#### "Cognitive" Deductive Shots @ R

Are Humans Rational?
RPI

Selmer Bringsjord & Atriya Sen

9.12.19

<sup>\*</sup>Makes use of such mental verbs as believing, knowing, etc.

If humans are as described in this thesis, then they can solve the forthcoming "cognitive" problems.

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Sorry Selmer & company, your thesis  $\mathcal R$  is false.

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#### Floridi's Continuum (augmented), and Claims

("Consciousness, Agents, and the Knowledge Game" Minds & Machines)

	False Belief Task	Wise Man Test (n)	Deafening Test	Torture Boots Test	Ultimate Sifter	Infinitary False Belief Task
Cutting-Edge AI	Yes	Yes	No	No	No	?
Zombies	Yes	Yes	Yes	Yes	No	?
Human Persons (s-conscious! p-conscious!)	Yes	Yes	Yes	Yes	Yes	Yes

## Cracking False-Belief Tasks ...

#### In SL, w/ real-time comm w/ ATP



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```
(Refutation
(Row 1
  (or (not (person ?x)) (not (object ?y))
(not (container ?z)) (not (in ?y ?z))
(bel-in ?x ?y ?z))
  assertion)
(Row 2
  (or (not (person ?x))
       (not (container ?y))
       (not (object ?z))
       (not (w-retrieve ?x ?z))
       (not (bel-in ?x ?z ?y))
       (t-retrieve ?x ?z ?y))
  assertion)
(Row 4
  (person subject)
  assertion)
(Row 6
  (container c2)
  assertion)
(Row 7
   (object teddybear)
  assertion)
(Row 8
```

```
(in teddybear c2)
   assertion)
(Row 9
   (w-retrieve subject teddybear)
   assertion)
(Row 10
   (not (t-retrieve subject teddybear ?x))
   negated conjecture
   Answer (looks-in ?x))
(Row 11
   (or (not (person ?x)) (bel-in ?x
teddybear c2))
   (rewrite (resolve 1 8) 6 7))
(Row 25
   (bel-in subject teddybear c2)
   (resolve 11 4))
(Row 28
   (t-retrieve subject teddybear c2)
   (rewrite (resolve 2 25) 9 7 6 4))
(Row 30
   false
   (resolve 10 28)
   Answer (looks-in c2)))
:PROOF-FOUND
SNARK-USER 15 > (answer t)
(LOOKS-IN C2)
```

```
(Refutation
(Row 1
   (or (not (person ?x))
       (not (container ?y))
       (not (object ?z))
       (not (w-retrieve ?x ?z))
       (not (bel-in ?x ?z ?y))
       (t-retrieve ?x ?z ?y))
   assertion)
(Row 2
   (or (not (person ?x)) (not (object ?
y)) (not (container ?z)) (not (p-in ?x ?y
?z)) (bel-in ?x ?y ?z))
   assertion)
(Row 4
   (person subject)
   assertion)
(Row 5
   (container c1)
   assertion)
(Row 7
   (object teddybear)
  assertion)
(Row 8
   (p-in subject teddybear c1)
```

