Motivating Paradoxes, Puzzles, and $\mathcal{R}$,

Part II

(Why Study Logic?)

Selmer Bringsjord

*Intro to (Formal) Logic*

1/25/18

Selmer.Bringsjord@gmail.com
What is Logic?
What is Logic?

• The key to becoming rational.
What is Logic?

• The key to becoming rational.
• “The science of reasoning.” — so the not-unreasonable slogan goes.
What is Logic?

• The key to becoming rational.
• “The science of reasoning.” — so the not-unreasonable slogan goes.
• The only invincible subject there is.
What is Logic?

- The key to becoming rational.
- “The science of reasoning.” — so the not-unreasonable slogan goes.
- The only invincible subject there is.
- The basis for the formal sciences (from mathematics to game theory to decision theory to probability calculi to axiomatic physics ….) — and hence the basis for disciplines based on the formal sciences (e.g., engineering, computer science).
What is Logic?

• The key to becoming rational.
• “The science of reasoning.” — so the not-unreasonable slogan goes.
• The only invincible subject there is.
• The basis for the formal sciences (from mathematics to game theory to decision theory to probability calculi to axiomatic physics ….) — and hence the basis for disciplines based on the formal sciences (e.g., engineering, computer science).
• The way of escape from shallow content and context to pure, immaterial, and immortal form and structure (which is why the exotic, imaginary, and seemingly non-sensical is so pedagogically useful).
What is Logic?

- The key to becoming rational.
- “The science of reasoning.” — so the not-unreasonable slogan goes.
- The only invincible subject there is.
- The basis for the formal sciences (from mathematics to game theory to decision theory to probability calculi to axiomatic physics ….) — and hence the basis for disciplines based on the formal sciences (e.g., engineering, computer science).
- The way of escape from shallow content and context to pure, immaterial, and immortal form and structure (which is why the exotic, imaginary, and seemingly non-sensical is so pedagogically useful).
- The most challenging subject there is.
What is Logic?

• The key to becoming rational.
• “The science of reasoning.” — so the not-unreasonable slogan goes.
• The only invincible subject there is.
• The basis for the formal sciences (from mathematics to game theory to decision theory to probability calculi to axiomatic physics ….) — and hence the basis for disciplines based on the formal sciences (e.g., engineering, computer science).
• The way of escape from shallow content and context to pure, immaterial, and immortal form and structure (which is why the exotic, imaginary, and seemingly non-sensical is so pedagogically useful).
• The most challenging subject there is.
• One of the chief differentiators between dogs and monkeys versus you (let alone bears and you); and mindless machines (like Deep Blue & Watson) versus you.
What is Logic?

• The key to becoming rational.
• “The science of reasoning.” — so the not-unreasonable slogan goes.
• The only invincible subject there is.
• The basis for the formal sciences (from mathematics to game theory to decision theory to probability calculi to axiomatic physics ….) — and hence the basis for disciplines based on the formal sciences (e.g., engineering, computer science).
• The way of escape from shallow content and context to pure, immaterial, and immortal form and structure (which is why the exotic, imaginary, and seemingly non-sensical is so pedagogically useful).
• The most challenging subject there is.
• One of the chief differentiators between dogs and monkeys versus you (let alone bears and you); and mindless machines (like Deep Blue & Watson) versus you.
• A key to riches.
What is Logic?

• The key to becoming rational.
• “The science of reasoning.” — so the not-unreasonable slogan goes.
• The only invincible subject there is.
• The basis for the formal sciences (from mathematics to game theory to decision theory to probability calculi to axiomatic physics ….) — and hence the basis for disciplines based on the formal sciences (e.g., engineering, computer science).
• The way of escape from shallow content and context to pure, immaterial, and immortal form and structure (which is why the exotic, imaginary, and seemingly non-sensical is so pedagogically useful).
• The most challenging subject there is.
• One of the chief differentiators between dogs and monkeys versus you (let alone bears and you); and mindless machines (like Deep Blue & Watson) versus you.
• A key to riches.
• The key to divining the meaning of life (and other such big questions).
What is Logic?

