Propositional Calculus III: 
*Reductio ad Absurdum*

Selmer Bringsjord

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Troy, New York 12180 USA

Intro to Logic  
2/8/2018; & again 2/12/18 by Rini Palamittam
Logistics ...
Any takers?

Logistics ...
Any takers? NO

Logistics ...
Any takers? NO
Logistics ...
Any takers? NO

Logistics ...

Note: Should now have laptop with you and ready to go with Slate installed.
Any takers? NO

Logistics ... 

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And ... HyperGrader will debut in class on Feb 12, led by Rini.
Any takers? NO

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http://www.logicamodernapproach.com
Logistics ...

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Plenty of problems there to *already* learn/practice on!
Your code for Slate & HyperGrader for the semester:
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18450
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To submit purportedly correct proofs to HyperGrader, you will authenticate with your email address & your code.
Save sleeve & CD, snapshot sleeve & archive!!
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- larry_lucy_preview.slt (799 bytes, Document)
- oracle1_and_oracle2_firsttime.slt (2 KB, Document)
- Pand-RandQ--2--PandQ-andR.slt (472 bytes, Document)
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- **Mac OS**
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- HyperGrader

Complete, sign, email pdf to Selmer.Bringsjord@gmail.com
Again: Initial Steps
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• Snapshot sleeve with code, and archive.
Again: Initial Steps

- Snapshot sleeve with code, and archive.
- Copy the folder from CD to your laptop.
Again: Initial Steps

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- Eject CD and “bank-vault”-save both!
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• Depending upon whether you’re Windows or MacOS, expand the relevant zipped file to obtain Slate.
• Open Slate
• Today I’ll continue to explain and show inference rules in Slate.
Propositional Calculus III: 
*Reductio ad Absurdum*

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Intro to Logic
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Reductio ...
“Reductio ad absurdum, which Euclid loved so much, is one of a mathematician's finest weapons. It is a far finer gambit than any chess gambit: a chess player may offer the sacrifice of a pawn or even a piece, but a mathematician offers the game.”

—G. H. Hardy
A Greek-shocking Example …
\frac{p}{q}
What are rational numbers?

\[ \frac{p}{q} \]
What are rational numbers?

• Any number that can be expressed in the form of \( \frac{p}{q} \) such that we have a numerator \( p \) and a non-zero denominator.

• Rational numbers are a subset of real numbers!

• Examples of irrational numbers?
What are rational numbers?

• Any number that can be expressed in the form of \( \frac{p}{q} \) such that we have a numerator \( p \) and a non-zero denominator.

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• Examples of irrational numbers?

\[ \sqrt{2} \]
What are rational numbers?

- Any number that can be expressed in the form of $\frac{p}{q}$ such that we have a numerator $p$ and a non-zero denominator.

- Rational numbers are a subset of real numbers!

- Examples of irrational numbers?

\[ \sqrt{2} \quad e \]
Prove that:
Prove that:

$$\sqrt{2}$$
Prove that:

$$\sqrt{2} \text{ is irrational}$$
Suppose $\sqrt{2}$ is rational. That means it can be written as the ratio of two integers $p$ and $q$

$$\sqrt{2} = \frac{p}{q} \tag{1}$$

where we may assume that $p$ and $q$ have no common factors. (If there are any common factors we cancel them in the numerator and denominator.) Squaring in (1) on both sides gives

$$2 = \frac{p^2}{q^2} \tag{2}$$

which implies

$$p^2 = 2q^2 \tag{3}$$

Thus $p^2$ is even. The only way this can be true is that $p$ itself is even. But then $p^2$ is actually divisible by 4. Hence $q^2$ and therefore $q$ must be even. So $p$ and $q$ are both even which is a contradiction to our assumption that they have no common factors. The square root of 2 cannot be rational!
What are prime numbers?
What are prime numbers?

• A number that can be divided by only two numbers, one and itself.

• Must be a whole number.

• Example: 2, 3, 5, 7….
And recall: Euclidean “Magic”

**Theorem:** There are infinitely many primes.

**Proof:** We take an indirect route. Let $\Pi = p_1 = 2, p_2 = 3, p_3 = 5, \ldots, p_k$ be a finite, exhaustive consecutive sequence of prime numbers. Next, let $M_{\Pi}'$ be $p_1 \times p_2 \times \cdots \times p_k$, and set $M_{\Pi}'$ to $M_{\Pi} + 1$. Either $M_{\Pi}'$ is prime, or not; we thus have two (exhaustive) cases to consider.

**C1** Suppose $M_{\Pi}'$ is prime. In this case we immediately have a prime number beyond any in $\Pi$ — contradiction!

**C2** Suppose on the other hand that $M_{\Pi}'$ is not prime. Then some prime $p$ divides $M_{\Pi}'$. (Why?) Now, $p$ itself is either in $\Pi$, or not; we hence have two sub-cases. Supposing that $p$ is in $\Pi$ entails that $p$ divides $M_{\Pi}$. But we are operating under the supposition that $p$ divides $M_{\Pi}'$ as well. This implies that $p$ divides 1, which is absurd (a contradiction). Hence the prime $p$ is outside $\Pi$.

Hence for any such list $\Pi$, there is a prime outside the list. That is, there are infinitely many primes. **QED**
And recall: Euclidean “Magic”

**Theorem:** There are infinitely many primes.

**Proof:** We take an indirect route. Let $\Pi = p_1 = 2, p_2 = 3, p_3 = 5, \ldots, p_k$ be a finite, exhaustive consecutive sequence of prime numbers. Next, let $M_{\Pi}^\prime$ be $p_1 \times p_2 \times \cdots \times p_k$, and set $M_{\Pi}^\prime$ to $M_{\Pi} + 1$. Either $M_{\Pi}^\prime$ is prime, or not; we thus have two (exhaustive) cases to consider.

C1 Suppose $M_{\Pi}^\prime$ is prime. In this case we immediately have a prime number beyond any in $\Pi$ — contradiction!

C2 Suppose on the other hand that $M_{\Pi}^\prime$ is *not* prime. Then some prime $p$ divides $M_{\Pi}^\prime$. *(Why?)* Now, $p$ itself is either in $\Pi$, or not; we hence have two sub-cases. Supposing that $p$ is in $\Pi$ entails that $p$ divides $M_{\Pi}$. But we are operating under the supposition that $p$ divides $M_{\Pi}^\prime$ as well. This implies that $p$ divides 1, which is absurd (a contradiction). Hence the prime $p$ is outside $\Pi$.

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**C2** Suppose on the other hand that \( M'_{\Pi} \) is *not* prime. Then some prime \( p \) divides \( M'_{\Pi} \). (Why?) Now, \( p \) itself is either in \( \Pi \), or not; we hence have two sub-cases. Supposing that \( p \) is in \( \Pi \) entails that \( p \) divides \( M_{\Pi} \). But we are operating under the supposition that \( p \) divides \( M'_{\Pi} \) as well. This implies that \( p \) divides \( 1 \), which is absurd (a contradiction). Hence the prime \( p \) is outside \( \Pi \).

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C1 Suppose \( M''_\Pi \) is prime. In this case we immediately have a prime number \( p \) not in \( \Pi \). Hence for \( \Pi \), there is a prime outside the list. That is, there are infinitely many primes. QED

C2 Suppose on the other hand that \( M''_\Pi \) is not prime. Then some prime \( p \) divides \( M'_\Pi \). (Why?) Now, \( p \) itself is either in \( \Pi \), or not; we hence have two sub-cases. Supposing that \( p \) is in \( \Pi \) entails that \( p \) divides \( M'_\Pi \). But we are operating under the supposition that \( p \) divides \( M''_\Pi \) as well. This implies that \( p \) divides 1, which is absurd (a contradiction). Hence the prime \( p \) is outside \( \Pi \).

Study it word by work until you endorse it with your very soul!
From Algebra 2 in High School …
(Pearson Common-Core Compliant Textbook)
A proof involving indirect reasoning is an **indirect proof**. Often in an indirect proof, a statement and its negation are the only possibilities. When you see that one of these possibilities leads to a conclusion that contradicts a fact you know to be true, you can eliminate that possibility. For this reason, indirect proof is sometimes called _proof by contradiction_.

**TAKE NOTE  Key Concept**

**Writing an Indirect Proof**

**Step 1** State as a temporary assumption the opposite (negation) of what you want to prove.

**Step 2** Show that this temporary assumption leads to a contradiction.

**Step 3** Conclude that the temporary assumption must be false and that what you want to prove must be true.
Problem 3  Writing an Indirect Proof

Proof

Given:  \( \triangle ABC \) is scalene.

Prove:  \( \angle A, \angle B, \) and \( \angle C \) all have different measures.

THINK

Assume temporarily the opposite of what you want to prove.

Show that this assumption leads to a contradiction.

Conclude that the temporary assumption must be false and that what you want to prove must be true.

WRITE

Assume temporarily that two angles of \( \triangle ABC \) have the same measure. Assume that \( m\angle A = m\angle B \).

By the Converse of the Isosceles Triangle Theorem, the sides opposite \( \angle A \) and \( \angle B \) are congruent. This contradicts the given information that \( \triangle ABC \) is scalene.

The assumption that two angles of \( \triangle ABC \) have the same measure must be false. Therefore, \( \angle A, \angle B, \) and \( \angle C \) all have different measures.
Problem 3  Writing an Indirect Proof

Proof

Given:  \( \triangle ABC \) is scalene.

Prove:  \( \angle A, \angle B, \) and \( \angle C \) all have different measures.

THINK

Assume temporarily the opposite of what you want to prove.

Show that this assumption leads to a contradiction.

Conclude that the temporary assumption must be false and that what you want to prove must be true.

WRITE

Assume temporarily that two angles of \( \triangle ABC \) have the same measure. Assume that \( m\angle A = m\angle B \).

By the Converse of the Isosceles Triangle Theorem, the sides opposite \( \angle A \) and \( \angle B \) are congruent. This contradicts the given information that \( \triangle ABC \) is scalene.

The assumption that two angles of \( \triangle ABC \) have the same measure must be false. Therefore, \( \angle A, \angle B, \) and \( \angle C \) all have different measures.
Explosion

GIVEN. $\varphi \land \neg \varphi$

{GIVEN} Assume ✓

PC ← ✓

GOAL. $\psi$
{GIVEN}
Explosion: Partial Proof Plan

\[
\text{GIVEN. } \varphi \land \lnot \varphi \\
\{\text{GIVEN} \} \text{ Assume } \checkmark
\]

3. \(\lnot \psi\) \\
\{3\} \text{ Assume } \checkmark

4. \(\varphi\) \\
\land \text{ elim } \checkmark

5. \(\lnot \varphi\) \\
\land \text{ elim } \checkmark

\sim \text{ elim } \checkmark

\text{GOAL. } \psi
GreenCheeseMoon I

GIVEN. \( \neg(P \rightarrow (Q \rightarrow P)) \)

\{GIVEN\} Assume ✓

PC ✓

GOAL. G

\{GIVEN\}
GreenCheeseMoon1: Partial Proof Plan

GIVEN. \( \neg(P \rightarrow (Q \rightarrow P)) \)
\( \text{(GIVEN) Assume } \checkmark \)

5. \( P \)
\( \text{(5) Assume } \checkmark \)

3. \( P \rightarrow (Q \rightarrow P) \)
\( \text{PC } \checkmark \)

4. \( \neg G \)
\( \text{(4) Assume } \checkmark \)

GOAL. \( G \)
\( \neg \text{ elim } \checkmark \)