```
assertion)
(Row 9
   (w-retrieve subject teddybear)
   assertion)
(Row 10
   (not (t-retrieve subject teddybear ?
x))
   negated conjecture
  Answer (looks-in ?x))
(Row 11
   (bel-in subject teddybear c1)
   (rewrite (resolve 2 8) 5 7 4))
(Row 25
   (t-retrieve subject teddybear c1)
   (rewrite (resolve 1 11) 9 7 5 4))
(Row 26
   false
   (resolve 10 25)
  Answer (looks-in c1))
:PROOF-FOUND
SNARK-USER 13 > (answer t)
(LOOKS-IN C1)
```

"The present account of the false belief transition is incomplete in important ways. After all, our agent had only to choose the best of two known models. This begs an understanding of the dynamics of rational revision near threshold and when the space of possible models is far larger. Further, a single formal model ought ultimately to be applicable to many false belief tasks, and to reasoning about mental states more generally. Several components seem necessary to extend a particular theory of mind into such a framework theory: a richer representation for the propositional content and attitudes in these tasks, extension of the implicit quantifier over trials to one over situations and people, and a broader view of the probability distributions relating mental state variables. Each of these is an important direction for future research."

"Intuitive Theories of Mind: A Rational Approach to False Belief" Goodman et al.

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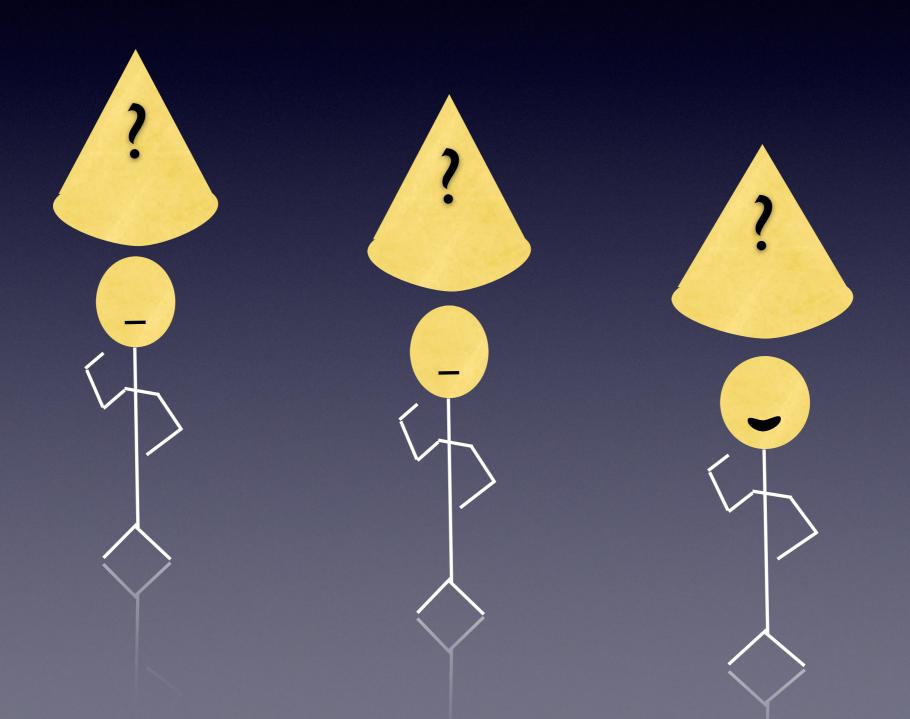
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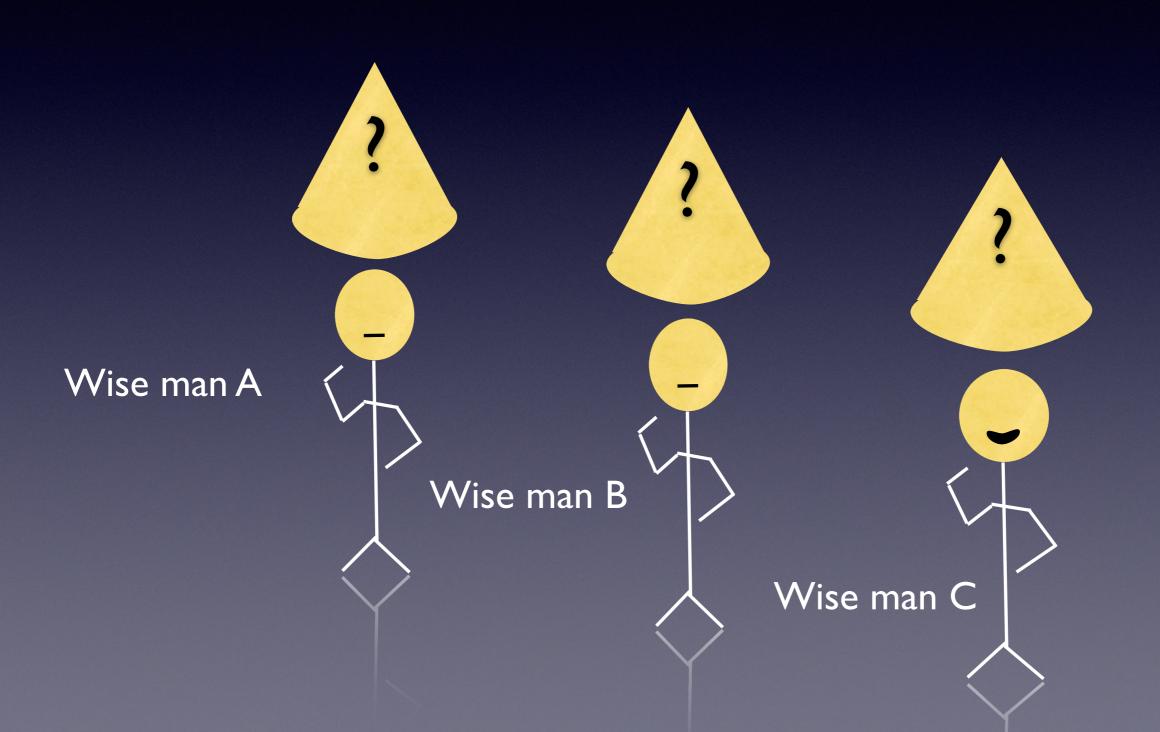
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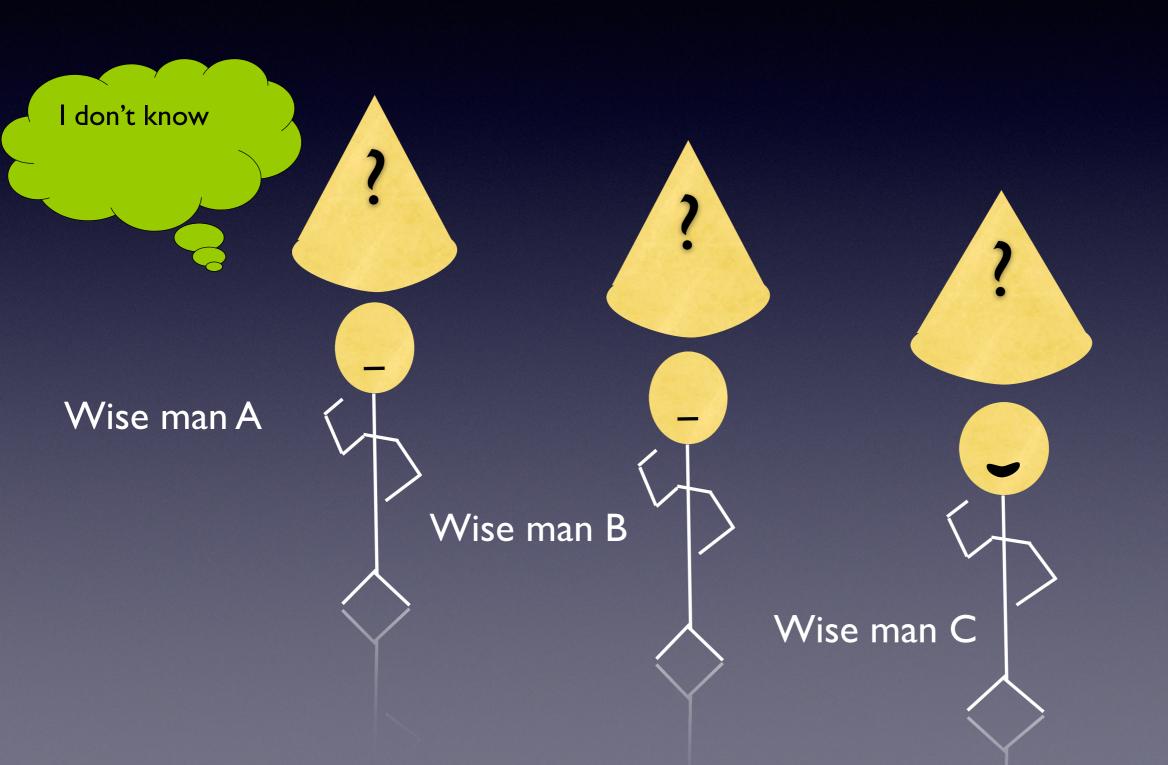
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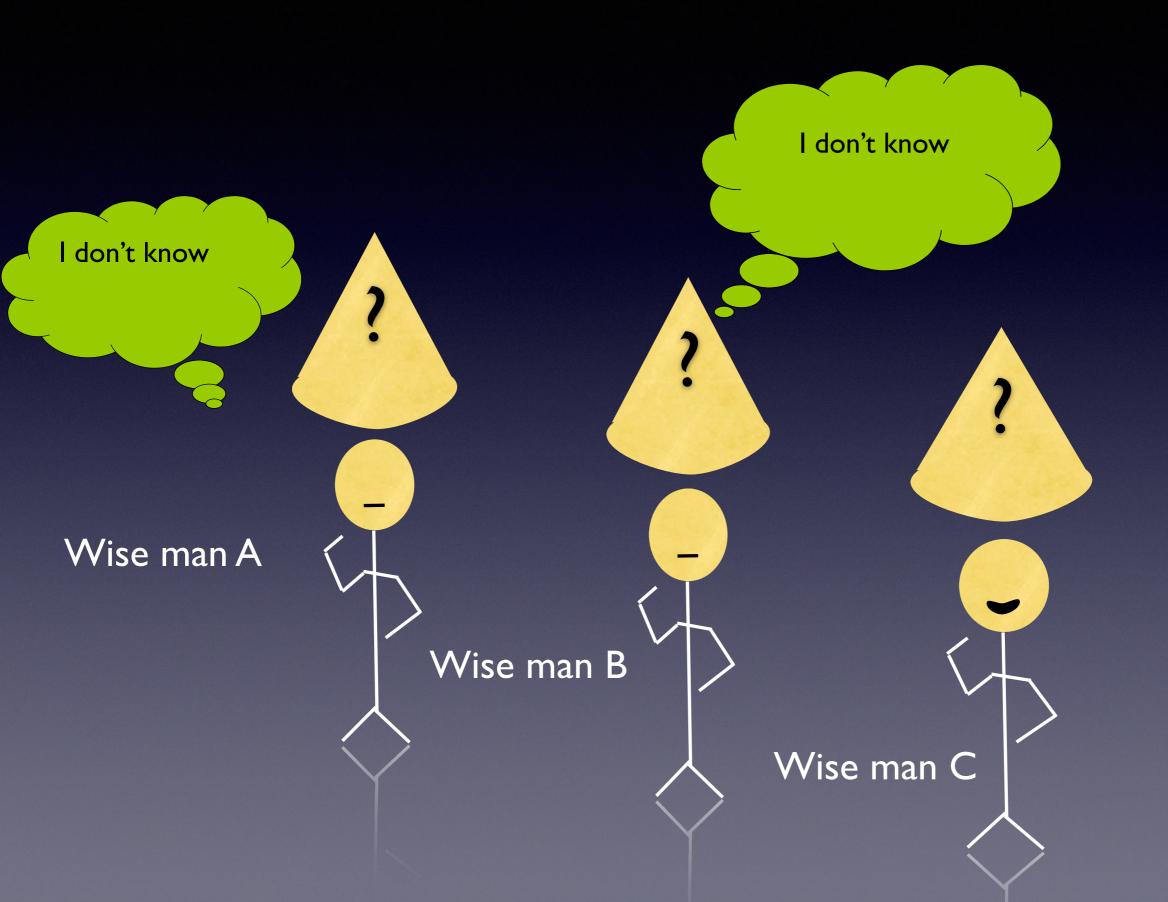
Done.

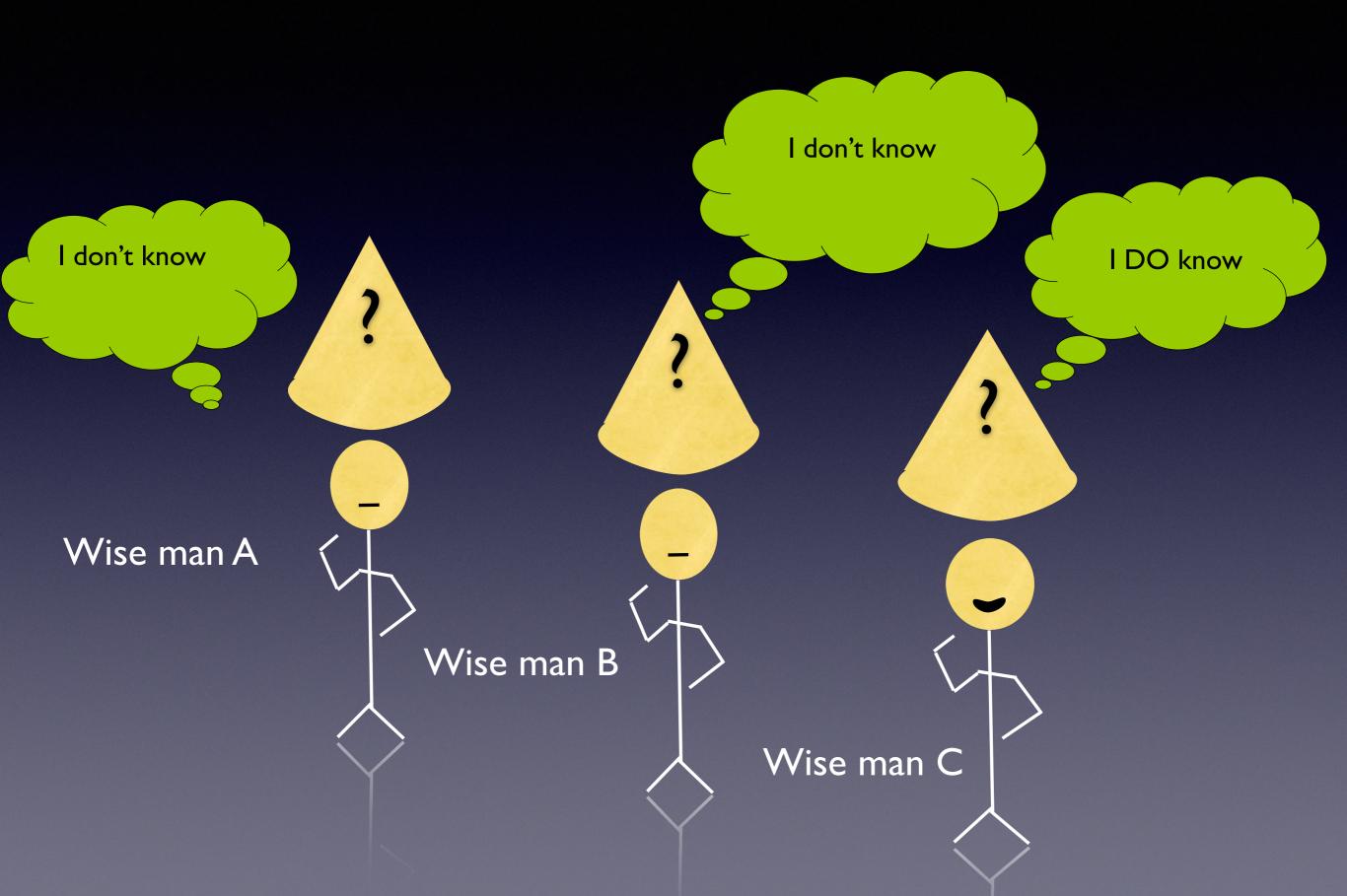
# Cracking Wise Man Tests ...

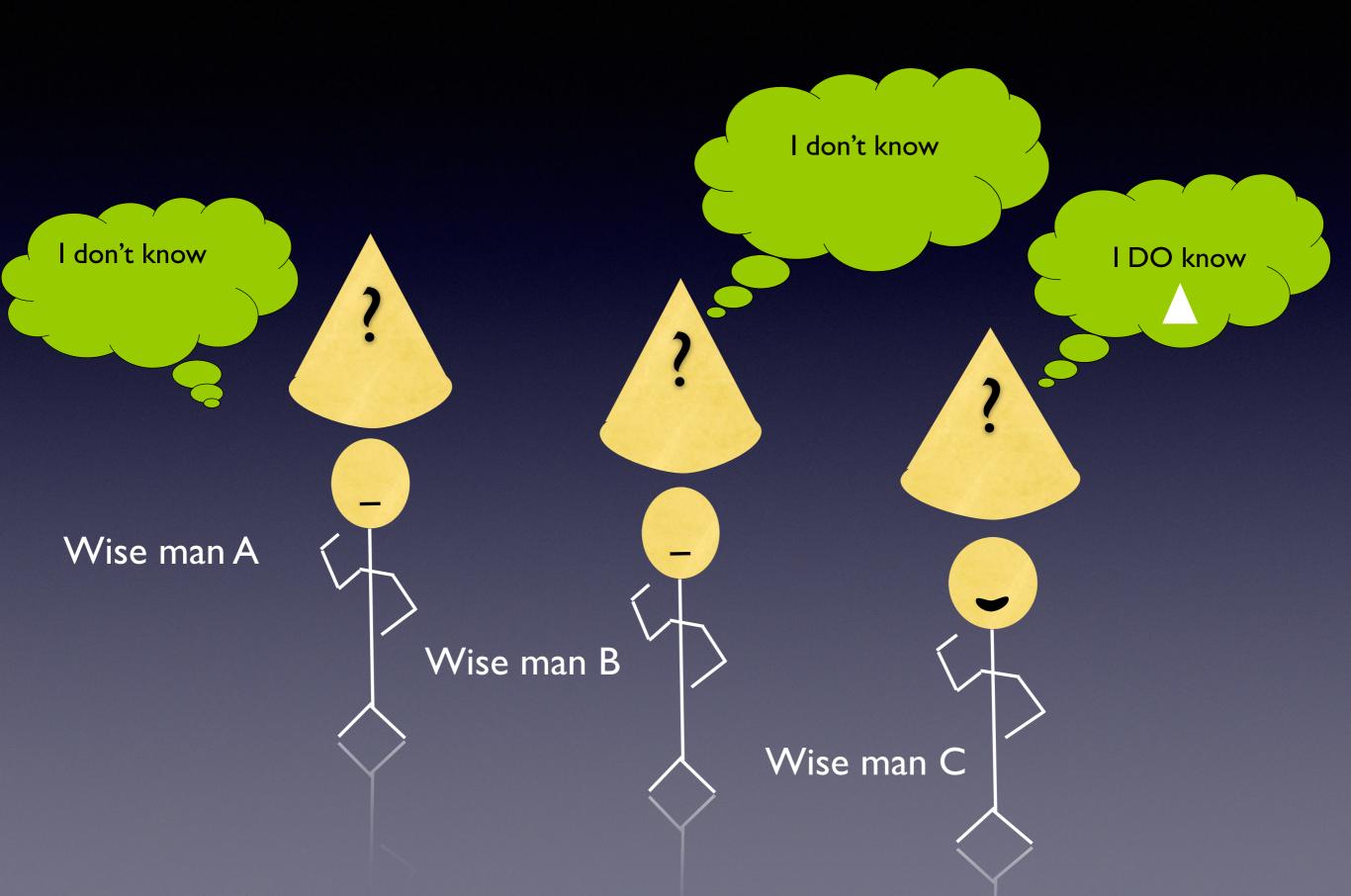


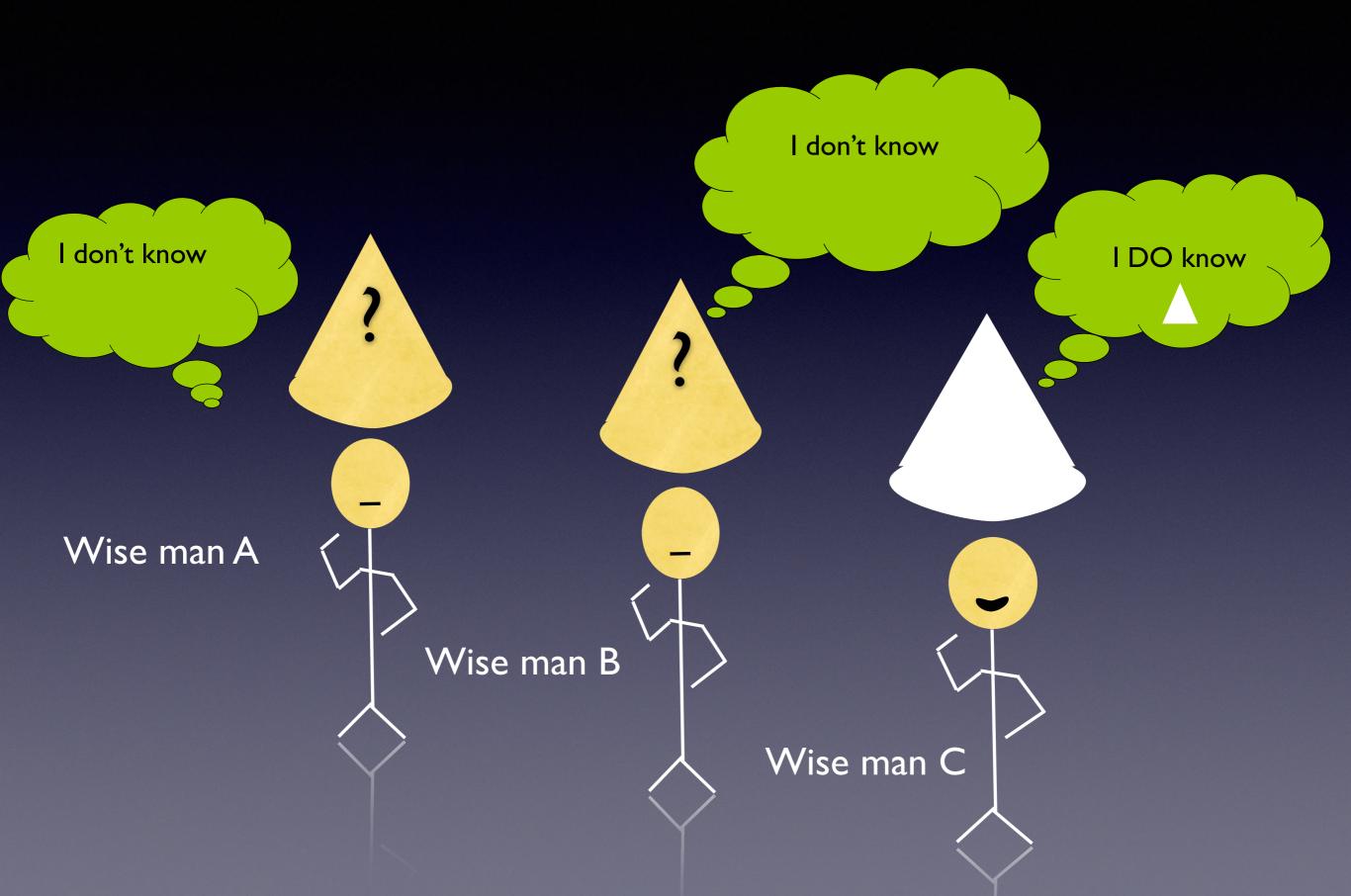












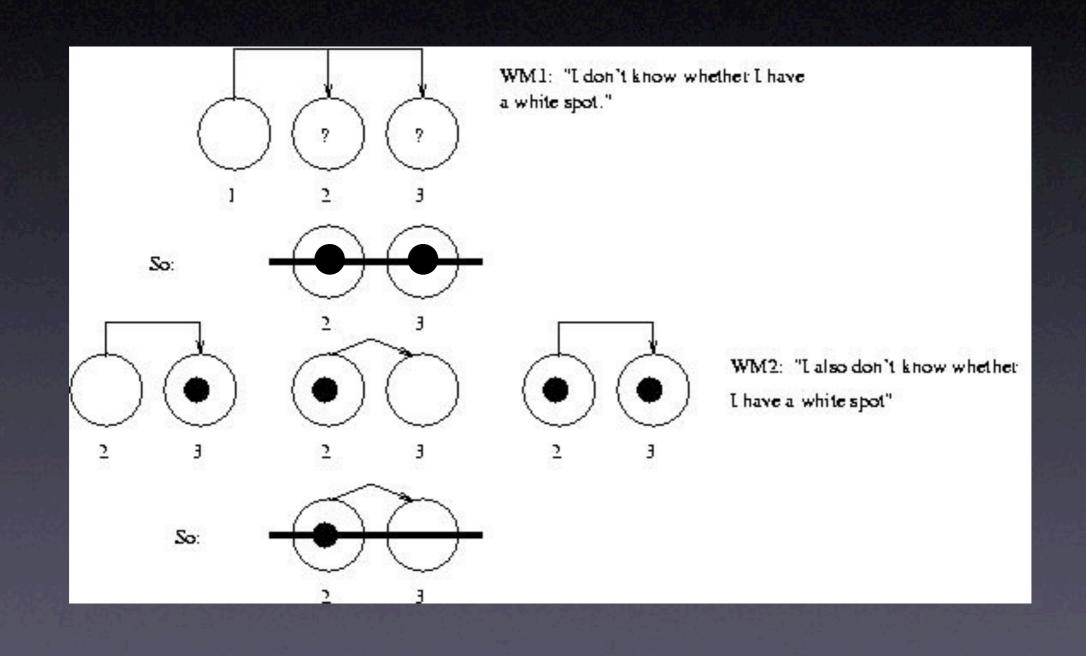
#### Proof from WM3

**Proposition**: I have a white fez.

Proof: Remember as a first fact that we all know that at least one of us has a white fez. When the first wise man says that he doesn't know, I immediately know that either WM2 has a white fez, or I do, or both of us does. I know this because if neither of us has a whilte fez, WMI would have said immediately that in light of our first fact, he has a white fez. My next piece of info comes from what WM2 says; he says that he is also ignorant. Now, if he had seen no white fez on my head, he would have immediately said "I have a white fez!" (He would have said this because after WMI spoke, he carried out the same reasoning I did, and hence ruled out the (WM2-bf & WM3-bf) permutation.) But this isn't what he said. Hence, I do have a white fez on my head. QED

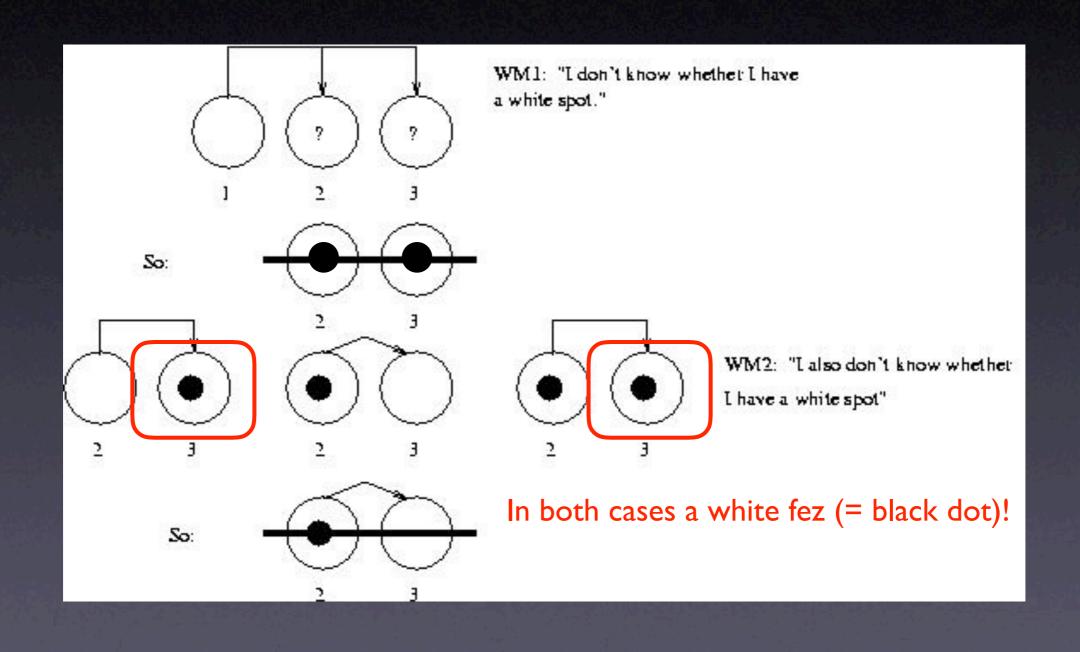
#### Diagrammatic Version of Reasoning in WMP3

(pov of truly wise man; easy for rational humans)