• The key to becoming rational.
• “The science of reasoning.” — so the not-unreasonable slogan goes.
• The only invincible subject there is.
• The basis for the formal sciences (from mathematics to game theory to decision theory to probability calculi to axiomatic physics ….) — and hence the basis for disciplines based on the formal sciences (e.g., engineering, computer science).
• The way of escape from shallow content and context to pure, immaterial, and immortal form and structure (which is why the exotic, imaginary, and seemingly non-sensical is so pedagogically useful).
• The most challenging subject there is.
• One of the chief differentiators between dogs and monkeys versus you (let alone bears and you); and mindless machines (like Deep Blue & Watson) versus you.
• A key to riches.
• The key to divining the meaning of life (and other such big questions).
• The better way to program computers; and fundamentally the only way to reliably program computers.
What is Logic?

• The key to becoming rational.
• “The science of reasoning.” — so the not-unreasonable slogan goes.
• The only invincible subject there is.
• The basis for the formal sciences (from mathematics to game theory to decision theory to probability calculi to axiomatic physics ….) — and hence the basis for disciplines based on the formal sciences (e.g., engineering, computer science).
• The way of escape from shallow content and context to pure, immaterial, and immortal form and structure (which is why the exotic, imaginary, and seemingly non-sensical is so pedagogically useful).
• The most challenging subject there is.
• One of the chief differentiators between dogs and monkeys versus you (let alone bears and you); and mindless machines (like Deep Blue & Watson) versus you.
• A key to riches.
• The key to divining the meaning of life (and other such big questions).
• The better way to program computers; and fundamentally the only way to reliably program computers.
• One of two fundamental approaches to studying minds, and replicating/simulating minds in machines…
What is Logic?

• The key to becoming rational.
• “The science of reasoning.” — so the not-unreasonable slogan goes.
• The only invincible subject there is.
• The basis for the formal sciences (from mathematics to game theory to decision theory to probability calculi to axiomatic physics ….) — and hence the basis for disciplines based on the formal sciences (e.g., engineering, computer science).
• The way of escape from shallow content and context to pure, immaterial, and immortal form and structure (which is why the exotic, imaginary, and seemingly non-sensical is so pedagogically useful).
• The most challenging subject there is.
• One of the chief differentiators between dogs and monkeys versus you (let alone bears and you); and mindless machines (like Deep Blue & Watson) versus you.
• A key to riches.
• The key to divining the meaning of life (and other such big questions).
• The better way to program computers; and fundamentally the only way to reliably program computers.
• One of two fundamental approaches to studying minds, and replicating/simulating minds in machines…
• The thing many creatures of fiction have mastered — have you (as a New Yorker)? …
What is Logic?

• The key to becoming rational.
• “The science of reasoning.” — so the not-unreasonable slogan goes.
• The only invincible subject there is.
• The basis for the formal sciences (from mathematics to game theory to decision theory to probability calculi to axiomatic physics …) — and hence the basis for disciplines based on the formal sciences (e.g., engineering, computer science).
• The way of escape from shallow content and context to pure, immaterial, and immortal form and structure (which is why the exotic, imaginary, and seemingly non-sensical is so pedagogically useful).
• The most challenging subject there is.
• One of the chief differentiators between dogs and monkeys versus you (let alone bears and you); and mindless machines (like Deep Blue & Watson) versus you.
• A key to riches.
• The key to divining the meaning of life (and other such big questions).
• The better way to program computers; and fundamentally the only way to reliably program computers.
• One of two fundamental approaches to studying minds, and replicating/simulating minds in machines…
• The thing many creatures of fiction have mastered — have you (as a New Yorker)?

What is Logic?

