteq \text{NEXPTIME} \subseteq \text{EXPSPACE} \]
Measuring Intelligence, Period

Analytical Hierarchy

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</tr>
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</table>

P ⊆ NP ⊆ PSPACE = NPSPACE ⊆ EXPTIME ⊆ NEXPTIME ⊆ EXPSPACE

“Hey, do these two Java programs compute the very same function?”
This, all of this, is derived from consideration of first-order logic and second-order logic, with an emphasis on quantification and proof.

“Hey, do these two Java programs compute the very same function?”
An “Advanced” Topic for Measuring Intelligence …
An “Advanced” Topic for Measuring Intelligence …

• FOL formulae that (only) enforce domain size:
An “Advanced” Topic for Measuring Intelligence …

- FOL formulae that (only) enforce domain size:

\[ \exists x \exists y (x \neq y) \]
An “Advanced” Topic for Measuring Intelligence …

- FOL formulae that (only) enforce domain size:

\[ \exists x \exists y (x \neq y) \] at least two things
An “Advanced” Topic for Measuring Intelligence ...

- FOL formulae that (only) enforce domain size:

\[
\exists x \exists y (x \neq y) \text{ at least two things}
\]

\[
\exists x \exists y \exists z (x \neq y \land y \neq z \land x \neq z)
\]
An “Advanced” Topic for Measuring Intelligence …

• FOL formulae that (only) enforce domain size:

\[ \exists x \exists y \left( x \neq y \right) \text{ at least two things} \]
\[ \exists x \exists y \exists z \left( x \neq y \land y \neq z \land x \neq z \right) \text{ at least three things} \]
An “Advanced” Topic for Measuring Intelligence …

• FOL formulae that (only) enforce domain size:

\[
\exists x \forall y (x \neq y) \text{ at least two things} \\
\exists x \exists y \exists z (x \neq y \land y \neq z \land x \neq z) \text{ at least three things} \\
\vdots
\]
An “Advanced” Topic for Measuring Intelligence ...

- FOL formulae that (only) enforce domain size:

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\[ \phi_n \]
An “Advanced” Topic for Measuring Intelligence ...

- FOL formulae that (only) enforce domain size:

\[ \exists x \exists y (x \neq y) \] at least two things

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\[ \vdots \]

\[ \phi_n \] domain of at least \( n \) things
An “Advanced” Topic for Measuring Intelligence …

- FOL formulae that (only) enforce domain size:

\[ \exists x \exists y (x \neq y) \] at least two things
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\[ \vdots \]
\[ \phi_n \] domain of at least \( n \) things

\[ \exists x \forall y (y = x) \]
An “Advanced” Topic for Measuring Intelligence ...

- FOL formulae that (only) enforce domain size:

\[
\begin{align*}
\exists x \exists y (x \neq y) & \text{ at least two things} \\
\exists x \exists y \exists z (x \neq y \land y \neq z \land x \neq z) & \text{ at least three things} \\
\vdots & \\
\phi_n & \text{ domain of at least } n \text{ things} \\
\exists x \forall y (y = x) & \text{ at most one thing}
\end{align*}
\]
An “Advanced” Topic for Measuring Intelligence ...

• FOL formulae that (only) enforce domain size:

\[ \exists x \exists y (x \neq y) \text{ at least two things} \]
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\[ \phi_n \text{ domain of at least } n \text{ things} \]
\[ \exists x \forall y (y = x) \text{ at most one thing} \]
\[ \exists x \exists y \forall z (z = x \lor z = y) \]
An “Advanced” Topic for Measuring Intelligence ...

- FOL formulae that (only) enforce domain size:

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An “Advanced” Topic for Measuring Intelligence …

- FOL formulae that (only) enforce domain size:

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\exists x \exists y (x \neq y) \quad \text{at least two things}
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\vdots
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\exists x \exists y \forall z (z = x \lor z = y) \quad \text{at most two things}
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\exists x_1 \exists x_2 \exists x_3 \forall y (y = x_1 \lor y = x_2 \lor y = x_3)
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An “Advanced” Topic for Measuring Intelligence …

• FOL formulae that (only) enforce domain size:

\[\exists x \exists y (x \neq y)\] at least two things

\[\exists x \exists y \exists z (x \neq y \land y \neq z \land x \neq z)\] at least three things

\[\vdots\]

\[\phi_n\] domain of at least \(n\) things

\[\exists x \forall y (y = x)\] at most one thing

\[\exists x \exists y \forall z (z = x \lor z = y)\] at most two things

\[\exists x_1 \exists x_2 \exists x_3 \forall y (y = x_1 \lor y = x_2 \lor y = x_3)\] at most three things
An “Advanced” Topic for Measuring Intelligence …

- FOL formulae that (only) enforce domain size:

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\[
\exists x_1 \exists x_2 \exists x_3 \forall y (y = x_1 \lor y = x_2 \lor y = x_3) \quad \text{at most three things}
\]
\[
\vdots
\]
\[
\phi_n
\]
Chandler’s Addition to RAIR-Lab
Interoperability for AI …
“Are there more than two spheres? Answer & justify.”
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“Are there more than two spheres? Answer & justify.”