#### Diagrammatic Version of Reasoning in WMP3

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Metareasoning for multi-agent epistemic logics

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Abstract. We present an encoding of a sequent calculus for a multiagent epistemic logic in Athena, an interactive theorem proving system for many-sorted first-order logic. We then use Athena as a metalanguage in order to reason about the multi-agent logic an as object language. This facilitates theorem proving in the multi-agent logic in several ways. First, it lets us marshal the highly efficient theorem provers for classical first-order logic that are integrated with Athena for the purpose of doing proofs in the multi-agent logic. Second, unlike model-theoretic embeddings of modal logics into classical first-order logic, our proofs are directly convertible into native epistemic logic proofs. Third, because we are able to quantify over propositions and agents, we get nuch of the generality and power of higher-order logic even though we are in a firstorder setting. Finally, we are able to use Athena's versatile tactics for proof automation in the multi-agent logic. We illustrate by developing a tactic for solving the generalized version of the wise men problem.

#### 1 Introduction

Multi-agent modal logies are widely used in Computer Science and AI. Multi-agent epistemic logies, in particular, have found applications in fields ranging from AI domains such as roboties, planning, and motivation analysis in natural language [13]; to negotiation and game theory in economics, to distributed systems analysis and protocol authentication in computer security [16, 31]. The reason is simple—intelligent agents must be able to reason about knowledge. It is therefore important to have efficient means for performing machine reasoning in such logies. While the validity problem for most propositional modal logies is of intractable theoretical complexity<sup>1</sup>, several approaches have been investigated in recent years that have resulted in systems that appear to work well in practice. These approaches include tableau-based provers, SAT-based algorithms, and translations to first-order logic coupled with the use of resolution-based automated theorem provers (ATPs). Some representative systems are FaCT [24], KsatC [14], TA [25], LWB [23], and MSPASS [37].

Translation-based approaches (such as that of MSPASS) have the advantage

Translation-based approaches (such as that of MSPASS) have the advantage of leveraging the tremendous implementation progress that has occurred over

All our human-authored proofs machine-checked.

#### Arkoudas-Proved-Sound Algorithm for Generating Proof-Theoretic Solution to WMP<sub>n</sub>

Metareasoning for multi-agent epistemic logics

5

Metareasoning for multi-agent epistemic logics

7

