

# Motivating Paradoxes, Puzzles, and $\mathcal{R}$ ,

## Part II

(Why Study Logic?)

**Selmer Bringsjord**

*Intro to (Formal) Logic*

1/25/18

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# Background Claim

$\mathcal{R}$  Humans, at least neurobiologically normal ones, are fundamentally rational, where rationality is constituted by certain logico-mathematically based reasoning and decision-making in response to real-world stimuli, including stimuli given in the form of focused tests; but mere animals are not fundamentally rational, since, *contra* Darwin, their minds are fundamentally qualitatively inferior to the human mind. As to whether computing machines/robots are fundamentally rational, the answer is “No.” For starters, if  $x$  can’t read, write, and create,  $x$  can’t be rational; computing machines/robots can neither read nor write nor create; ergo, they aren’t fundamentally rational.



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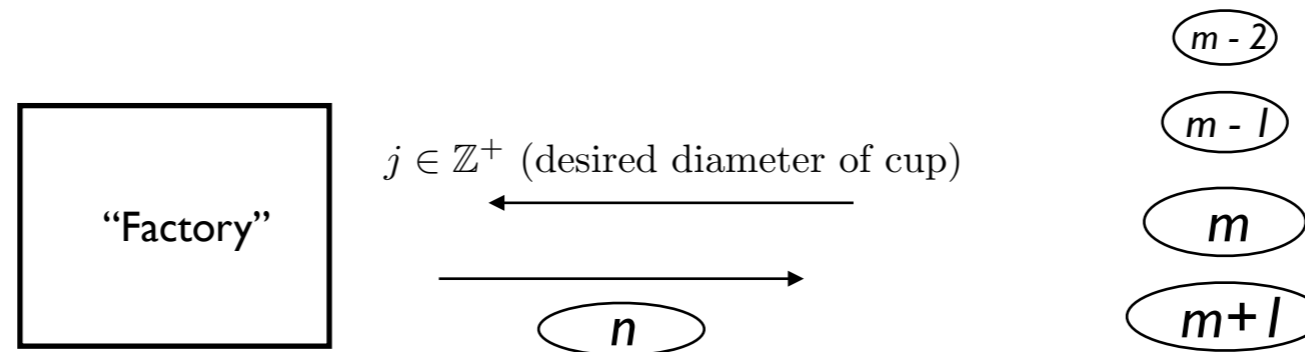
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# Selmer's Seriated Cup Challenge, Part I

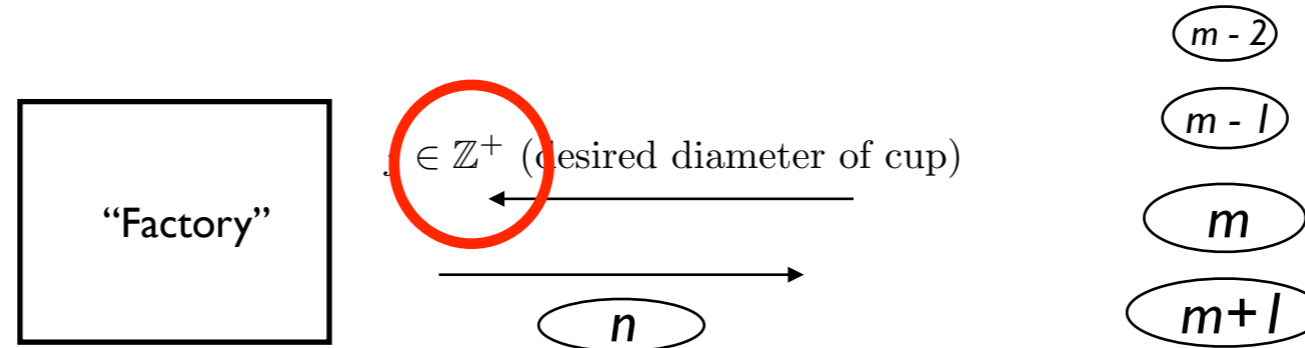
Suppose you have at your disposal a “factory” that, upon hearing you announce a number  $j$ , can quickly output a cup having a diameter of precisely  $j$  units. Can you insert a new cup between two of the seriated cups in the tower shown here? — where the  $j$  you send in *must* be a positive integer,  $m$  is likewise a positive integer, and every cup in every tower must be more in diameter than the one immediately above it, and less in diameter than the one immediately below it? \*\* Prove that your answer is correct.