“Yes! And here’s the proof.”
“Are there more than two spheres? Answer & justify.”

“Yes! And here’s the proof.”
Measuring AI Intelligence via (in part) Logic: Quantification

Toby Walsh: “The Singularity May Never Be Near”
Toby Walsh: “The Singularity May Never Be Near”

“I will not tackle here head on what we mean by measuring the intelligence of machines (or of humans). I will simply suppose there is such a property as intelligence, that it can be measured and compared, and that the technological singularity is when this measure increases exponentially fast in an appropriate and reasonable scale.” (p. 1)
But logico-mathematical definitions of intelligence for animals, humans, machines, aliens, gods ... *are* possible; recall our consideration of the *Entscheidungsproblem*. We can specifically challenge today’s AI on the basis of simple quantification and simple deduction ...
First, need Chandler’s quantifiers:
First, need Chandler’s quantifiers:

$$\exists x \forall y (y = x \land \phi(x)) \text{ will be } \exists^=1 x \phi(x)$$
First, need Chandler’s quantifiers:

\[ \exists x \forall y (y = x \land \phi(x)) \text{ will be } \exists^{=1} x \phi(x) \]
\[ \exists x \exists y \exists z (x \neq y \land y \neq z \land x \neq z \land \phi(x, y, z)) \text{ will be } \exists^{\geq 3} x \phi(x) \]
First, need Chandler’s quantifiers:

$$\exists x \forall y (y = x \land \phi(x))$$ will be $$\exists^=1 x \phi(x)$$

$$\exists x \exists y \exists z (x \neq y \land y \neq z \land x \neq z \land \phi(x, y, z))$$ will be $$\exists^\geq3 x \phi(x)$$

How do we define formulae of this type: $$\exists^=k x \psi(x)$$
First, need Chandler’s quantifiers:

\[ \exists x \forall y (y = x \land \phi(x)) \text{ will be } \exists^{=1} x \phi(x) \]
\[ \exists x \exists y \exists z (x \neq y \land y \neq z \land x \neq z \land \phi(x, y, z)) \text{ will be } \exists^{\geq 3} x \phi(x) \]

How do we define formulae of this type: \( \exists^{=k} x \psi(x) \)

How do we define formulae of this type: \( \exists^{\leq n} x \psi(x) \)
First, need Chandler’s quantifiers:

$$\exists x \forall y (y = x \land \phi(x))$$ will be $$\exists^{=1} x \phi(x)$$

$$\exists x \exists y \exists z (x \neq y \land y \neq z \land x \neq z \land \phi(x, y, z))$$ will be $$\exists^{\geq 3} x \phi(x)$$

How do we define formulae of this type: $$\exists^{=k} x \psi(x)$$

How do we define formulae of this type: $$\exists^{\leq n} x \psi(x)$$

\vdots
First, need Chandler’s quantifiers:

\[ \exists x \forall y (y = x \land \phi(x)) \text{ will be } \exists^1 x \phi(x) \]
\[ \exists x \exists y \exists z (x \neq y \land y \neq z \land x \neq z \land \phi(x, y, z)) \text{ will be } \exists^3 x \phi(x) \]

How do we define formulae of this type: \( \exists^k x \psi(x) \)

How do we define formulae of this type: \( \exists^n x \psi(x) \)

\[ \vdots \]

Okay, now AI:
First, need Chandler’s quantifiers:

\[ \exists x \forall y (y = x \land \phi(x)) \text{ will be } \exists^{=1} x \phi(x) \]

\[ \exists x \exists y \exists z (x \neq y \land y \neq z \land x \neq z \land \phi(x, y, z)) \text{ will be } \exists^{\geq 3} x \phi(x) \]

How do we define formulae of this type: \( \exists^{=k} x \psi(x) \)

How do we define formulae of this type: \( \exists^{\leq n} x \psi(x) \)

: 

Okay, now AI:

At least seven kenspeckle blookers are red.
First, need Chandler’s quantifiers:

\[ \exists x \forall y (y = x \land \phi(x)) \]  
will be \[ \exists^{=1} x \phi(x) \]

\[ \exists x \exists y \exists z (x \neq y \land y \neq z \land x \neq z \land \phi(x, y, z)) \]  
will be \[ \exists^{\geq 3} x \phi(x) \]

How do we define formulae of this type: \[ \exists^{=k} x \psi(x) \]
How do we define formulae of this type: \[ \exists^{\leq n} x \psi(x) \]

Okay, now AI:

At least seven kenspeckle blookers are red.

Given this, is it true that there are two red blookers? Why, exactly?
Letter Grades ...