```
\begin{array}{c} \Gamma \vdash |K_{\alpha}(P \Rightarrow Q)| \Rightarrow [K_{\alpha}(P) \Rightarrow K_{\alpha}(Q)] \end{array}^{[K]} & \overline{\Gamma \vdash K_{\alpha}(P) \Rightarrow P}^{[T]} \\ & \frac{\emptyset \vdash P}{\Gamma \vdash C(P)} [C\text{-}I] & \overline{\Gamma \vdash C(P) \Rightarrow K_{\alpha}(P)} [C\text{-}E] \\ \\ & \overline{\Gamma \vdash |C(P \Rightarrow Q)|} \Rightarrow [C(P) \Rightarrow C(Q)] \end{array}^{[C_{K}]} & \overline{\Gamma \vdash C(P) \Rightarrow C(K_{\alpha}(P))}^{[R]} \end{array}
```

Fig. 2. Inference rules for the epistemic operators.

is  $\Gamma \vdash P$ . Intuitively, this is a judgment stating that P follows from  $\Gamma$ . We will write  $P, \Gamma$  (or  $\Gamma, P$ ) as an abbreviation for  $\Gamma \cup \{P\}$ . The sequent calculus that we will use consists of a collection of inference rules for deriving judgments of the form  $\Gamma \vdash P$ . Figure 1 shows the inference rules that deal with the standard propositional connectives. This part is standard (e.g., it is very similar to the sequent calculus of Ebbinghaus et al. [15]). In addition, we have some rules pertaining to  $K_{\alpha}$  and C, shown in Figure 2.

Rule [K] is the sequent formulation of the well-known Kripke axiom stating that the knowledge operator distributes over conditionals. Rule  $[C_K]$  is the corresponding principle for the common knowledge operator. Rule [T] is the "truth axiom": an agent cannot know false propositions. Rule  $[C_I]$  is an introduction rule for common knowledge: if a proposition P follows from the empty set of hypotheses, i.e., if it is a tautology, then it is commonly known. This is the common-knowledge version of the "omniscience axiom" for single-agent knowledge which says that  $\Gamma \vdash K_{\alpha}(P)$  can be derived from  $\emptyset \vdash P$ . We do not need to postulate that axiom in our formulation, since it follows from [C-I] and [C-E]. The latter says that if it is common knowledge that P then any (every) agent knows P, while [R] says that if it is common knowledge that P then it is common knowledge.

$$C(P \Rightarrow E(P)) \Rightarrow [P \Rightarrow C(P)]$$

where E is the shared-knowledge operator. Since we do not need E for our purposes, we omit its formalization and "unfold" C via rule [R] instead. We state a few lemmas that will come handy later:

Lemma 1 (Cut). If  $\Gamma_1 \vdash P_1$  and  $\Gamma_2, P_1 \vdash P_2$  then  $\Gamma_1 \cup \Gamma_2 \vdash P_2$ .

Proof: Assume  $\Gamma_1 \vdash P_1$  and  $\Gamma_2, P_1 \vdash P_2$ . Then, by  $[\Rightarrow -I]$ , we get  $\Gamma_2 \vdash P_1 \Rightarrow P_2$ . Further, by dilution, we have  $\Gamma_1 \cup \Gamma_2 \vdash P_1 \Rightarrow P_2$  and  $\Gamma_1 \cup \Gamma_2 \vdash P_1$ . Hence, by  $[\Rightarrow -E]$ , we obtain  $\Gamma_1 \cup \Gamma_2 \vdash P_2$ .

The proofs of the remaining lemmas are equally simple exercises:

```
R_1 \wedge R_2 \wedge R_3 \} \vdash R_1
                                                                        [Reflex], \land-E_1
R_1 \wedge R_2 \wedge R_3 \} \vdash R_2
                                                                         [Reflex], \land-E<sub>1</sub>, \land-E<sub>2</sub>
R_1 \wedge R_2 \wedge R_3 \} \vdash R_3
                                                                         [Reflex], \land-E_2
R_1 \wedge R_2 \wedge R_3} \vdash K_{\alpha}(\neg Q) \Rightarrow K_{\alpha}(P)
                                                                        2, [K], \Rightarrow -E
                                                                        3, 4, Lemma 2
R_1 \wedge R_2 \wedge R_3} \vdash \neg Q \Rightarrow K_{\alpha}(P)
R_1 \wedge R_2 \wedge R_3} \vdash \neg K_{\alpha}(P) \Rightarrow \neg \neg Q
                                                                        Lemma 3
R_1 \wedge R_2 \wedge R_3 \} \vdash \neg \neg Q
                                                                        6, 1, ⇒-E
\{l_1 \land R_2 \land R_3\} \vdash Q
                                                                        7, [¬-E]
```

at the above proof is not entirely low-level because most steps combine nore inference rule applications in the interest of brevity.

a 7. Consider any agent α and propositions P,Q. Define R<sub>1</sub> and R<sub>3</sub> emma 6, let R<sub>2</sub> = P ∨ Q, and let S<sub>i</sub> = C(R<sub>i</sub>) for i = 1,2,3. Then S<sub>3</sub>} ⊢ C(Q).

Let  $R'_2 = \neg Q \Rightarrow P$  and consider the following derivation:

```
S_1, S_2, S_3\} \vdash S_1
                                                                                             Reflex
S_1, S_2, S_3 \} \vdash S_2
                                                                                             Reflex
                                                                                             Reflex
S_1, S_2, S_3\} \vdash S_3
\vdash (P \lor Q) \Rightarrow (\neg Q \Rightarrow P)
                                                                                            Lemma 4a
S_1, S_2, S_3\} \vdash C((P \lor Q) \Rightarrow (\neg Q \Rightarrow P))
                                                                                            4, [C-I]
                                                                                            5, [C_K], [\Rightarrow -E]
S_1, S_2, S_3} \vdash C(P \lor Q) \Rightarrow C(\neg Q \Rightarrow P)
S_1, S_2, S_3} \vdash C(\neg Q \Rightarrow P)
                                                                                            6, 2, [\Rightarrow -E]
S_1, S_2, S_3} \vdash C(\neg Q \Rightarrow P) \Rightarrow C(K_{\alpha}(\neg Q \Rightarrow P))
S_1, S_2, S_3} \vdash C(K_{\alpha}(\neg Q \Rightarrow P))
                                                                                            8, 7, [\Rightarrow -E]
R_1 \wedge K_{\alpha}(\neg Q \Rightarrow P) \wedge R_3\} \vdash Q
                                                                                            Lemma 6
\vdash (R_1 \land K_{\alpha}(\neg Q \Rightarrow P) \land R_3) \Rightarrow Q
                                                                                            10. [⇒-I]
                                                                                            11, [C-I]
S_1, S_2, S_3} \vdash C((R_1 \land K_{\alpha}(\neg Q \Rightarrow P) \land R_3) \Rightarrow Q)
S_1, S_2, S_3} \vdash C(R_1 \land K_{\alpha}(\neg Q \Rightarrow P) \land R_3) \Rightarrow C(Q)
                                                                                            12, [C_K], [\Rightarrow -E]
S_1, S_2, S_3} \vdash C(R_1 \land K_{\alpha}(\neg Q \Rightarrow P) \land R_3)
                                                                                             1, 3, 9, Lemma 5, [∧-I]
S_1, S_2, S_3\} \vdash C(Q)
                                                                                             13, 14, [⇒-E]
```

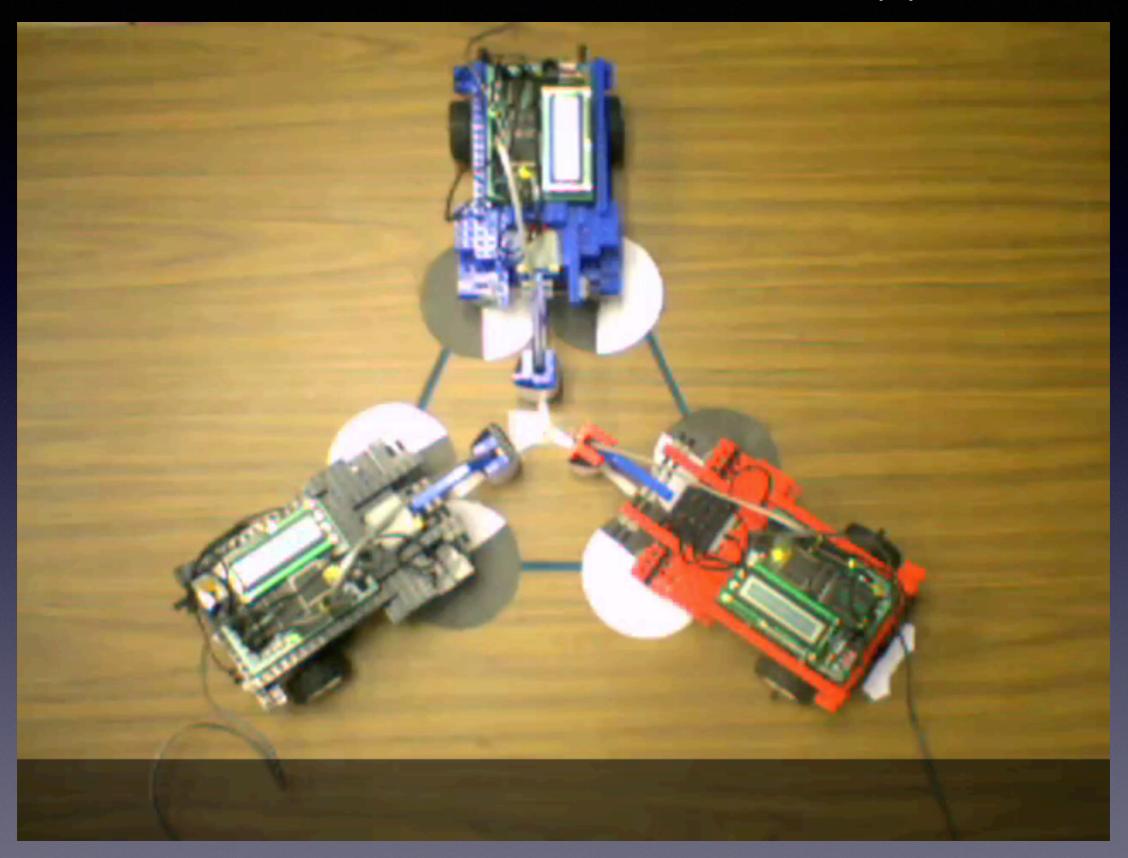