• The key to becoming rational.
• “The science of reasoning.” — so the not-unreasonable slogan goes.
• The only invincible subject there is.
• The basis for the formal sciences (from mathematics to game theory to decision theory to probability calculi to axiomatic physics ….) — and hence the basis for disciplines based on the formal sciences (e.g., engineering, computer science).
• The way of escape from shallow content and context to pure, immaterial, and immortal form and structure (which is why the exotic, imaginary, and seemingly non-sensical is so pedagogically useful).
• The most challenging subject there is.
• One of the chief differentiators between dogs and monkeys versus you (let alone bears and you); and mindless machines (like Deep Blue & Watson) versus you.
• A key to riches.
• The key to divining the meaning of life (and other such big questions).
• The better way to program computers; and fundamentally the only way to reliably program computers.
• One of two fundamental approaches to studying minds, and replicating/simulating minds in machines…
• The thing many creatures of fiction have mastered — have you (as a New Yorker)?
What is Logic?

• The key to becoming rational.
• “The science of reasoning.” — so the not-unreasonable slogan goes.
• The only invincible subject there is.
• The basis for the formal sciences (from mathematics to game theory to decision theory to probability calculi to axiomatic physics …) — and hence the basis for disciplines based on the formal sciences (e.g., engineering, computer science).
• The way of escape from shallow content and context to pure, immaterial, and immortal form and structure (which is why the exotic, imaginary, and seemingly non-sensical is so pedagogically useful).
• The most challenging subject there is.

• One of the chief differentiators between dogs and monkeys versus you (let alone bears and you); and mindless machines (like Deep Blue & Watson) versus you.
• A key to riches.
• The key to divining the meaning of life (and other such big questions).
• The better way to program computers; and fundamentally the only way to reliably program computers.
• One of two fundamental approaches to studying minds, and replicating/simulating minds in machines…
• The thing many creatures of fiction have mastered — have you (as a New Yorker)? …
What is Logic?

- The key to becoming rational.
- “The science of reasoning.” — so the not-unreasonable slogan goes.
- The only invincible subject there is.
- The basis for the formal sciences (from mathematics to game theory to decision theory to probability calculi to axiomatic physics ….) — and hence the basis for disciplines based on the formal sciences (e.g., engineering, computer science).
- The way of escape from shallow content and context to pure, immaterial, and immortal form and structure (which is why the exotic, imaginary, and seemingly non-sensical is so pedagogically useful).
- The most challenging subject there is.
- One of the chief differentiators between dogs and monkeys versus you (let alone bears and you); and mindless machines (like Deep Blue & Watson) versus you.
- A key to riches.
- The key to divining the meaning of life (and other such big questions).
- The better way to program computers; and fundamentally the only way to reliably program computers.
- One of two fundamental approaches to studying minds, and replicating/simulating minds in machines…
- The thing many creatures of fiction have mastered — have you (as a New Yorker)? …
Humans, at least neurobiologically normal ones, are fundamentally rational, where rationality is constituted by certain logico-mathematically based reasoning and decision-making in response to real-world stimuli, including stimuli given in the form of focused tests; but mere animals are not fundamentally rational, since, *contra* Darwin, their minds are fundamentally qualitatively inferior to the human mind. As to whether computing machines/robots are fundamentally rational, the answer is “No.” For starters, if $x$ can’t read, write, and create, $x$ can’t be rational; computing machines/robots can neither read nor write nor create; ergo, they aren’t fundamentally rational.
Background Claim

Humans, at least neurobiologically normal ones, are fundamentally rational, where rationality is constituted by certain logico-mathematically based reasoning and decision-making in response to real-world stimuli, including stimuli given in the form of focused tests; but mere animals are not fundamentally rational, since, *contra* Darwin, their minds are fundamentally qualitatively inferior to the human mind. As to whether computing machines/robots are fundamentally rational, the answer is “No.” For starters, if *x* can’t read, write, and create, *x* can’t be rational; computing machines/robots can neither read nor write nor create; ergo, they aren’t fundamentally rational.
Humans, at least neurobiologically normal ones, are fundamentally rational, where rationality is constituted by certain logico-mathematically based reasoning and decision-making in response to real-world stimuli, including stimuli given in the form of focused tests; but mere animals are not fundamentally rational, since, contra Darwin, their minds are fundamentally qualitatively inferior to the human mind. As to whether computing machines/robots are fundamentally rational, the answer is “No.” For starters, if x can’t read, write, and create, x can’t be rational; computing machines/robots can neither read nor write nor create; ergo, they aren’t fundamentally rational.

