Motivating Paradoxes, Puzzles, and $\mathbb{R}$, Part II

(Why Study Logic?)

Selmer Bringsjord

*Intro to Logic*

1/26/17

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What’s on the CD?

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What is Logic?

• The key to becoming rational.

• “The science of reasoning.” — so the not-unreasonable slogan goes.

• The only invincible subject there is.

• The basis for the formal sciences (from mathematics to game theory to decision theory to probability calculi to axiomatic physics ….) — and hence the basis for disciplines based on the formal sciences (e.g., engineering, computer science).

• The way of escape from shallow content and context to pure, immaterial, and immortal form and structure (which is why the exotic, imaginary, and seemingly non-sensical is so pedagogically useful).

• The most challenging subject there is.

• One of the chief differentiators between dogs and monkeys versus you (let alone bears and you); and mindless machines (like Deep Blue & Watson) versus you.

• A key to riches.

• The key to divining the meaning of life (and other such big questions).

• The better way to program computers; and fundamentally the only way to reliably program computers.

• One of two fundamental approaches to studying minds, and replicating/simulating minds in machines…

• The thing many creatures of fiction have mastered — have you (as a New Yorker)? …
What is Logic?

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• The thing many creatures of fiction have mastered — have you (as a New Yorker)? …
Suppose that the following premise is true:

*If there is a king in the hand, then there is an ace in the hand; or if there isn’t a king in the hand, then there is an ace; but not both of these if-then statements are true.*

What can you infer from this premise?
Suppose that the following premise is true:

*If there is a king in the hand, then there is an ace in the hand; or if there isn’t a king in the hand, then there is an ace; but not both of these if-then statements are true.*

What can you infer from this premise?

There is an ace in the hand.
Suppose that the following premise is true:

If there is a king in the hand, then there is an ace in the hand; or if there isn’t a king in the hand, then there is an ace; but not both of these if-then statements are true.

What can you infer from this premise?

There is an ace in the hand.
King-Ace 2

Suppose that the following premise is true:

*If there is a king in the hand, then there is an ace in the hand; or if there isn’t a king in the hand, then there is an ace; but not both of these if-then statements are true.*

What can you infer from this premise?

**NO! There is an ace in the hand.**
Suppose that the following premise is true:

*If there is a king in the hand, then there is an ace in the hand; or if there isn’t a king in the hand, then there is an ace; but not both of these if-then statements are true.*

What can you infer from this premise?

**NO!** There is an ace in the hand. **NO!**
Suppose that the following premise is true:

*If there is a king in the hand, then there is an ace in the hand; or if there isn’t a king in the hand, then there is an ace; but not both of these if-then statements are true.*

What can you infer from this premise?

**NO! There is an ace in the hand. NO!**

In fact, what you *can* infer is that there *isn’t* an ace in the hand!
A criminal genius nearly a match for Sherlock Holmes (Do you recognize the Dr?) has built a massive hydrogen bomb, and life on Earth is hanging in the balance, hinging on whether you make the rational prediction. Dr M gives you a sporting chance to: make the right prediction, snip or not snip accordingly, and prove that you’re right …
If one of the following assertions is true then so is the other:

(1) If the red wire runs to the bomb, then the blue wire runs to the bomb; and, if the blue wire runs to the bomb, then the red wire runs to the bomb.

(2) The red wire runs to the bomb.

Given this perfectly reliable clue from Dr Moriarty, if either wire is more likely to run to the bomb, that wire does run to the bomb, and the bomb is ticking, with only a minute left! If both are equiprobable, neither runs to the bomb, and you are powerless. Make your prediction as to what will happen when a wire is snipped, and then make your selected snip by clicking on the wire you want to snip! Or leave well enough alone!

Red more likely.

Blue more likely.

Equiprobable.
Life on Earth has ended.

advance one more slide to see a proof that you indeed made an irrational decision…
Proposition: The blue wire is more likely!

Proof: (1) can be treated as a biconditional, obviously (R <=> B).

There are two top-level cases to consider: (1) and (2) are both true; or both are false. In the case where they are both true, it’s trivial to deduce both R and B. So far, then, R and B are equiprobable. What happens in the case where (1) and (2) are both false? We immediately have ~R from the denial of (2). But a biconditional is true just in case both sides are true, or both sides are false; so we have two sub-cases to consider.

Consider first the case where R is true and B is false. We have an immediate contradiction in this sub-case, so both R and B can both be deduced here, and we have not yet departed from equiprobable. So what about the case where R is false and B is true? The falsity of R is not new information (we already have that from the denial of (2)), but we can still derive B. Hence the blue wire is more likely. QED
Life on Earth is saved!

if you can now hand Dr M a proof that your decision was the rational one!

Advance one more slide to see a proof from Bringsjord that yours had better match up to …
Proposition: The blue wire is more likely!

Proof: (1) can be treated as a biconditional, obviously (R <=> B).

There are two top-level cases to consider: (1) and (2) are both true; or both are false. In the case where they are both true, it’s trivial to deduce both R and B. So far, then, R and B are equiprobable. What happens in the case where (1) and (2) are both false? We immediately have ~R from the denial of (2). But a biconditional is true just in case both sides are true, or both sides are false; so we have two sub-cases to consider.

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Life on Earth has ended.

advance one more slide to see a proof that you indeed made an irrational decision...
**Proposition:** The blue wire is more likely!

**Proof:** (1) can be treated as a biconditional, obviously \((R \iff B)\).

There are two top-level cases to consider: (1) and (2) are both true; or both are false. In the case where they are both true, it’s trivial to deduce both \(R\) and \(B\). So far, then, \(R\) and \(B\) are equiprobable. What happens in the case where (1) and (2) are both false? We immediately have \(\sim R\) from the denial of (2). But a biconditional is true just in case both sides are true, or both sides are false; so we have two sub-cases to consider.

Consider first the case where \(R\) is true and \(B\) is false. We have an immediate contradiction in this sub-case, so both \(R\) and \(B\) can both be deduced here, and we have not yet departed from equiprobable. So what about the case where \(R\) is false and \(B\) is true? The falsity of \(R\) is not new information (we already have that from the denial of (2)), but we can still derive \(B\). Hence the blue wire is more likely. **QED**
If one of the following assertions is true then so is the other:

(1) There is a king in the hand if and only if there is an ace in the hand.

(2) There is a king in the hand.

Which is more likely to be in the hand, if either: the king or the ace?