all  $n \ge 1$ , it turns out that the last— $(n+1)^{st}$ —wise man knows he is . The case of two wise men is simple. The reasoning runs essentially by iction. The second wise man reasons as follows:

pose I were not marked. Then  $w_1$  would have seen this, and knowing : at least one of us is marked, he would have inferred that he was marked one. But  $w_1$  has expressed ignorance; therefore, I must be

r now the case of n=3 wise men  $w_1,w_2,w_3$ . After  $w_1$  amounces that not know that he is marked,  $w_2$  and  $w_3$  both infer that at least one of marked. For if neither  $w_2$  nor  $w_3$  were marked,  $w_1$  would have seen this uld have concluded—and stated—that he was the marked one, since he hat at least one of the three is marked. At this point the puzzle reduces wo-men case: both  $w_2$  and  $w_3$  know that at least one of them is marked,

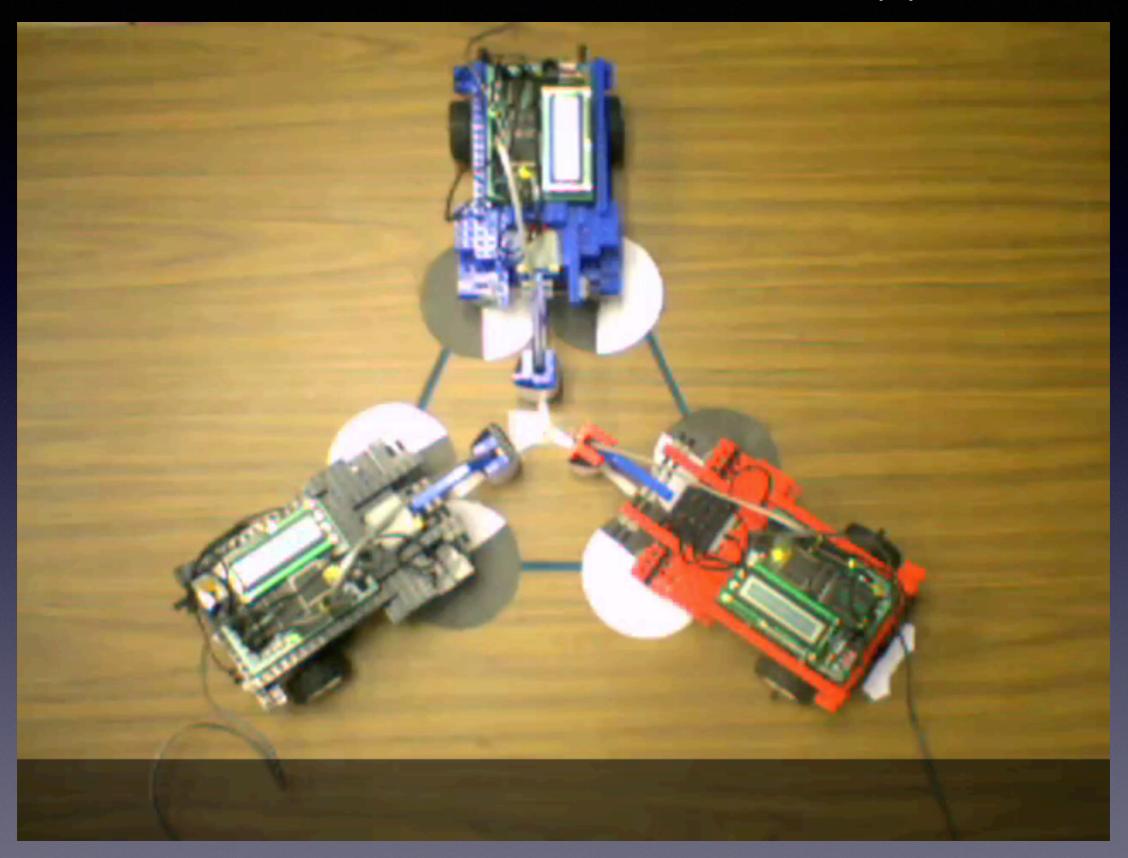
<sup>&</sup>lt;sup>1</sup> For instance, the validity problem for multi-agent propositional epistemic logic is PSPACE-complete [18]; adding a common knowledge operator makes the problem EXPTIME-complete [21].

#### "Life and Death" Wise Man Test (3)



\*Again: Object-level reasoning, reasoning that *produces* object-level reasoning (e.g., methods), and direct, "dirty," purely computational procedures.

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# Now, harder ...

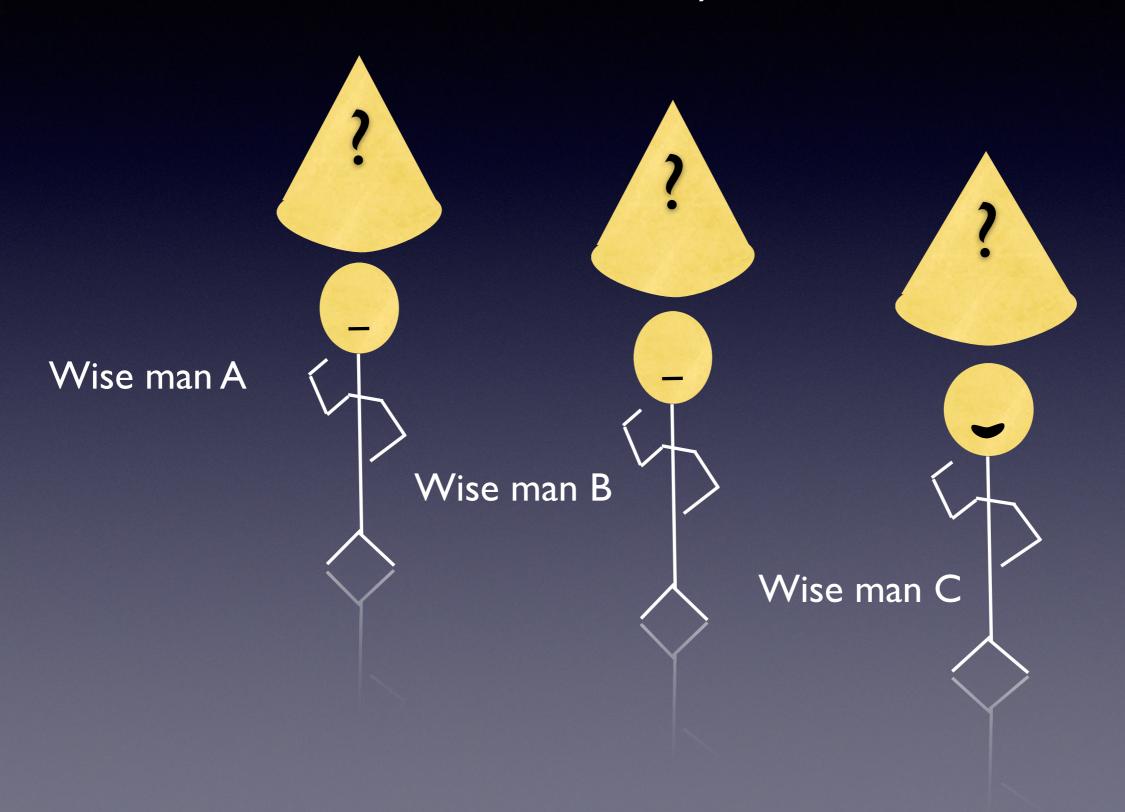
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Human Persons (s-conscious! p-conscious!)	Yes	Yes	Yes	Yes	Yes	Yes

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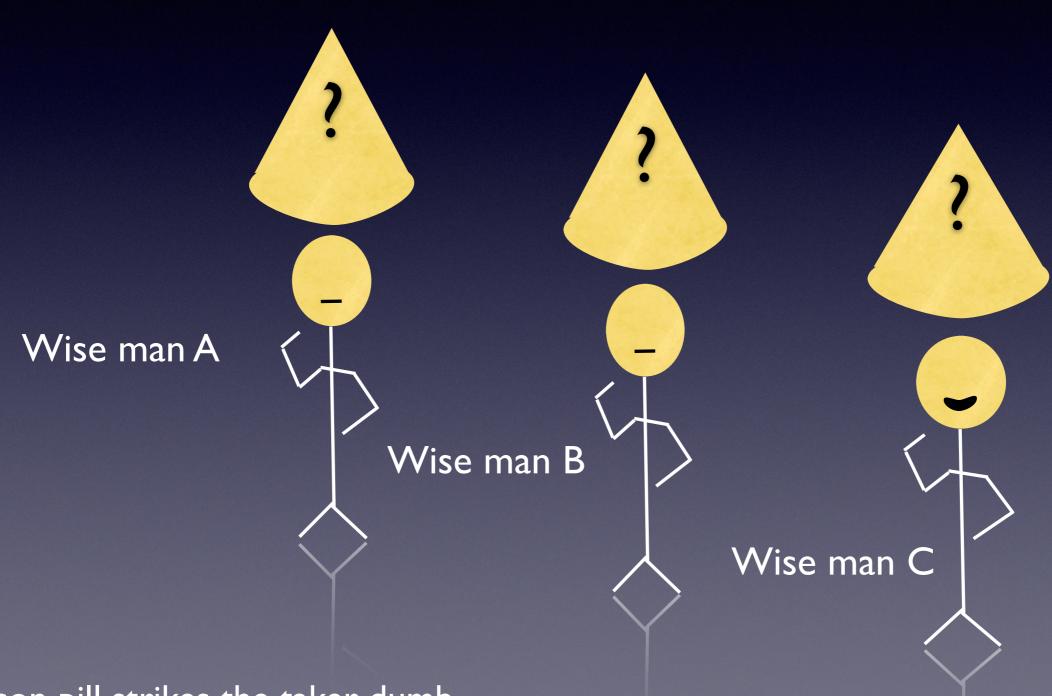
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# Floridi's "Ultimate (s- and p-consciousness) Sifter"

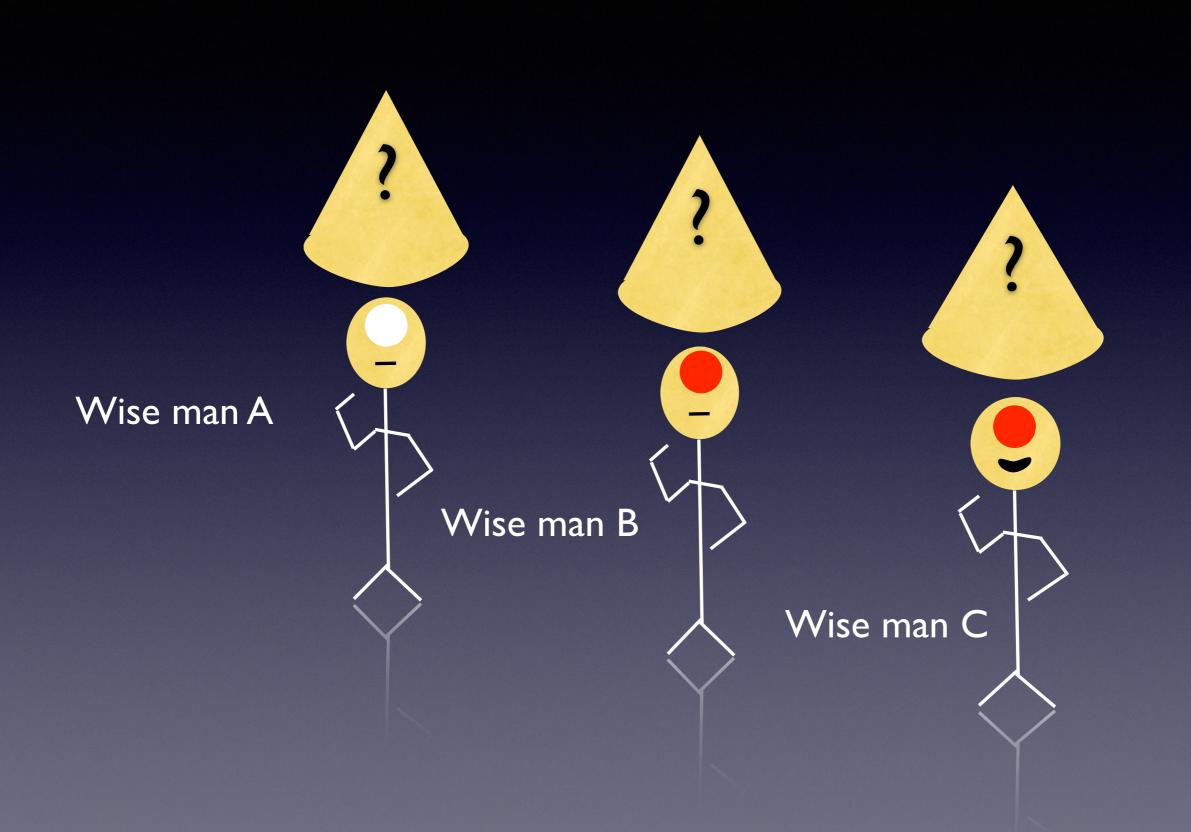


#### poison innocuous

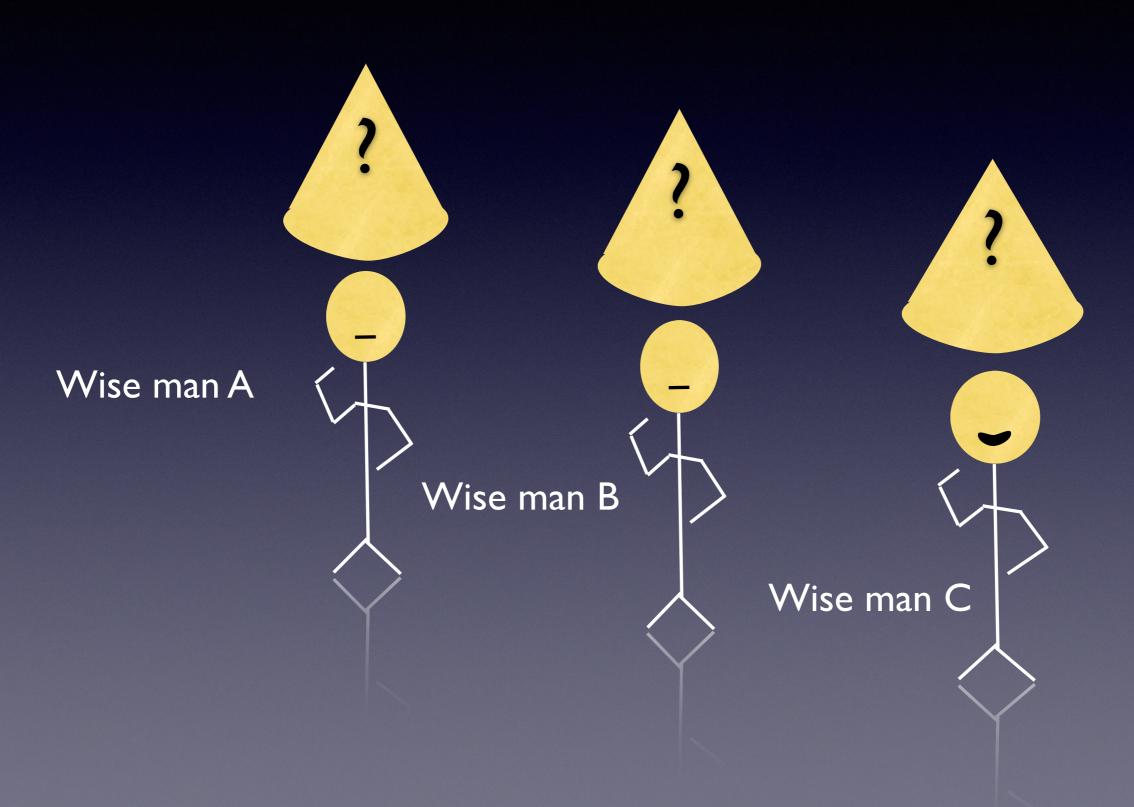




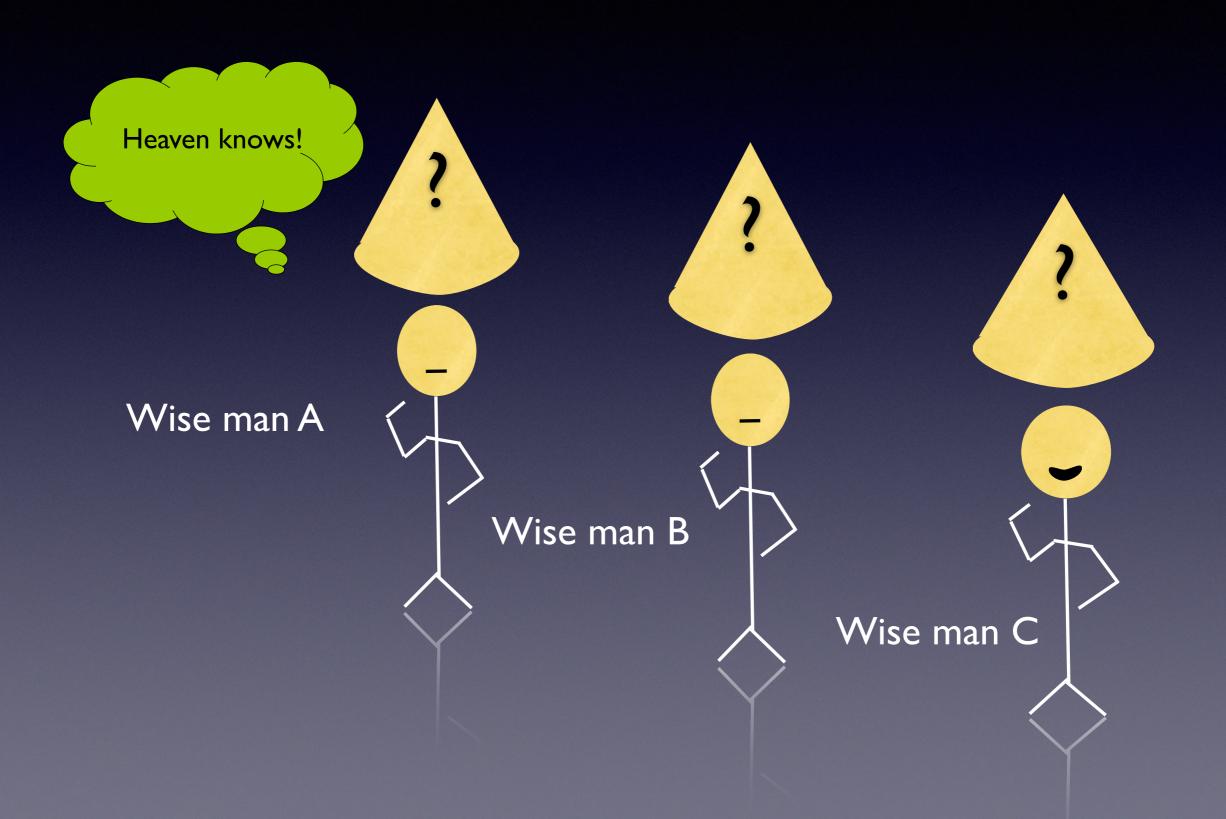
Poison pill strikes the taker dumb.



# "Have you been struck dumb? As always: Prove it!"



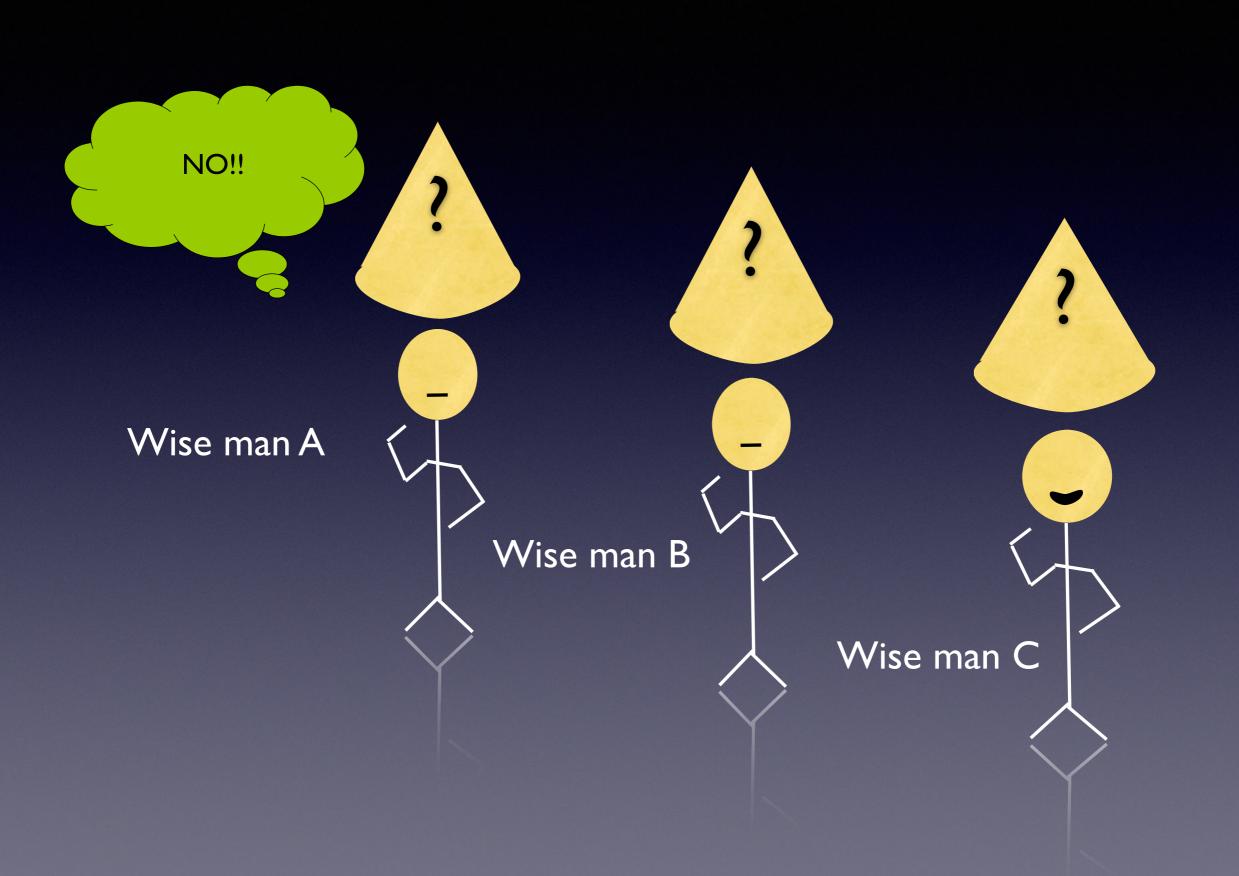
#### "Have you been struck dumb?"



#### Two possibilities:

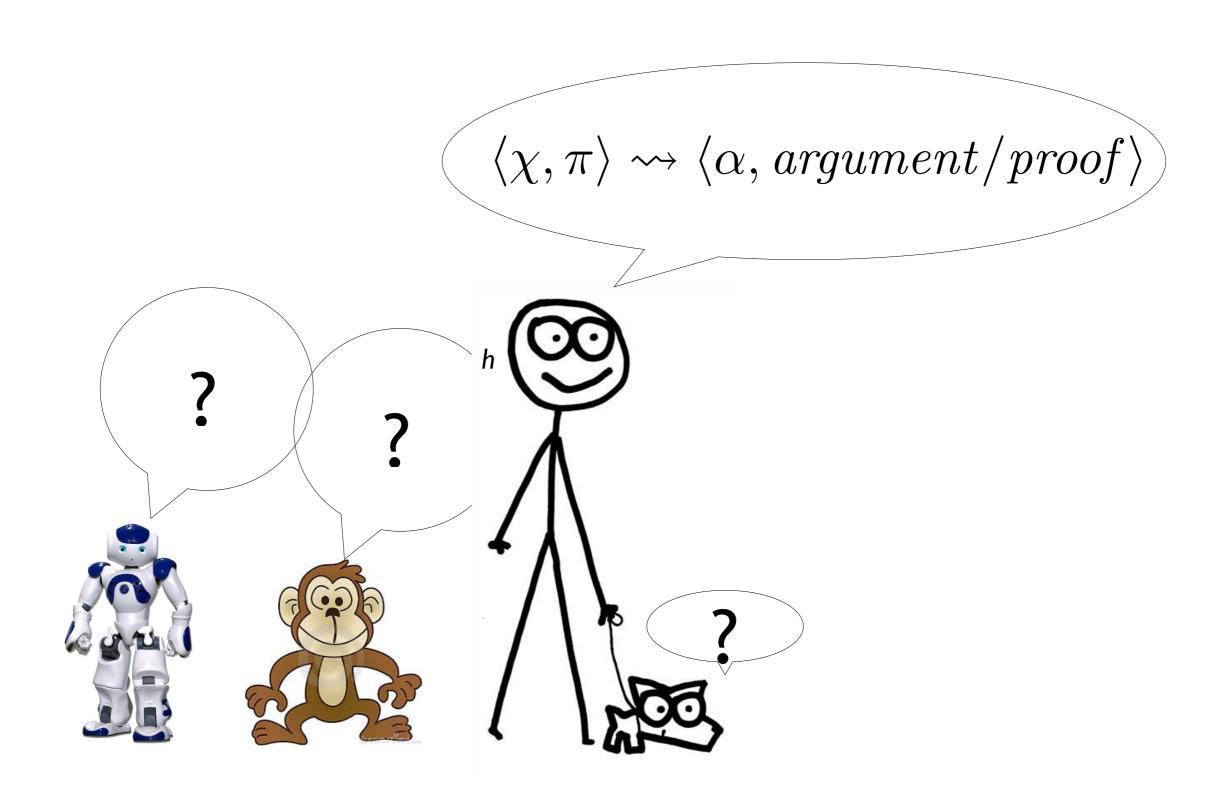
Subsequent silence: failure/death.

Or ...

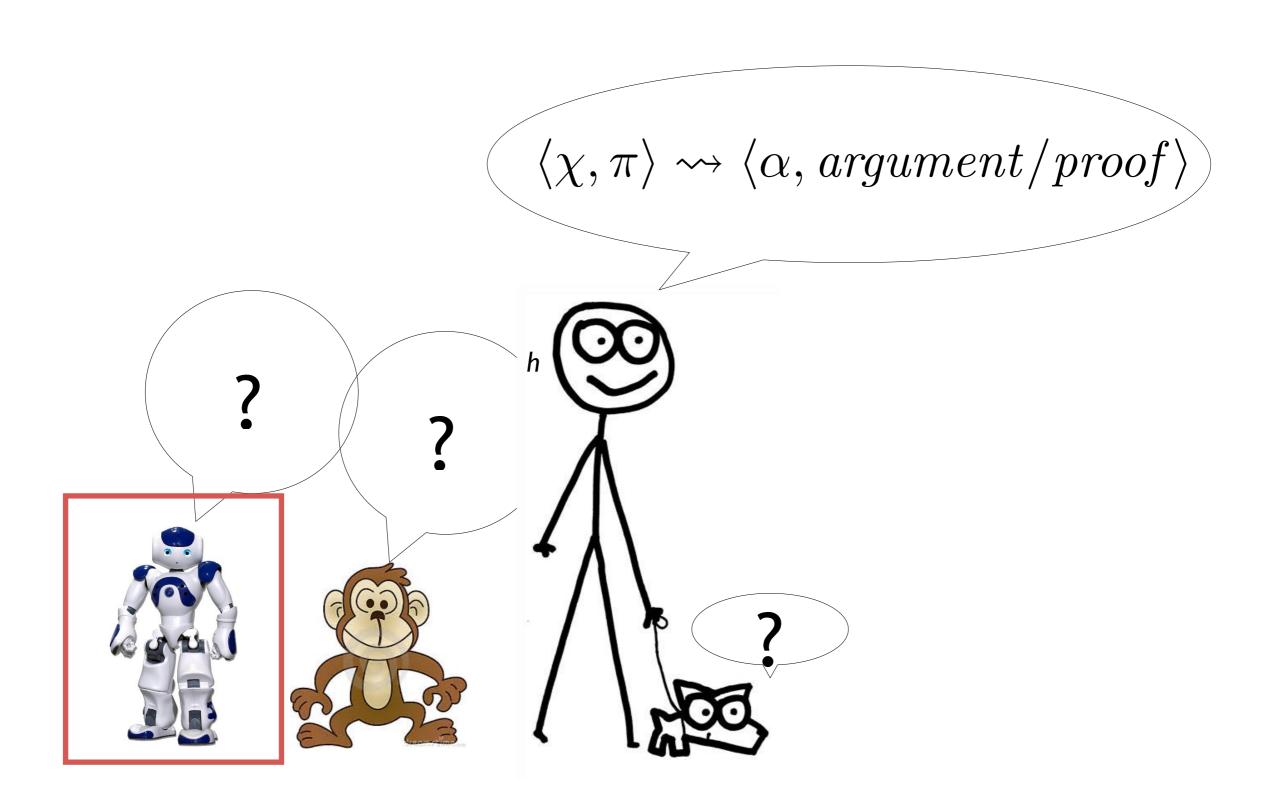


"Had I taken the dumbing tablet I would not have been able to report orally my state of ignorance about my dumb/non-dumb state, but I have been, and I know that I have been, as I have heard myself speaking and saw the guard reacting to my speaking, but this (my oral report) is possible only if I did not take the dumbing tablet, so I know I know I am in the non-dumb state, hence I know that ..."