To infinity and beyond! — routinely
Humans, at least neurobiologically normal ones, are fundamentally rational, where rationality is constituted by certain logico-mathematically based reasoning and decision-making in response to real-world stimuli, including stimuli given in the form of focused tests; but mere animals are not fundamentally rational, since, contra Darwin, their minds are fundamentally qualitatively inferior to the human mind. As to whether computing machines/robots are fundamentally rational, the answer is “No.” For starters, if $x$ can’t read, write, and create, $x$ can’t be rational; computing machines/robots can neither read nor write nor create; ergo, they aren’t fundamentally rational.

self-reference

To infinity and beyond! — routinely
Background

Claim

Humans, at least neurobiologically normal ones, are fundamentally rational, where rationality is constituted by certain logico-mathematically based reasoning and decision-making in response to real-world stimuli, including stimuli given in the form of focused tests; but mere animals are not fundamentally rational, since, contra Darwin, their minds are fundamentally qualitatively inferior to the human mind. As to whether their natural robotic instincts are fundamentally rational, the answer is “No.” For starters, if a robot were to be rational, it, x, can't be rational; computing machines/robots can neither read nor write nor create; ergo, they aren't fundamentally rational.

recursion

self-reference

To infinity and beyond! — routinely
Background Claim

intensional reasoning

 recursion

self-reference

To infinity and beyond! — routinely
Humans, at least neurobiologically normal ones, are fundamentally rational, where rationality is
constituted by certain logico-mathematically based reasoning and decision-making in response to
real-world stimuli, including stimuli given in the form of focused tests; but mere animals are not
fundamentally rational, since, contra Darwin, their minds are fundamentally qualitatively inferior
to the human mind. As to what counts as a fundamental test are fundamentally rational, the
answer is “No.” For starters, if a machine can’t write or read, x can’t be rational; computing
machines/robots can neither read nor write nor create; ergo, they aren’t fundamentally rational.

To infinity and beyond! — routinely
abstract-and-valid inference schemata

quantification

intensional reasoning

recursion

self-reference

To infinity and beyond! — routinely
Selmer’s Seriated Cup Challenge, Part 1

Suppose you have at your disposal a “factory” that, upon hearing you announce a number \( j \), can quickly output a cup having a diameter of precisely \( j \) units. Can you insert a new cup between two of the seriated cups in the tower shown here? — where the \( j \) you send in must be a positive integer, \( m \) is likewise a positive integer, and every cup in every tower must be more in diameter than the one immediately above it, and less in diameter than the one immediately below it?** Prove that your answer is correct.

**E.g., if \( m = 3 \), the tower in that case will have a base cup 4 units in diameter, immediately above that a cup 3 units in diameter, then a cup 2 units in diameter, and then finally a top cup of 1 unit in diameter.
Selmer’s Seriated Cup Challenge, Part 1

Suppose you have at your disposal a “factory” that, upon hearing you announce a number \( j \), can quickly output a cup having a diameter of precisely \( j \) units. Can you insert a new cup between two of the seriated cups in the tower shown here? — where the \( j \) you send in must be a positive integer, \( m \) is likewise a positive integer, and every cup in every tower must be more in diameter than the one immediately above it, and less in diameter than the one immediately below it?** Prove that your answer is correct.

**E.g., if \( m = 3 \), the tower in that case will have a base cup 4 units in diameter, immediately above that a cup 3 units in diameter, then a cup 2 units in diameter, and then finally a top cup of 1 unit in diameter.
Selmer’s Seriated Cup Challenge, Part II

Suppose you have at your disposal a “factory” that, upon hearing you announce a number \( j \), can quickly output a cup having a diameter of precisely \( j \) units. Can you insert a new cup between two of the seriated cups in the tower shown here? — where the \( j \) you send in must be a positive rational number; \( k, k', k'', k''' \ldots \) are likewise positive rational numbers, and every cup in every tower must be more in diameter than the one immediately above it, and less in diameter than the one immediately below it?** Prove that your answer is correct.

**E.g., if \( k = \frac{1}{7} \), the tower in that case will have a base cup \( \frac{1}{2} \) units in diameter, immediately above that there could be a cup \( \frac{2}{3} \) units in diameter, then perhaps a cup \( \frac{3}{4} \) units in diameter, and then perhaps finally a top cup of \( \frac{3}{2} \) units in diameter.
Selmer’s Seriated Cup Challenge, Part II

Suppose you have at your disposal a “factory” that, upon hearing you announce a number \( j \), can quickly output a cup having a diameter of precisely \( j \) units. Can you insert a new cup between two of the seriated cups in the tower shown here? — where the \( j \) you send in must be a positive rational number; \( k, k', k'', k''' \ldots \) are likewise positive rational numbers, and every cup in every tower must be more in diameter than the one immediately above it, and less in diameter than the one immediately below it?** Prove that your answer is correct.

**E.g., if \( k = \frac{1}{3} \), the tower in that case will have a base cup \( \frac{1}{3} \) units in diameter, immediately above that there could be a cup \( \frac{2}{3} \) units in diameter, then perhaps a cup \( \frac{1}{4} \) units in \( \frac{7}{8} \) diameter, and then perhaps finally a top cup of \( \frac{3}{2} \) units in diameter.
What is Logic?

• The key to becoming rational.

• “The science of reasoning.” — so the not-unreasonable slogan goes.

• The only invincible subject there is.

• The basis for the formal sciences (from mathematics to game theory to decision theory to probability calculi to axiomatic physics ….) — and hence the basis for disciplines based on the formal sciences (e.g., engineering, computer science).

• The way of escape from shallow content and context to pure, immaterial, and immortal form and structure (which is why the exotic, imaginary, and seemingly nonsensical is so pedagogically useful).

• The most challenging subject there is.

• One of the chief differentiators between dogs and monkeys versus you (let alone bears and you); and mindless machines (like Deep Blue & Watson) versus you.

• A key to riches.

• The key to divining the meaning of life (and other such big questions).

• The better way to program computers; and fundamentally the only way to reliably program computers.

• One of two fundamental approaches to studying minds, and replicating/simulating minds in machines…

• The thing many creatures of fiction have mastered — have you (as a New Yorker)?…

• …
What is Logic?

• The key to becoming rational. Or are you already rational? ... 

• “The science of reasoning.” — so the not-unreasonable slogan goes.

• The only invincible subject there is.

• The basis for the formal sciences (from mathematics to game theory to decision theory to probability calculi to axiomatic physics ….) — and hence the basis for disciplines based on the formal sciences (e.g., engineering, computer science).

• The way of escape from shallow content and context to pure, immaterial, and immortal form and structure (which is why the exotic, imaginary, and seemingly nonsensical is so pedagogically useful).

• The most challenging subject there is.

• One of the chief differentiators between dogs and monkeys versus you (let alone bears and you); and mindless machines (like Deep Blue & Watson) versus you.

• A key to riches.

• The key to divining the meaning of life (and other such big questions).

• The better way to program computers; and fundamentally the only way to reliably program computers.

• One of two fundamental approaches to studying minds, and replicating/simulating minds in machines...

• The thing many creatures of fiction have mastered — have you (as a New Yorker)?…

• …
It’s White’s turn. What move did Black just make?
Aha! (Beyond Deep Blue?)
Aha! (Beyond Deep Blue?)
Simple Selection Task

E  T  4  7

Suppose I claim that the following rule is true.

If a card has a vowel on one side, it has an even number on the other side.

Which card or cards, if any, should you turn over in order to try to efficiently decide whether the rule is true or false?
Suppose I claim that the following rule is true.

If a card has a vowel on one side, it has an even number on the other side.

Which card or cards, if any, should you turn over in order to try to efficiently decide whether the rule is true or false?
Suppose I claim that the following rule is true.

If a card has a vowel on one side, it has an even number on the other side.

Which card or cards, if any, should you turn over in order to try to efficiently decide whether the rule is true or false?
Another Simple Selection Task

D  U  9  4

Suppose I claim that the following rule is true.

If a card has a vowel on one side, it has an even number on the other side.