\*\*E.g., if  $m = 3$ , the tower in that case will have a base cup 4 units in diameter, immediately above that a cup 3 units in diameter, then a cup 2 units in diameter, and then finally a top cup of 1 unit in diameter.

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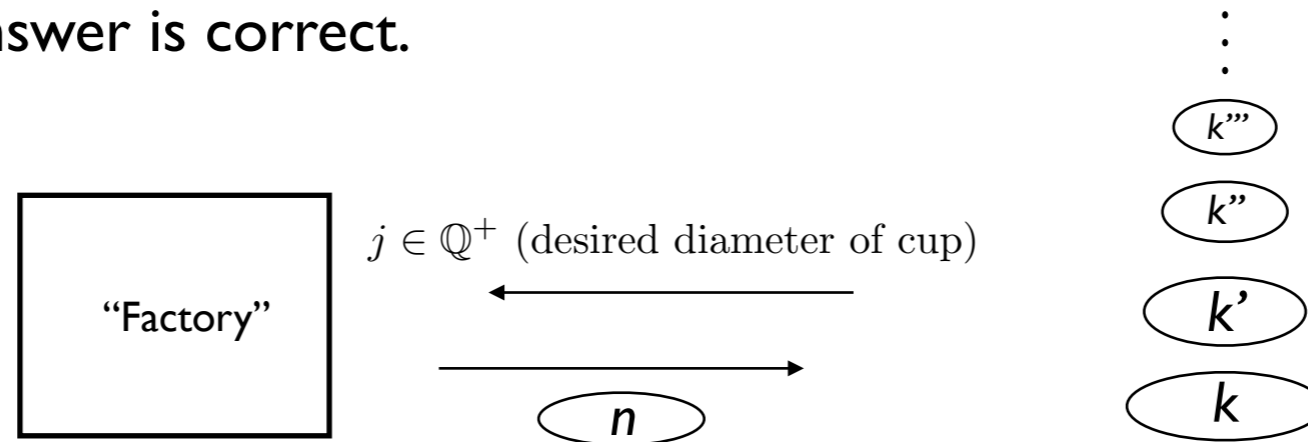
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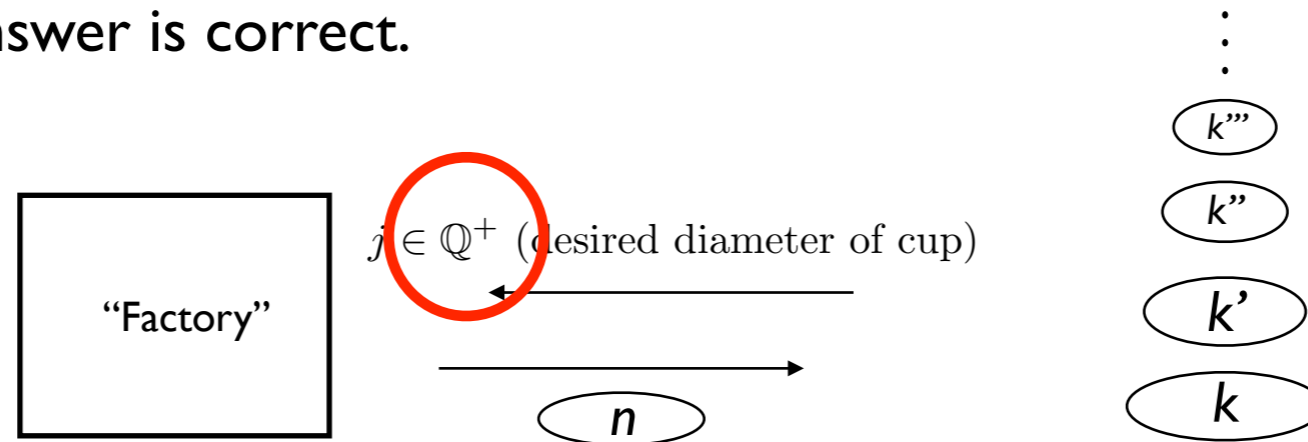
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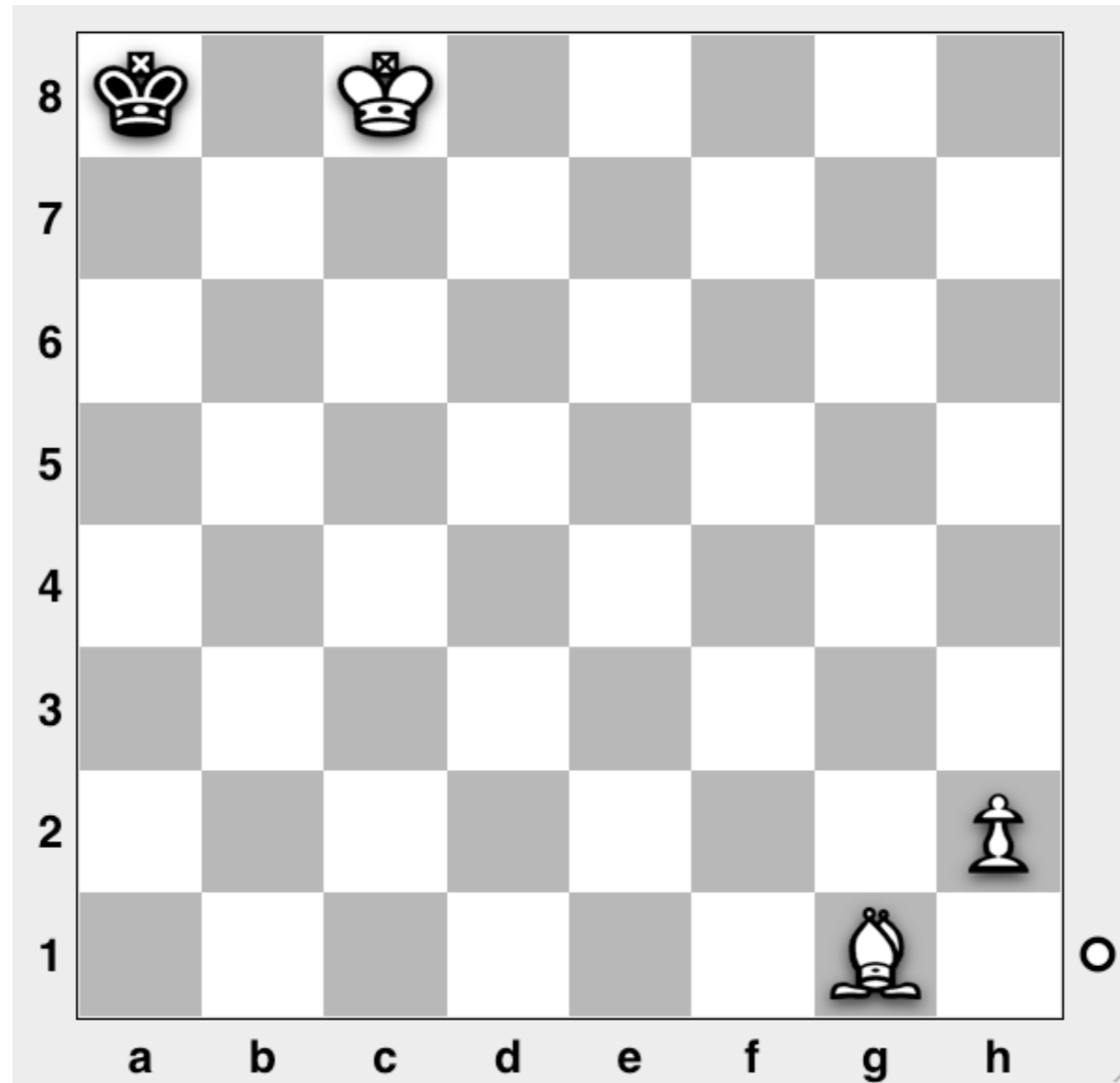
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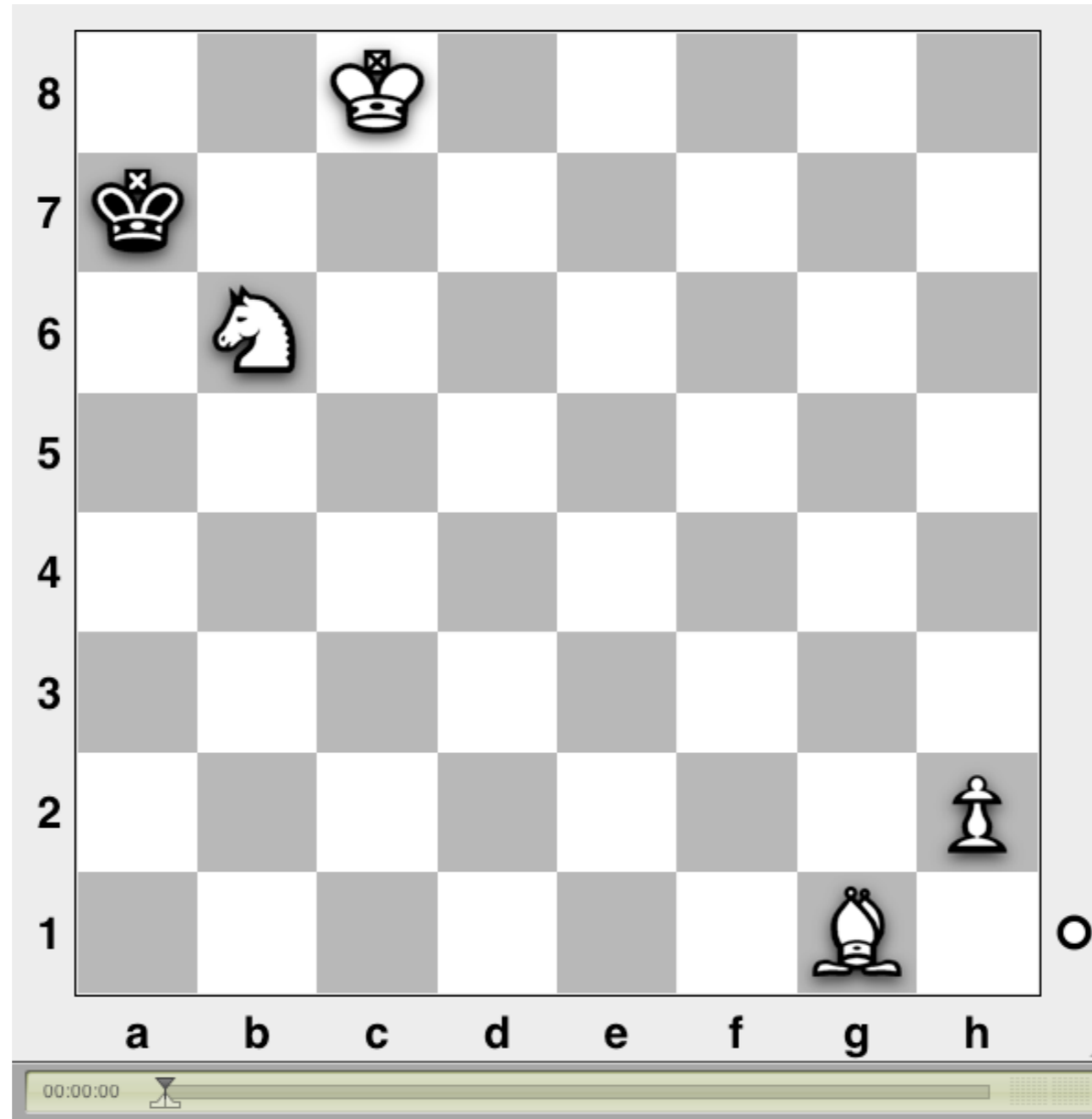
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It's White's turn. What move did Black just make?

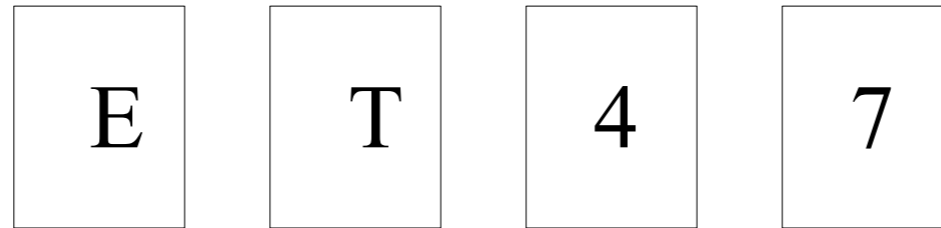


**Aha! (Beyond Deep Blue?)**

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# Simple Selection Task

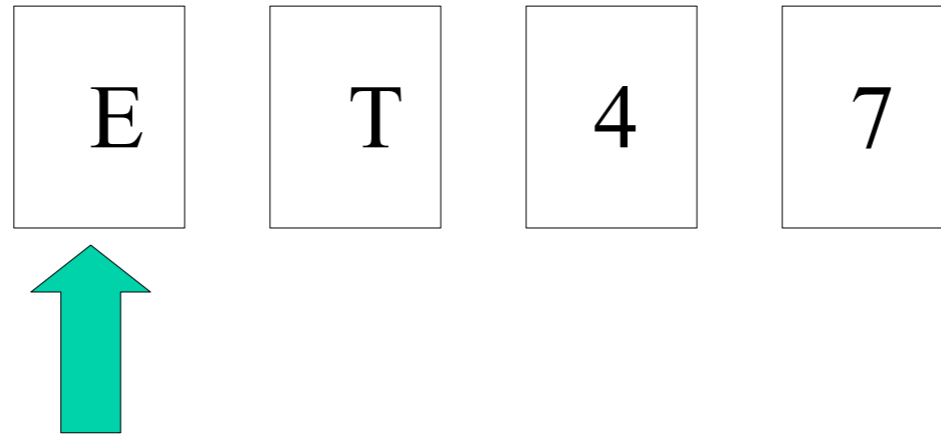


Suppose I claim that the following rule is true.

If a card has a vowel on one side, it has an even number on the other side.

Which card or cards, if any, should you turn over in order to try to efficiently decide whether the rule is true or false?

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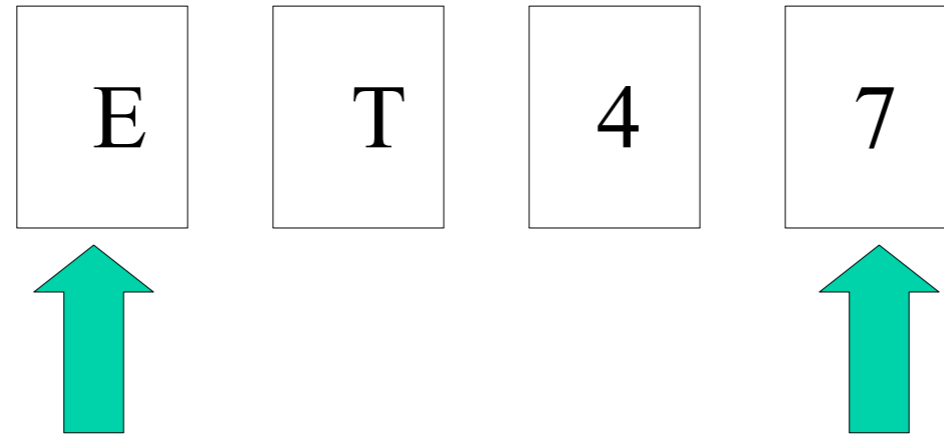


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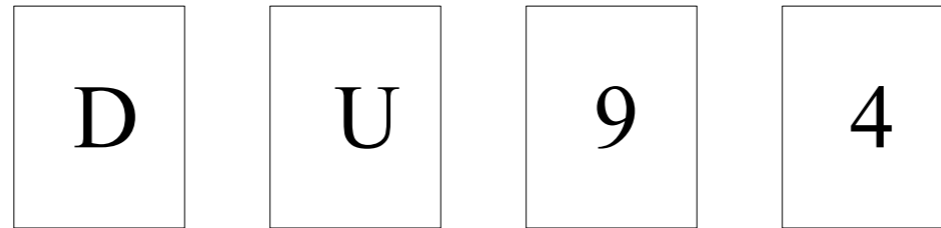


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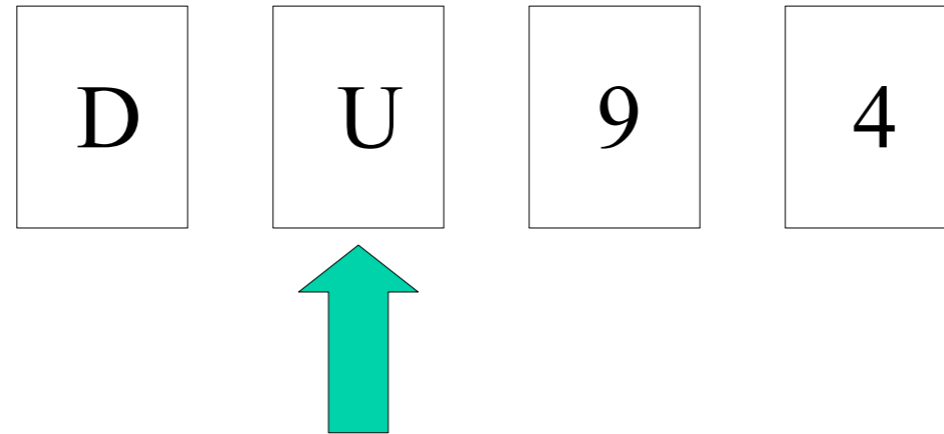
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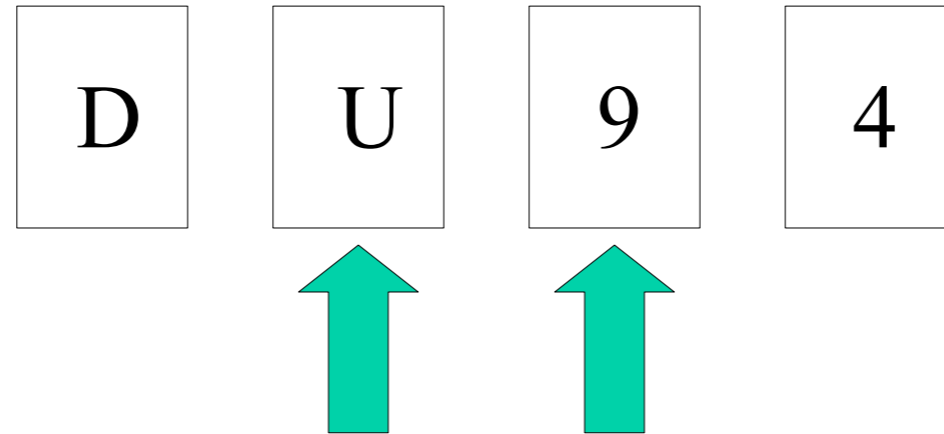


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Given the statements

$$\neg a \vee \neg b$$

$b$

$$c \rightarrow a$$

which one of the following statements must also be true?

$c$

$\neg b$

$\neg c$

$h$

$a$

none of the above

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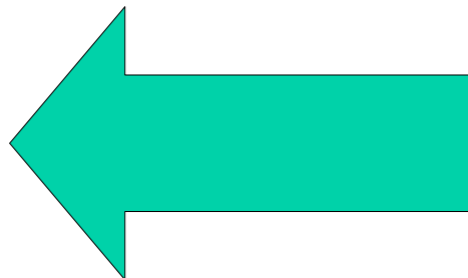
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# “NYS 2”

Which one of the following statements is logically equivalent to the following statement: “If you are not part of the solution, then you are part of the problem.”

If you are part of the solution, then you are not part of the problem.

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# “NYS 3”

Given the statements

$$\neg\neg c$$

$$c \rightarrow a$$

$$\neg a \vee b$$

$$b \rightarrow d$$

$$\neg(d \vee e)$$

which one of the following statements must also be true?

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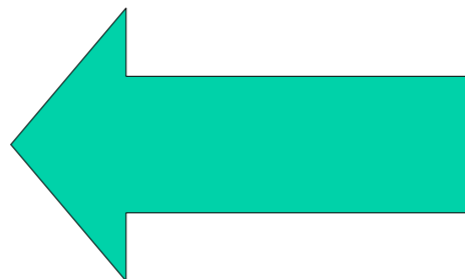
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all of the above





# “NYS 3”

Given the statements

$$\neg\neg c$$

$$c \rightarrow a$$

$$\neg a \vee b$$

$$b \rightarrow d$$

$$\neg(d \vee e)$$

which one of the following statements must also be true?

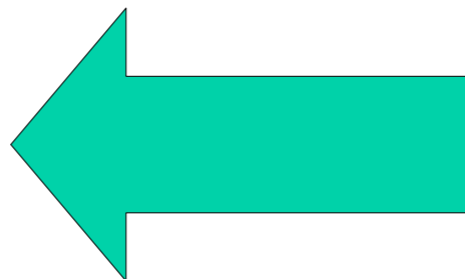
$$\neg c$$

e

h

$$\neg a$$

all of the above



# The Original King-Ace

Suppose that the following premise is true:

If there is a king in the hand, then there is an ace in the hand, or else if there isn't a king in the hand, then there is an ace.

What can you infer from this premise?

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NO! ~~There is an ace in the hand.~~ NO!

In fact, what you *can* infer is that there *isn't* an ace in the hand!

# King-Ace 2

Suppose that the following premise is true:

*If there is a king in the hand, then there is an ace in the hand; or if there isn't a king in the hand, then there is an ace; but not both of these if-then statements are true.*

What can you infer from this premise?



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# King-Ace Solved

(informal proof)

**Proposition:** There is *not* an ace in the hand.

**Proof:** We know that at least one of the if-thens (i.e., at least one of the **conditionals**) is false. So we have two cases to consider, viz., that  $K \Rightarrow A$  is false, and that  $\neg K \Rightarrow A$  is false. Take first the first case; accordingly, suppose that  $K \Rightarrow A$  is false. Then it follows that  $K$  is true (since when a conditional is false, its antecedent holds but its consequent doesn't), and  $A$  is false. Now consider the second case, which consists in  $\neg K \Rightarrow A$  being false. Here, in a direct parallel, we know  $\neg K$  and, once again,  $\neg A$ . In both of our two cases, which are exhaustive, there is no ace in the hand. The proposition is established. **QED**

# Train-to-Princeton Problem

Everyone loves anyone who loves someone.

Larry loves Lucy.

Can you infer that everyone loves Lucy?

ANSWER:

PROOF:

# Train-to-Princeton Problem

Everyone loves anyone who loves someone.

Larry loves Lucy.

Can you infer that everyone loves Lucy?

ANSWER: Yup.

PROOF: ??



# Bringsjord I

(1) The following three assertions are either all true or all false:

If Billy helped, Doreen helped.

If Doreen helped, Frank did as well.

If Frank helped, so did Emma.

(2) The following assertion is definitely true: Billy helped.

Can it be inferred from (1) and (2) that Emma helped?

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Can it be inferred from (1) and (2) that Emma helped?

**YUP! — & now prove it!**



A **criminal genius** nearly a match for Sherlock Holmes (Do you recognize the Dr?) has built a massive hydrogen bomb, and life on Earth is hanging in the balance, hinging on whether you make the rational prediction. Dr M gives you a sporting chance to: make the right prediction, snip or not snip accordingly, and prove that you're right ...



If one of the following assertions is true then so is the other:

(1) If the red wire runs to the bomb, then the blue wire runs to the bomb; and, if the blue wire runs to the bomb, then the red wire runs to the bomb.

(2) The red wire runs to the bomb.

Given this perfectly reliable clue from Dr Moriarty, if either wire is more likely to run to the bomb, that wire *does* run to the bomb, and the bomb is ticking, with only a minute left! If both are equiprobable, neither runs to the bomb, and you are powerless. Make your prediction as to what will happen when a wire is snipped, and then make your selected snip by clicking on the wire you want to snip! Or leave well enough alone!



Red more likely.

Blue more likely.

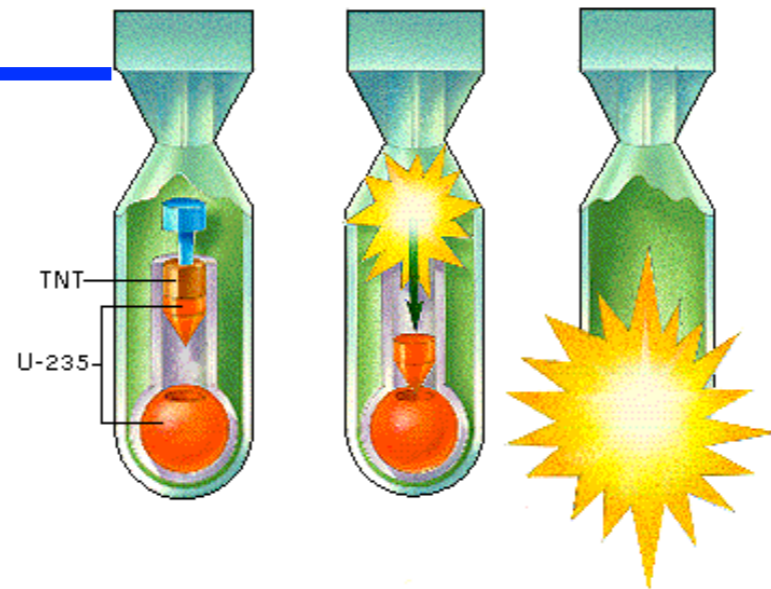
Equiprobable.

\_\_\_\_\_

\_\_\_\_\_

Snip

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Life  
on  
Earth  
has  
ended

•

advance one more  
slide to see a proof  
that you indeed made  
an irrational  
decision...

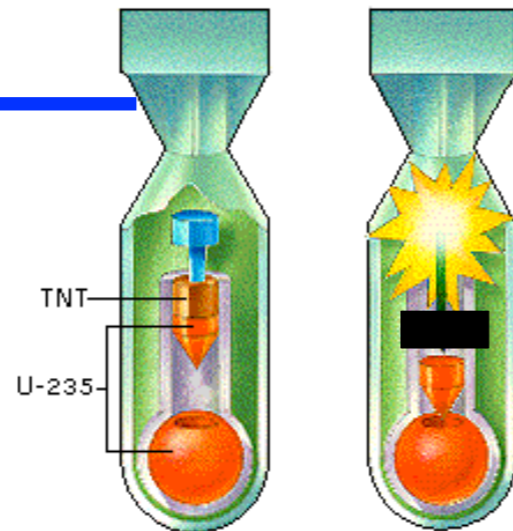
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Snip

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Life on  
Earth  
is  
saved!

*if you can now hand Dr  
M a proof that your  
decision was the rational  
one!*

Advance one more slide  
to see a proof from  
Bringsjord that yours  
had better match up to

...