—Luciano Floridi



http://ngm.nationalgeographic.com/2008/03/animal-minds/virginia-morell-text



Contrarian view on animal minds in Nat. Geo.:

http://ngm.nationalgeographic.com/2008/03/animal-minds/virginia-morell-text



http://kryten.mm.rpi.edu/SBringsjord\_etal\_self-con\_robots\_kg4\_0601151615NY.pdf

https://www.youtube.com/watch?v=MceJYhVD\_xY

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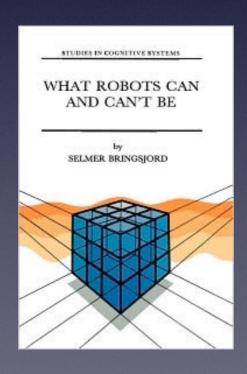
## Infinitary False Belief Task

http://kryten.mm.rpi.edu/PRES/COGSCI2019/infinitaryfalsebeliefprezCogSci2019.key



So ... despite the fact we can't build rational persons, apparently we can build Als that pass any short test. That's why Blade Runner (& Ex Machina?) is our future.

So ... despite the fact we can't build rational persons, apparently we can build Als that pass any short test. That's why Blade Runner (& Ex Machina?) is our future.



If humans are as described in this thesis, then they can solve the forthcoming cognitive problems.

But humans can't solve the problems in question.

Therefore:

If humans are as described in this thesis, then they FALSE can solve the forthcoming cognitive problems.

But humans can't solve the problems in question.

Therefore:

If humans are as described in this thesis, then they FALSE can solve the forthcoming cognitive problems.

If humans are as described in R, then humans can, given sufficient training, etc., eventually solve the cognitive problems in question.

But humans can't solve the problems in question.

Therefore:

If humans are as described in this thesis, then they FALSE can solve the forthcoming cognitive problems.

If humans are as described in R, then humans can, given sufficient training, etc., eventually solve the cognitive problems in question.

But humans can't solve the problems in question. FALSE

Therefore:

If humans are as described in this thesis, then they FALSE can solve the forthcoming cognitive problems.

If humans are as described in *R*, then humans can, given sufficient training, etc., eventually solve the cognitive problems in question.

But humans can't solve the problems in question. FALSE

Some humans can't, at present, solve the problems in question — & as it turns out, Als can do surprisingly well, at least until we get to the infinite case.

#### Therefore: