Motivating Paradoxes, Puzzles, and R,
Part I

(Why Study Logic?)

Selmer Bringsjord

Intro to *(Formal)* Logic

1/22/18

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Watch brainy zoo animals figure out a box puzzle to get at food
Plenty of Tests Out There for Nonhuman Animals

E.g. search in your browser for …

trap-tube task
Background Claim

Humans, at least neurobiologically normal ones, are fundamentally rational, where rationality is constituted by certain logico-mathematically based reasoning and decision-making in response to real-world stimuli, including stimuli given in the form of focused tests; but mere animals are not fundamentally rational, since, contra Darwin, their minds are fundamentally qualitatively inferior to the human mind. As to whether computing machines/robots are fundamentally rational, the answer is “No.” For starters, if x can’t read, write, and create, x can’t be rational; computing machines/robots can neither read nor write nor create; ergo, they aren’t fundamentally rational.
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To infinity and beyond! — routinely
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\textbf{self-reference}

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Humans, at least neurobiologically normal ones, are fundamentally rational, where rationality is constituted by certain logico-mathematically based reasoning and decision-making in response to real-world stimuli, including stimuli given in the form of focused tests; but mere animals are not fundamentally rational, since, contra Darwin, their minds are fundamentally qualitatively inferior to the human mind. As to whether computational or logical minds are fundamentally rational, the answer is “No.” For starters, if a mind is purely artificial, x cannot be rational; computing machines/robots can neither read nor write nor create; ergo, they aren’t fundamentally rational.

To infinity and beyond! — routinely
Background Claim

intensional reasoning

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recursion

self-reference

To infinity and beyond! — routinely
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abstract-and-valid inference schemata

quantification

intensional reasoning

recursion

self-reference

To infinity and beyond! — routinely
Problem!
Problem!
$< \theta, \pi > \to_{L_i} \text{answer } \oplus \text{ "proof" }$
\(< \vartheta, \pi > \rightarrow_{\text{Li}} \text{"proof"}\)
$< \vartheta, \pi > \rightarrow_{\text{Li}} " \text{proof}"$
Context: Assembly (Seriated)
Selmer’s Seriated Cup Challenge, Part 1

Suppose you have at your disposal a “factory” that, upon hearing you announce a number \(j\), can quickly output a cup having a diameter of precisely \(j\) units. Can you insert a new cup between two of the seriated cups in the tower shown here? — where the \(j\) you send in must be a positive integer, \(m\) is likewise a positive integer, and every cup in every tower must be more in diameter than the one immediately above it, and less in diameter than the one immediately below it?** Prove that your answer is correct.

**E.g., if \(m = 3\), the tower in that case will have a base cup 4 units in diameter, immediately above that a cup 3 units in diameter, then a cup 2 units in diameter, and then finally a top cup of 1 unit in diameter.
Selmer’s Seriated Cup Challenge, Part I

Suppose you have at your disposal a “factory” that, upon hearing you announce a number $j$, can quickly output a cup having a diameter of precisely $j$ units. Can you insert a new cup between two of the seriated cups in the tower shown here? — where the $j$ you send in must be a positive integer, $m$ is likewise a positive integer, and every cup in every tower must be more in diameter than the one immediately above it, and less in diameter than the one immediately below it?**  Prove that your answer is correct.

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Selmer’s Seriated Cup Challenge, Part II

Suppose you have at your disposal a “factory” that, upon hearing you announce a number \( j \), can quickly output a cup having a diameter of precisely \( j \) units. Can you insert a new cup between two of the seriated cups in the tower shown here? — where the \( j \) you send in must be a positive rational number; \( k, k', k'', k''' \ldots \) are likewise positive rational numbers, and every cup in every tower must be more in diameter than the one immediately above it, and less in diameter than the one immediately below it?** Prove that your answer is correct.

**E.g., if \( k = \frac{1}{7} \), the tower in that case will have a base cup \( \frac{1}{7} \) units in diameter, immediately above that there could be a cup \( \frac{2}{7} \) units in diameter, then perhaps a cup \( \frac{3}{7} \) units in diameter, and then perhaps finally a top cup of \( \frac{4}{7} \) units in diameter.
Selmer’s Seriated Cup Challenge, Part II

Suppose you have at your disposal a “factory” that, upon hearing you announce a number \( j \), can quickly output a cup having a diameter of precisely \( j \) units. Can you insert a new cup between two of the seriated cups in the tower shown here? — where the \( j \) you send in must be a positive rational number; \( k, k', k'', k''' \ldots \) are likewise positive rational numbers, and every cup in every tower must be more in diameter than the one immediately above it, and less in diameter than the one immediately below it?** Prove that your answer is correct.

\[ j \in \mathbb{Q}^+ \text{(desired diameter of cup)} \]

\[ \frac{1}{2} \]

\[ \frac{1}{3} \]

\[ \frac{1}{4} \]

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• …
It’s White’s turn. What move did Black just make?
Aha! (Beyond Deep Blue?)
Aha! (Beyond Deep Blue?)
Simple Selection Task

E  T  4  7

Suppose I claim that the following rule is true.

If a card has a vowel on one side, it has an even number on the other side.

Which card or cards, if any, should you turn over in order to try to efficiently decide whether the rule is true or false?
Simple Selection Task

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Suppose I claim that the following rule is true.

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Which card or cards, if any, should you turn over in order to try to efficiently decide whether the rule is true or false?
Given the statements

\( \neg a \lor \neg b \)
\( b \)
\( c \rightarrow a \)

which one of the following statements must also be true?

\( c \)
\( \neg b \)
\( \neg c \)
\( h \)
\( a \)
none of the above
Given the statements

\( \neg a \lor \neg b \)
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which one of the following statements must also be true?

\( c \)
\( \neg b \)
\( \neg c \)
\( h \)
\( a \)
none of the above
Which one of the following statements is logically equivalent to the following statement: “If you are not part of the solution, then you are part of the problem.”

If you are part of the solution, then you are not part of the problem.

If you are not part of the problem, then you are part of the solution.

If you are part of the problem, then you are not part of the solution.

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Which one of the following statements is logically equivalent to the following statement: "If you are not part of the solution, then you are part of the problem."

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If you are not part of the problem, then you are not part of the solution.
Given the statements
¬¬c
¬¬c
¬¬c → a
¬a ∨ b
b → d
¬(d ∨ e)

which one of the following statements must also be true?

¬c
e
h
¬a
all of the above
Given the statements
\neg \neg c
\neg a \lor b
b \rightarrow d
\neg (d \lor e)

which one of the following statements must also be true?

\neg c
e
h
\neg a
all of the above
The Original King-Ace

Suppose that the following premise is true:

If there is a king in the hand, then there is an ace in the hand, or else if there isn’t a king in the hand, then there is an ace.

What can you infer from this premise?
The Original King-Ace

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There is an ace in the hand.
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NO! There is an ace in the hand.
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What can you infer from this premise?

NO! There is an ace in the hand. NO!
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Suppose that the following premise is true:

If there is a king in the hand, then there is an ace in the hand, or else if there isn’t a king in the hand, then there is an ace.

What can you infer from this premise?

NO! There is an ace in the hand. NO!

In fact, what you can infer is that there isn’t an ace in the hand!
Suppose that the following premise is true:

*If there is a king in the hand, then there is an ace in the hand; or if there isn’t a king in the hand, then there is an ace; but not both of these if-then statements are true.*

What can you infer from this premise?
Suppose that the following premise is true:

If there is a king in the hand, then there is an ace in the hand; or if there isn’t a king in the hand, then there is an ace; but not both of these if-then statements are true.

What can you infer from this premise?

There is an ace in the hand.
Suppose that the following premise is true:

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- There is an ace in the hand.
King-Ace 2

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What can you infer from this premise?

**NO!** There is an ace in the hand.
Suppose that the following premise is true:

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What can you infer from this premise?

**NO!**  There is an ace in the hand.  **NO!**
Suppose that the following premise is true:

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What can you infer from this premise?

**NO!** There is an ace in the hand. **NO!**

In fact, what you *can* infer is that there isn’t an ace in the hand!