Which card or cards, if any, should you turn over in order to try to efficiently decide whether the rule is true or false?
Another Simple Selection Task

Suppose I claim that the following rule is true.

If a card has a vowel on one side, it has an even number on the other side.

Which card or cards, if any, should you turn over in order to try to efficiently decide whether the rule is true or false?
Another Simple Selection Task

Suppose I claim that the following rule is true.

If a card has a vowel on one side, it has an even number on the other side.

Which card or cards, if any, should you turn over in order to try to efficiently decide whether the rule is true or false?
Given the statements

\( \neg a \lor \neg b \)

\( b \)

\( c \rightarrow a \)

which one of the following statements must also be true?

\( c \)

\( \neg b \)

\( \neg c \)

\( h \)

\( a \)

none of the above
Given the statements

\( \neg a \lor \neg b \)

\( b \)

\( c \rightarrow a \)

which one of the following statements must also be true?

\( c \)

\( \neg b \)

\( \neg c \)

\( h \)

\( a \)

none of the above
Which one of the following statements is logically equivalent to the following statement: “If you are not part of the solution, then you are part of the problem.”

If you are part of the solution, then you are not part of the problem.

If you are not part of the problem, then you are part of the solution.

If you are part of the problem, then you are not part of the solution.

If you are not part of the problem, then you are not part of the solution.
Which one of the following statements is logically equivalent to the following statement: “If you are not part of the solution, then you are part of the problem.”

If you are part of the solution, then you are not part of the problem.

If you are not part of the problem, then you are part of the solution.

If you are part of the problem, then you are not part of the solution.

If you are not part of the problem, then you are not part of the solution.
Given the statements

¬¬c
c → a
¬a ∨ b
b → d
¬(d ∨ e)

which one of the following statements must also be true?

¬c
e
h
¬a
all of the above
Given the statements
\( \neg \neg c \)
\( c \rightarrow a \)
\( \neg a \lor b \)
\( b \rightarrow d \)
\( \neg (d \lor e) \)

which one of the following statements must also be true?

\( \neg c \)
\( e \)
\( h \)
\( \neg a \)
all of the above
Given the statements
¬¬c

¬¬a ∨ b

b → d

¬(d ∨ e)

which one of the following statements must also be true?

¬c
e
h
¬a
all of the above
The Original King-Ace

Suppose that the following premise is true:

If there is a king in the hand, then there is an ace in the hand, or else if there isn’t a king in the hand, then there is an ace.

What can you infer from this premise?
The Original King-Ace

Suppose that the following premise is true:

If there is a king in the hand, then there is an ace in the hand, or else if there isn’t a king in the hand, then there is an ace.

What can you infer from this premise?

There is an ace in the hand.
The Original King-Ace

Suppose that the following premise is true:

If there is a king in the hand, then there is an ace in the hand, or else if there isn’t a king in the hand, then there is an ace.

What can you infer from this premise?

There is an ace in the hand.
The Original King-Ace

Suppose that the following premise is true:

If there is a king in the hand, then there is an ace in the hand, or else if there isn’t a king in the hand, then there is an ace.

What can you infer from this premise?

NO! There is an ace in the hand.
The Original King-Ace

Suppose that the following premise is true:

If there is a king in the hand, then there is an ace in the hand, or else if there isn’t a king in the hand, then there is an ace.

What can you infer from this premise?

NO! There is an ace in the hand. NO!
The Original King-Ace

Suppose that the following premise is true:

If there is a king in the hand, then there is an ace in the hand, or else if there isn’t a king in the hand, then there is an ace.

What can you infer from this premise?

NO! There is an ace in the hand. NO!

In fact, what you can infer is that there isn’t an ace in the hand!
King-Ace 2

Suppose that the following premise is true:

*If there is a king in the hand, then there is an ace in the hand; or if there isn’t a king in the hand, then there is an ace; but not both of these if-then statements are true.*

What can you infer from this premise?
Suppose that the following premise is true:

*If there is a king in the hand, then there is an ace in the hand; or if there isn’t a king in the hand, then there is an ace; but not both of these if-then statements are true.*

What can you infer from this premise?

There is an ace in the hand.
Suppose that the following premise is true:

If there is a king in the hand, then there is an ace in the hand; or if there isn’t a king in the hand, then there is an ace; but not both of these if-then statements are true.

What can you infer from this premise?

There is an ace in the hand.
Suppose that the following premise is true:

If there is a king in the hand, then there is an ace in the hand; or if there isn’t a king in the hand, then there is an ace; but not both of these if-then statements are true.

What can you infer from this premise?

NO! There is an ace in the hand.
King-Ace 2

Suppose that the following premise is true:

*If there is a king in the hand, then there is an ace in the hand; or if there isn’t a king in the hand, then there is an ace; but not both of these if-then statements are true.*

What can you infer from this premise?

*NO! There is an ace in the hand. NO!*
King-Ace 2

Suppose that the following premise is true:

If there is a king in the hand, then there is an ace in the hand; or if there isn’t a king in the hand, then there is an ace; but not both of these if-then statements are true.

What can you infer from this premise?

NO! There is an ace in the hand. NO!

In fact, what you can infer is that there isn’t an ace in the hand!
King-Ace Solved
(informal proof)

**Proposition:** There is *not* an ace in the hand.

**Proof:** We know that at least one of the if-thens (i.e., at least one of the *conditionals*) is false. So we have two cases to consider, viz., that $K \Rightarrow A$ is false, and that $\neg K \Rightarrow A$ is false. Take first the first case; accordingly, suppose that $K \Rightarrow A$ is false. Then it follows that $K$ is true (since when a conditional is false, its antecedent holds but its consequent doesn’t), and $A$ is false. Now consider the second case, which consists in $\neg K \Rightarrow A$ being false. Here, in a direct parallel, we know $\neg K$ and, once again, $\neg A$. In both of our two cases, which are exhaustive, there is no ace in the hand. The proposition is established.  

**QED**
Train-to-Princeton Problem

Everyone loves anyone who loves someone.

Larry loves Lucy.

Can you infer that everyone loves Lucy?

ANSWER:

PROOF:
Train-to-Princeton Problem

Everyone loves anyone who loves someone.

Larry loves Lucy.

Can you infer that everyone loves Lucy?

ANSWER: Yup.

PROOF: ??
(1) The following three assertions are either all true or all false:

   If Billy helped, Doreen helped.
   If Doreen helped, Frank did as well.
   If Frank helped, so did Emma.

(2) The following assertion is definitely true: Billy helped.

Can it be inferred from (1) and (2) that Emma helped?
(1) The following three assertions are either all true or all false:

If Billy helped, Doreen helped.
If Doreen helped, Frank did as well.
If Frank helped, so did Emma.

(2) The following assertion is definitely true: Billy helped.

Can it be inferred from (1) and (2) that Emma helped?

YUP! — & now prove it!
A criminal genius nearly a match for Sherlock Holmes (Do you recognize the Dr?) has built a massive hydrogen bomb, and life on Earth is hanging in the balance, hinging on whether you make the rational prediction. Dr M gives you a sporting chance to: make the right prediction, snip or not snip accordingly, and prove that you're right …
If one of the following assertions is true then so is the other:

(1) If the red wire runs to the bomb, then the blue wire runs to the bomb; and, if the blue wire runs to the bomb, then the red wire runs to the bomb.

(2) The red wire runs to the bomb.

Given this perfectly reliable clue from Dr. Moriarty, if either wire is more likely to run to the bomb, that wire does run to the bomb, and the bomb is ticking, with only a minute left! If both are equiprobable, neither runs to the bomb, and you are powerless. Make your prediction as to what will happen when a wire is snipped, and then make your selected snip by clicking on the wire you want to snip! Or leave well enough alone!

Red more likely.

Blue more likely.

Equiprobable.
Life on Earth has ended.

advance one more slide to see a proof that you indeed made an irrational decision…
Life on Earth is saved! if you can now hand Dr M a proof that your decision was the rational one!

Advance one more slide to see a proof from Bringsjord that yours had better match up to ...