Informal Intro to "Gold"-Style Language Learning

—

With a Sample Target of RAIR-Lab Interest

(or: Human Language Acquisition is Hard!; Sorry Nonhuman Animals)

Selmer Bringsjord

Naveen Sundar Govindarajulu

Are Humans Rational?

RPI

11/18/19

See *Systems That Learn: An Introduction to Learning Theory* (2nd ed). Adapted from Bringsjord-Govindarajulu seminar *Computational Learning Theory & Science*.

How would *you* formalize/ model language learning in the human case?

Informal Intro to Gold-style Learning …

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5. *successful/unsuccessful behavior by a learner trying to learn a given, possible reality*.

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- *C* is a proper subset of *D* defined as follows: All sets that contain every positive integer, save for one.
  - E.g., here is a member of *C*: "all positive integers except for 2." I.e., {1, 3, 4, 5, ...}.

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- Nature: "I've selected a member of *C*. You, (language-learning) Child, must discover the set I have in mind. I shall give you clues, as follows. My secret member shall be ordered in a list *L* that contains all members of my secret set, and I shall then present the members of *L* one at a time to you. Each time I do so, you offer a hypothesis in the form of an English expression that uniquely describes a set of positive integers. You win the game iff you make only a finite number of conjectures, and the last one is correct."

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- Nature: "Let's play!"
- Child: "Okay!"

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- Is Child using some algorithm?

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Let *S* be the set of numbers presented so far. And let *m* be the smallest positive integer that isn't a member of *S*. Output the hypothesis: "All positive integers except for *m*." If this was your last hypothesis, remain silent.

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So, e.g., $G([4, 5, 8, 1]) = 2$.

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We stipulate that $\mathbf{Z}^+$ is in C.

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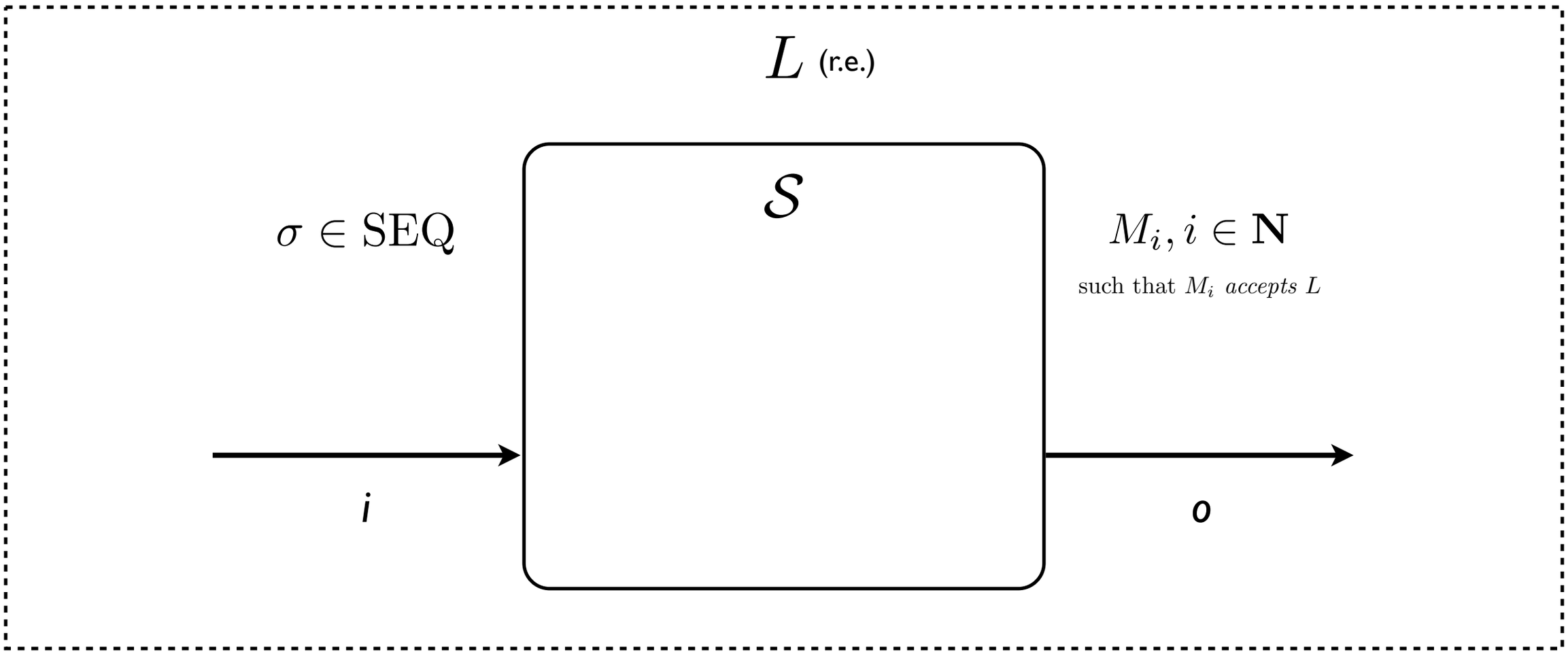
    - Of course, given our Theorems 1 & 2 from above, *we* can in *some* sense know that the system Child is using will win. But the Child himself/herself doesn't know.

CLT-based Model of Language Learning



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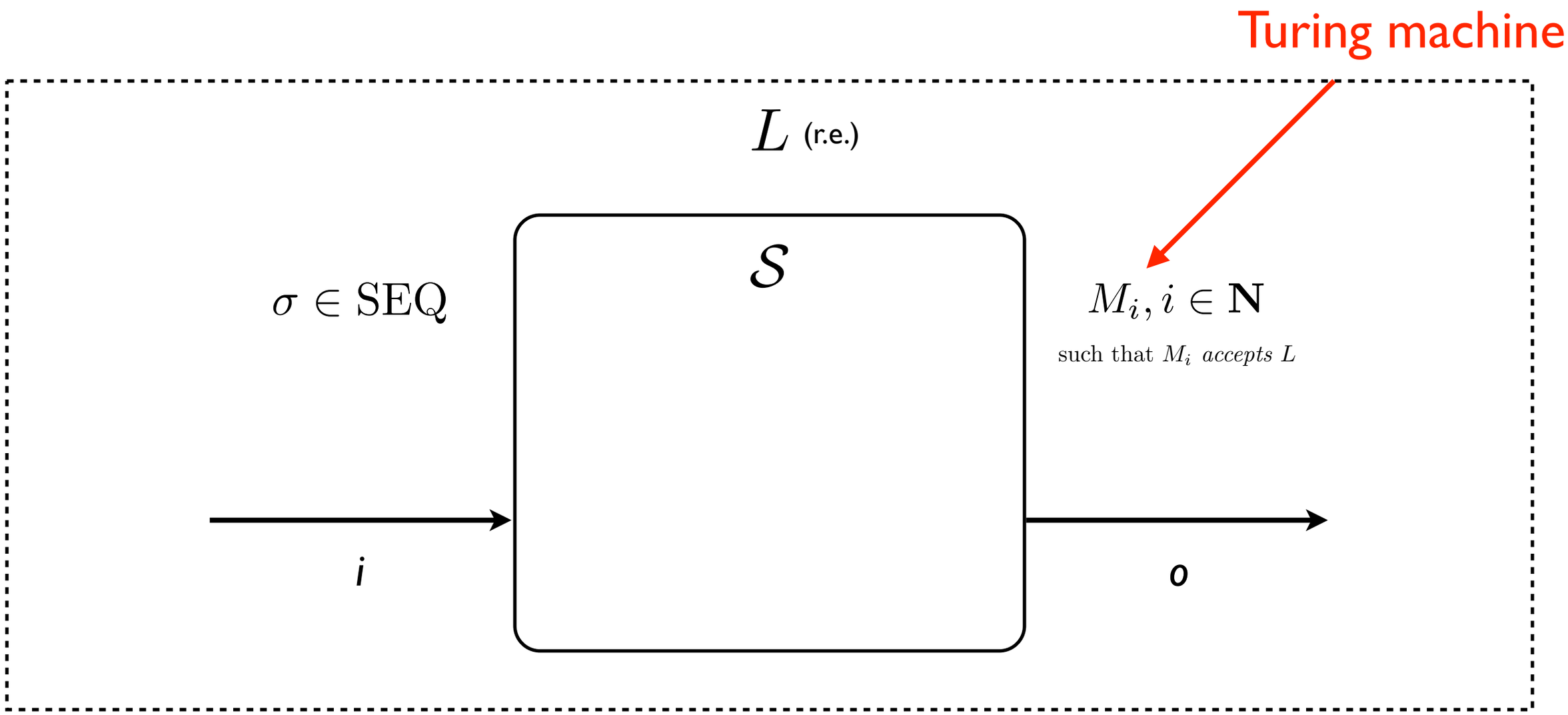

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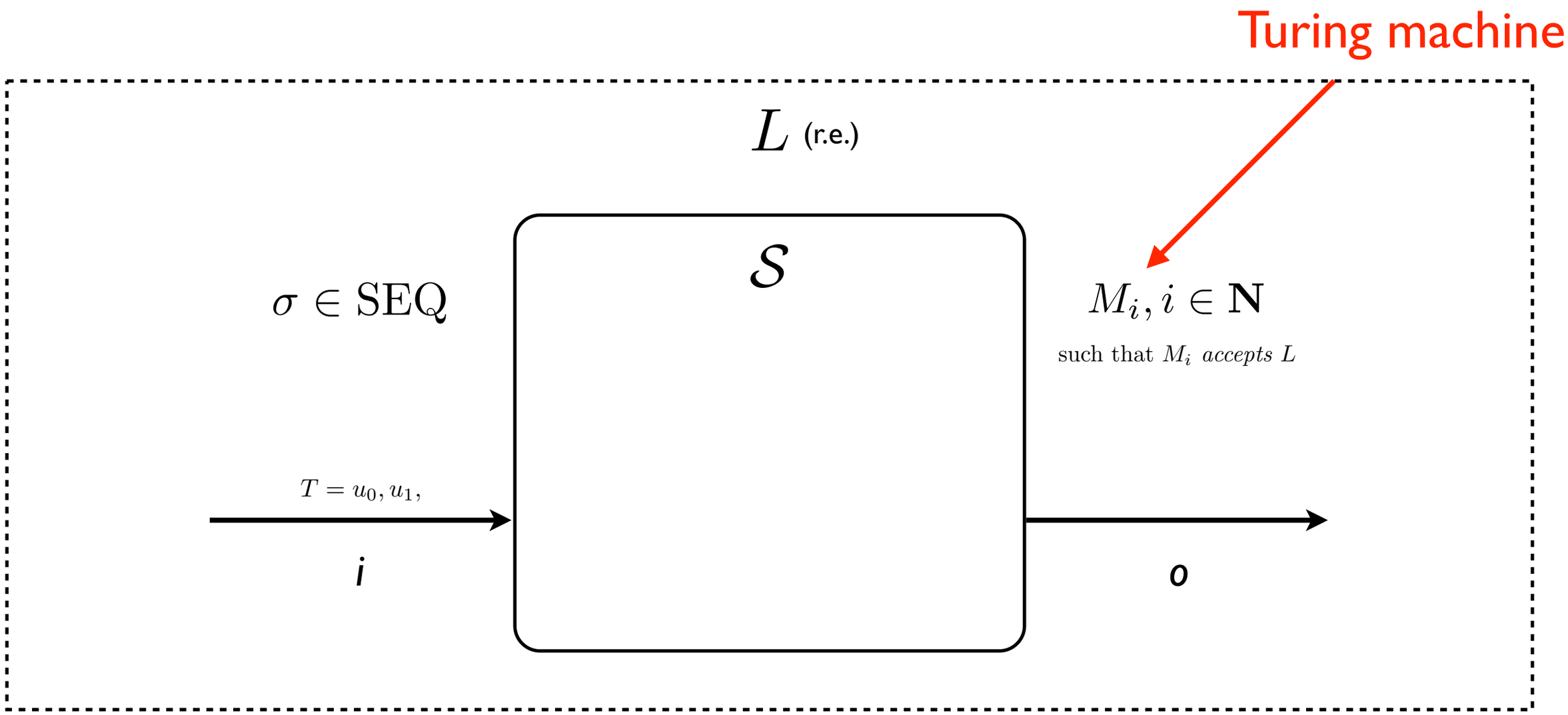

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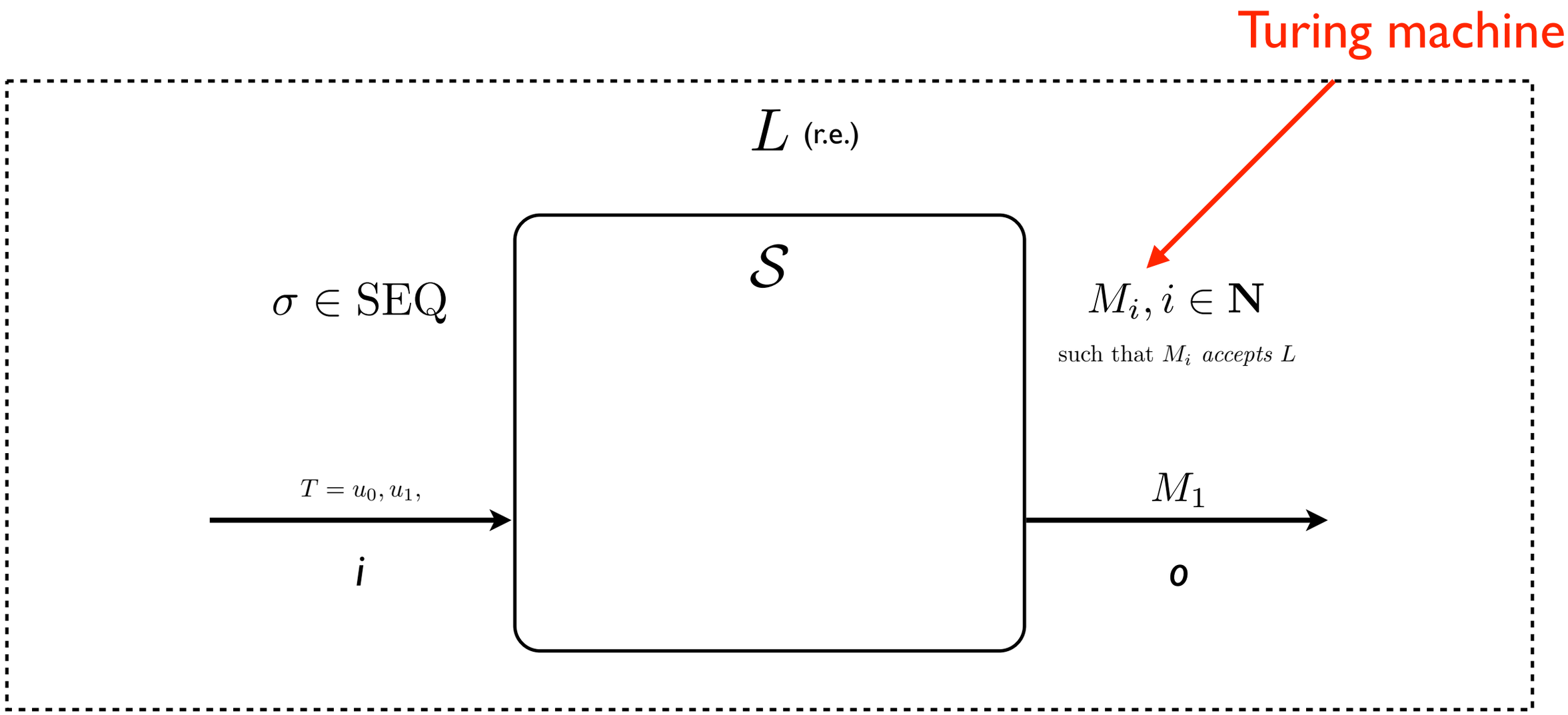

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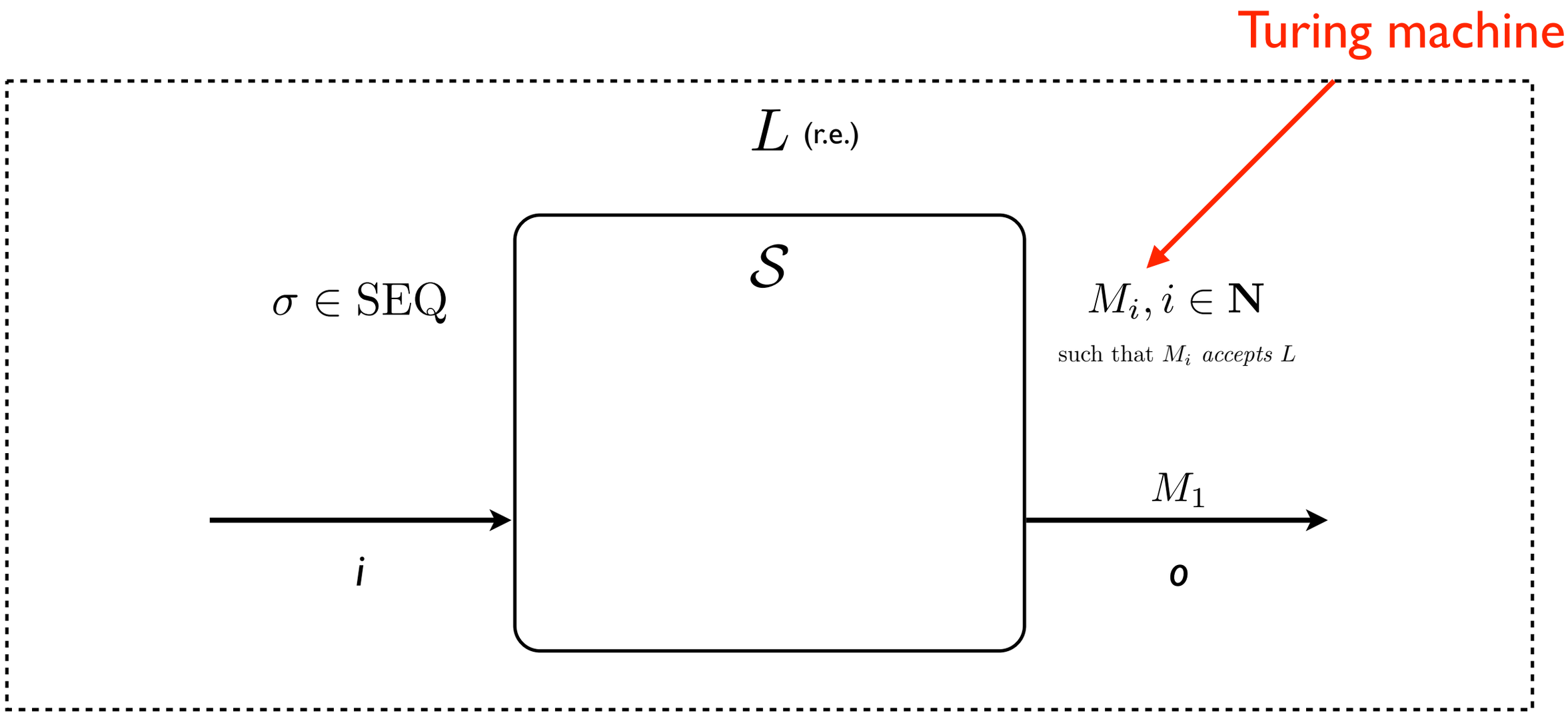

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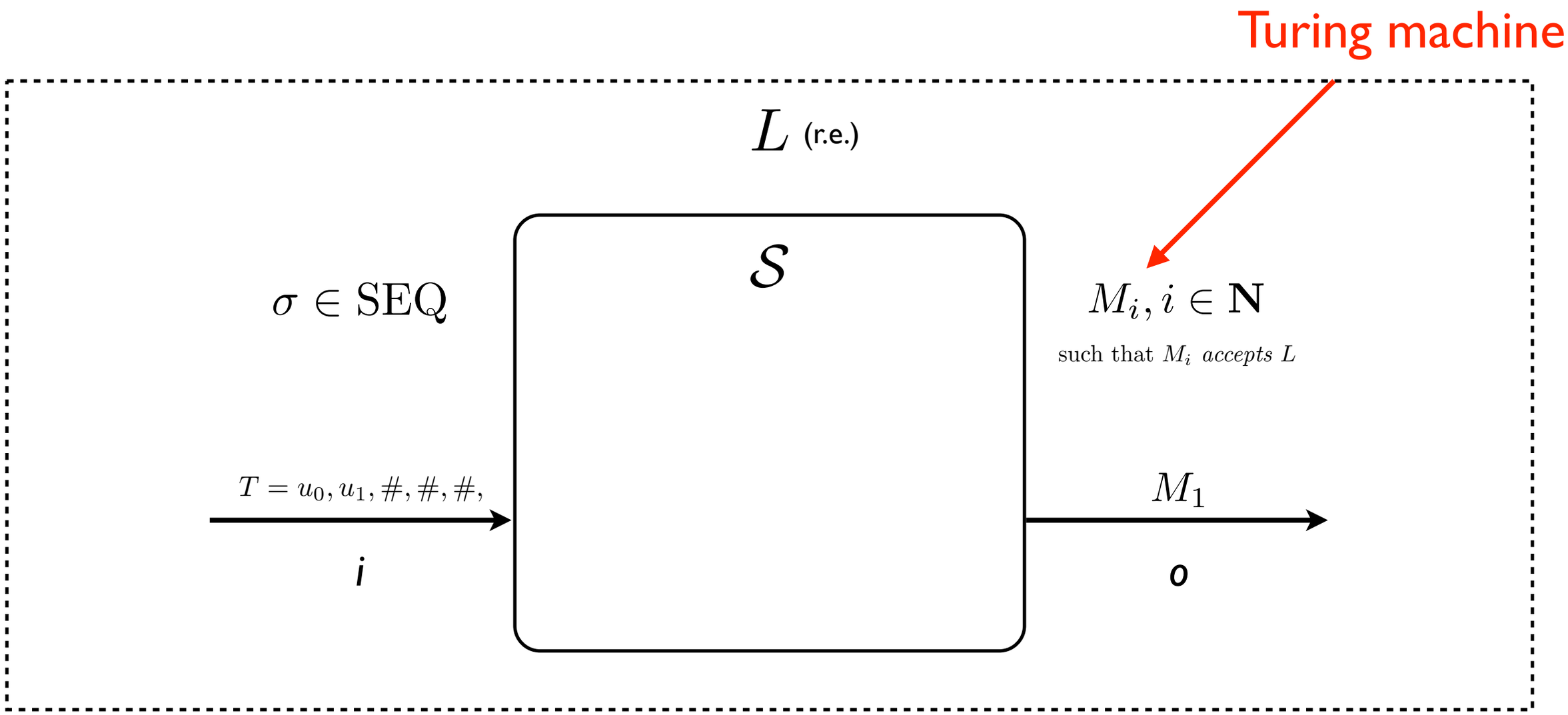

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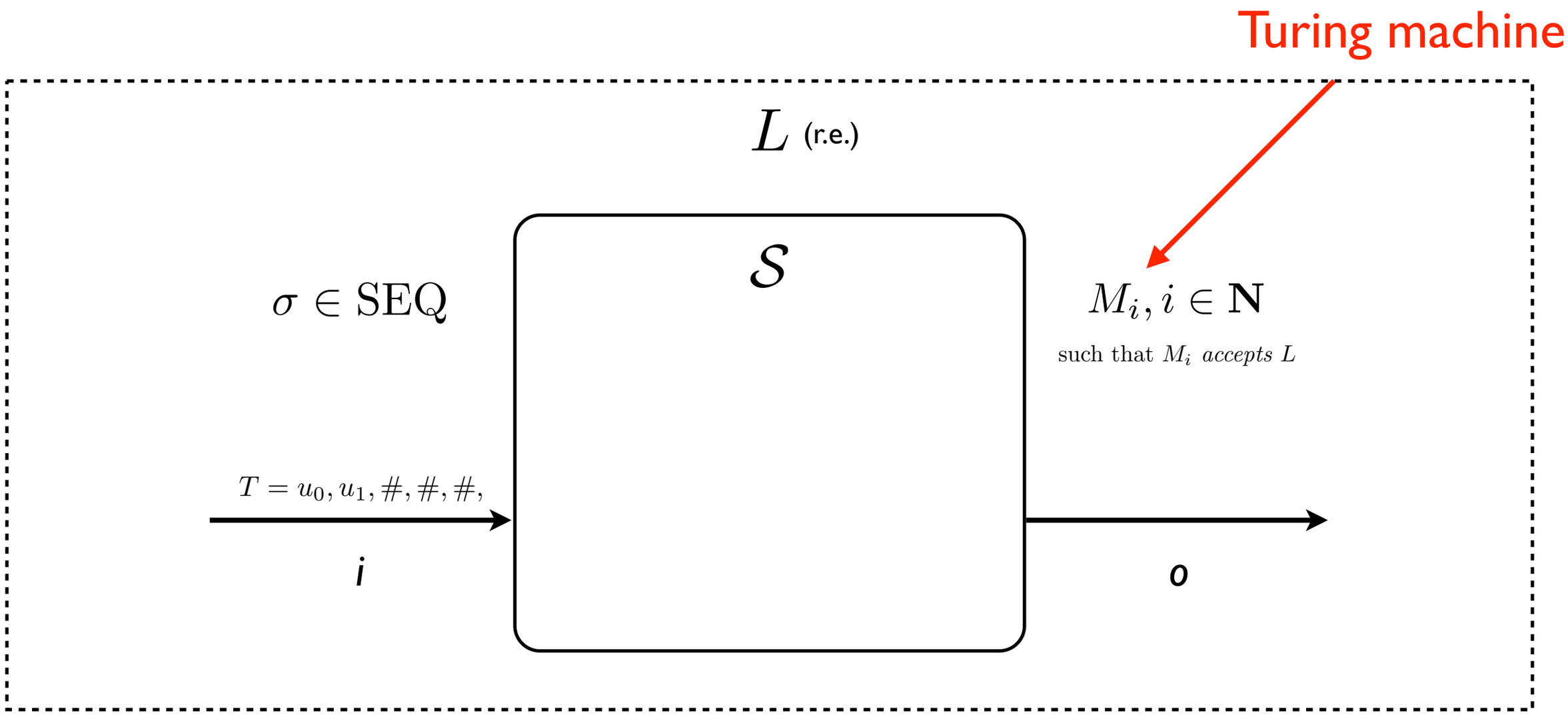

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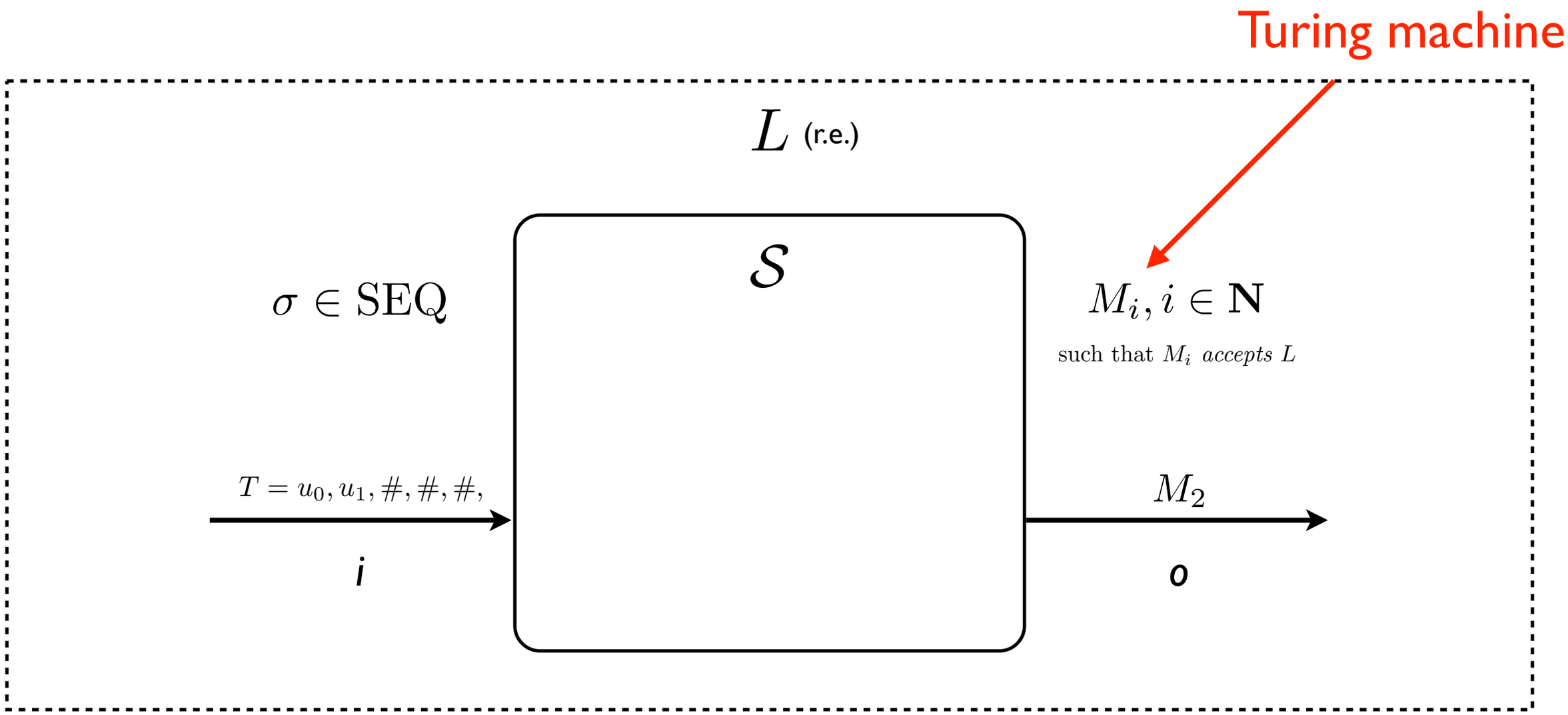

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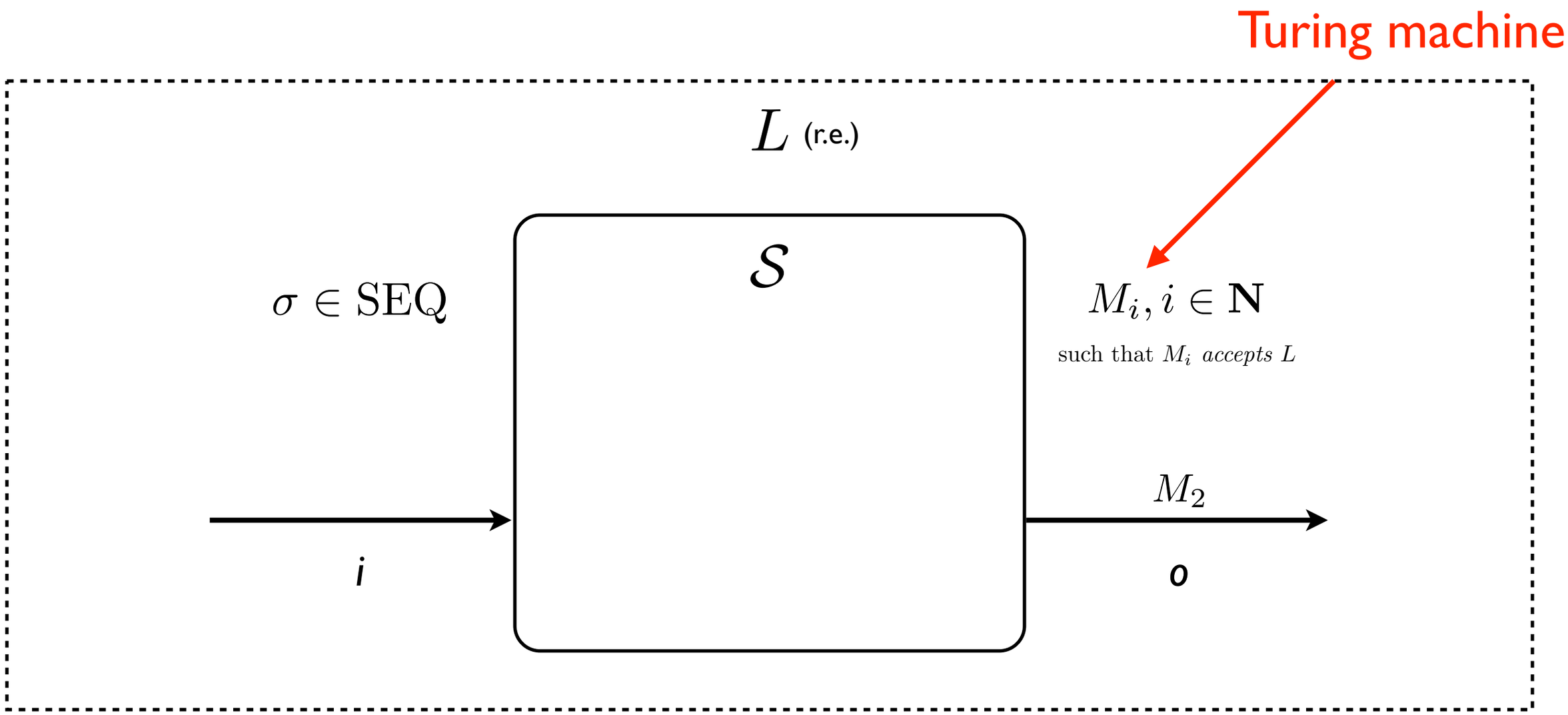

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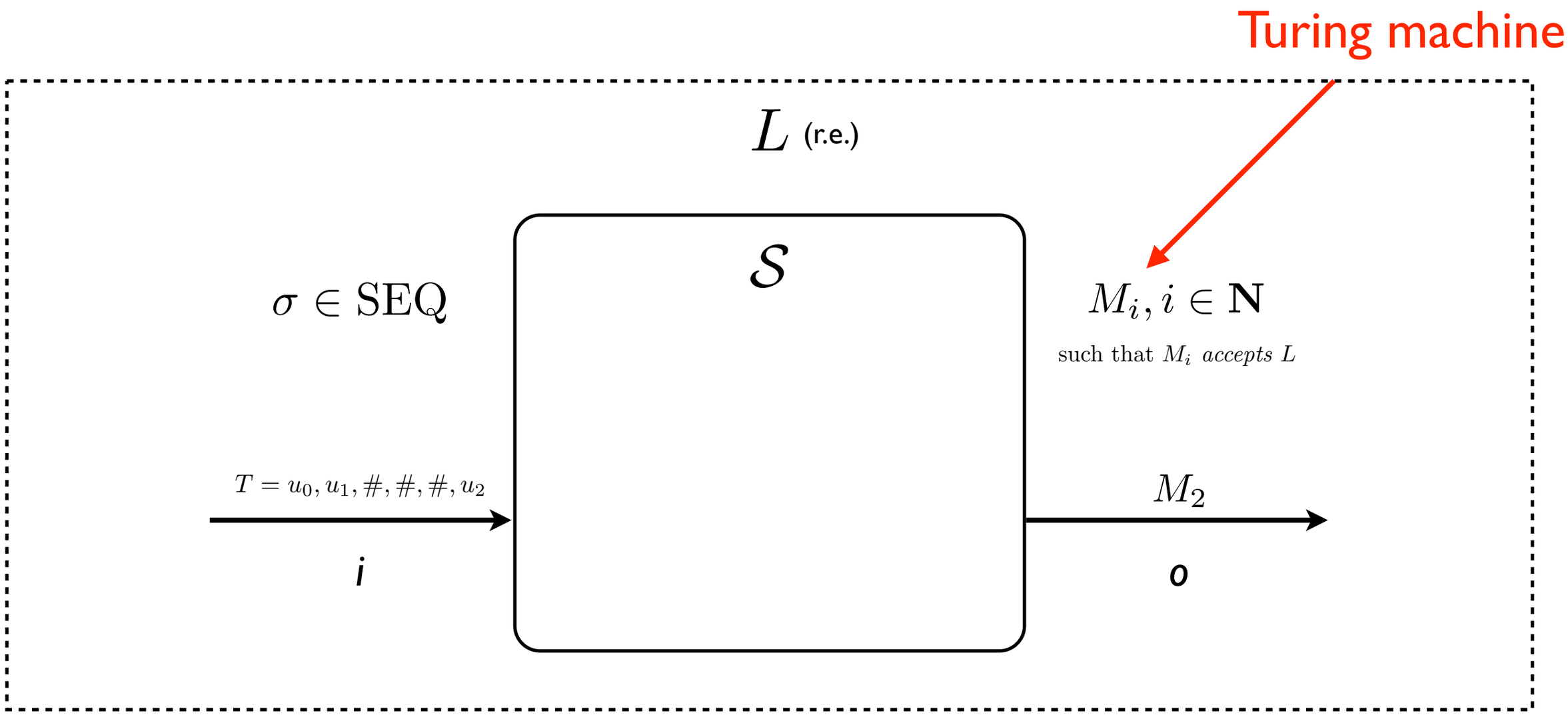

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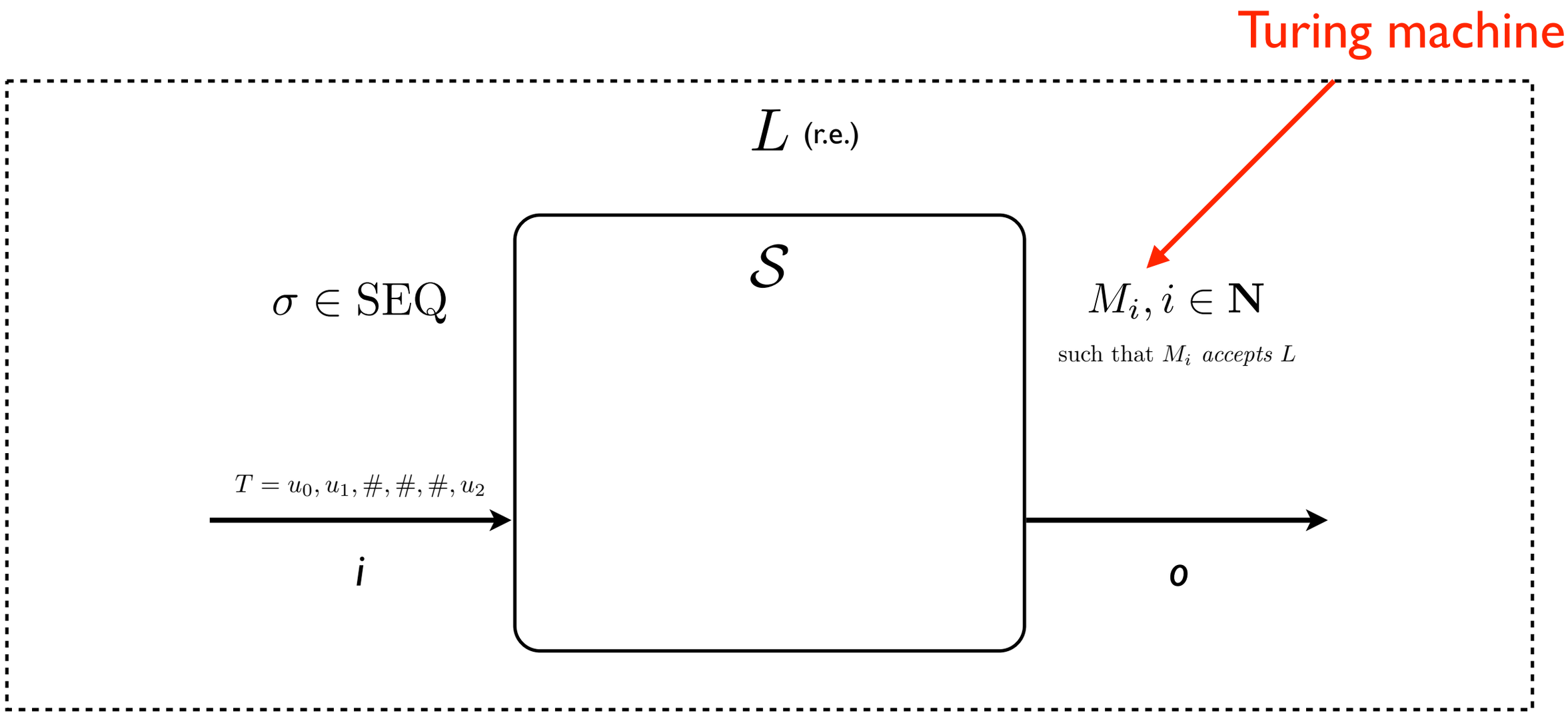

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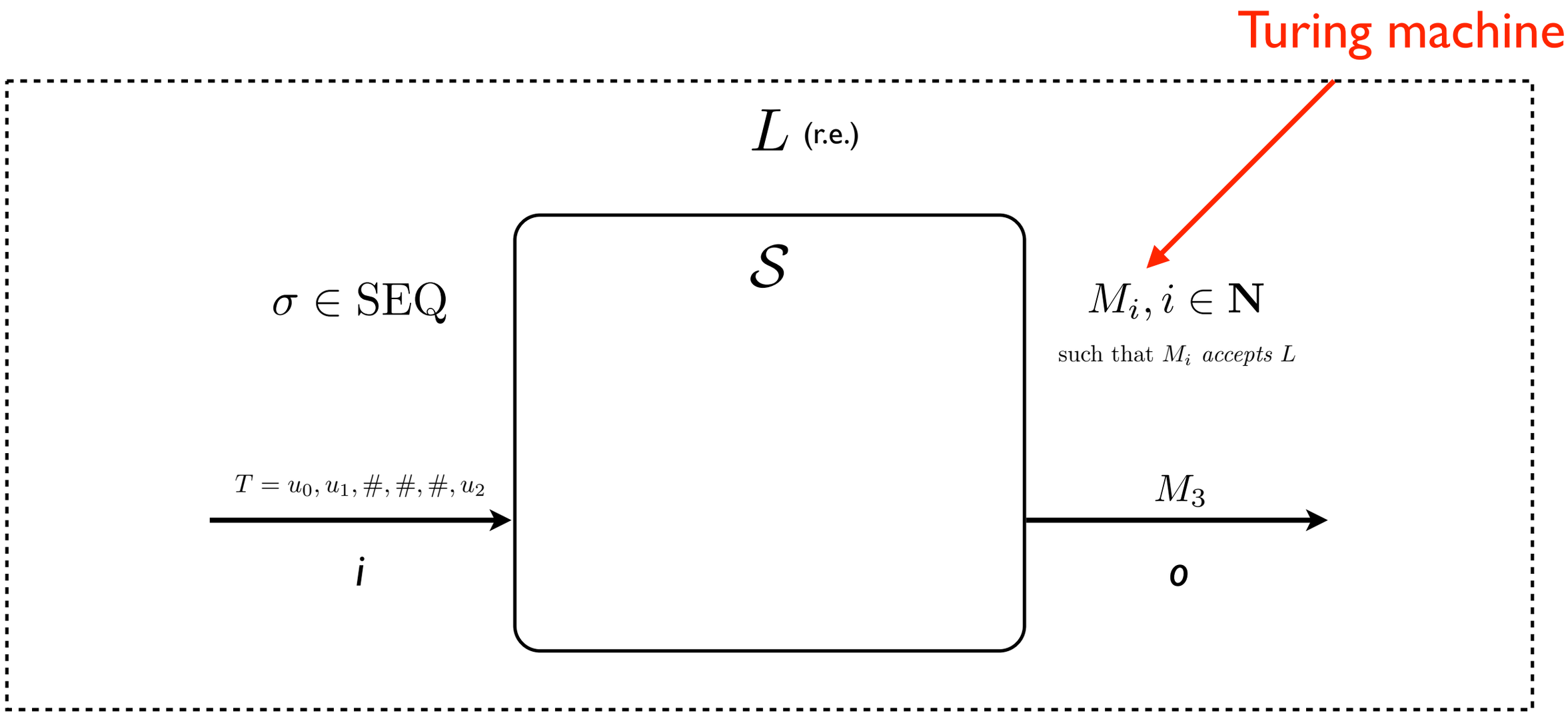

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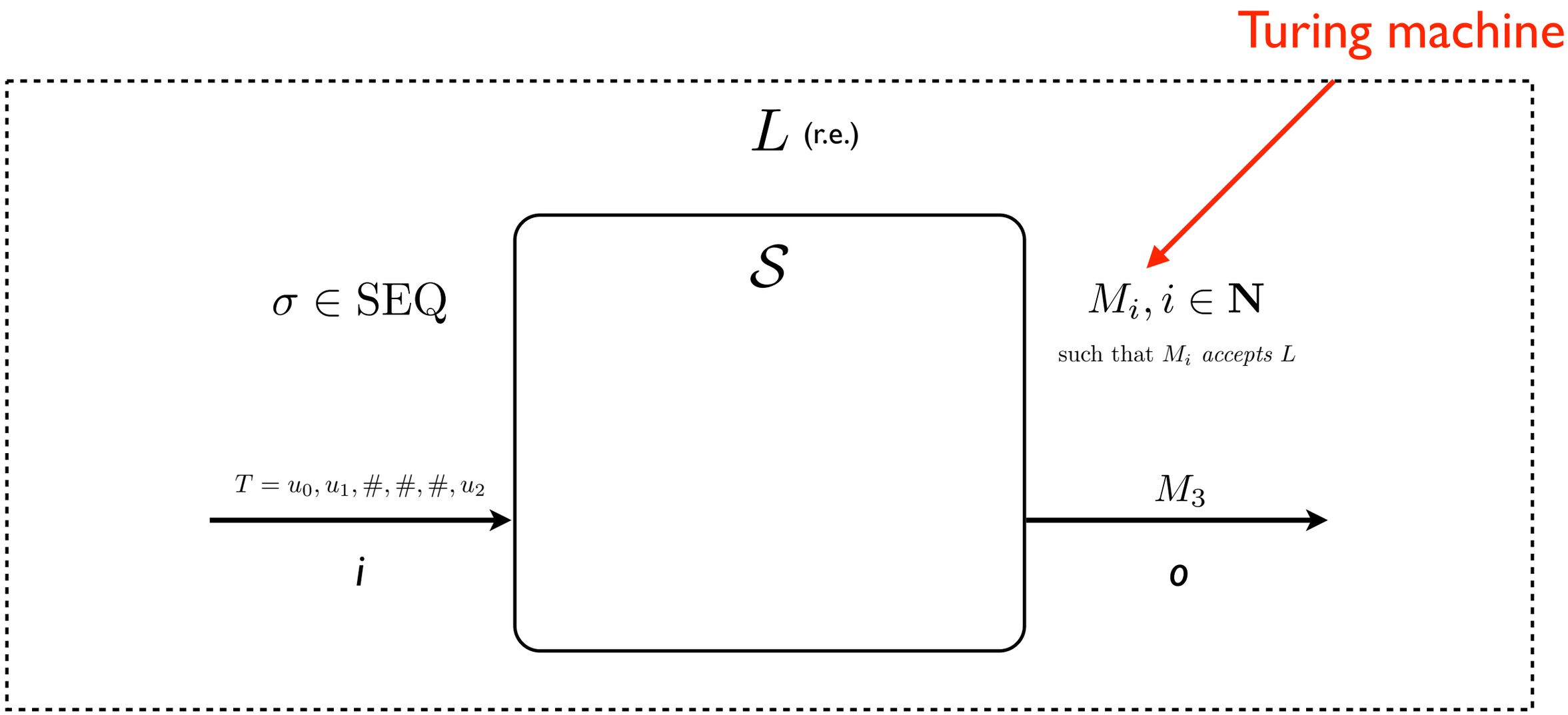

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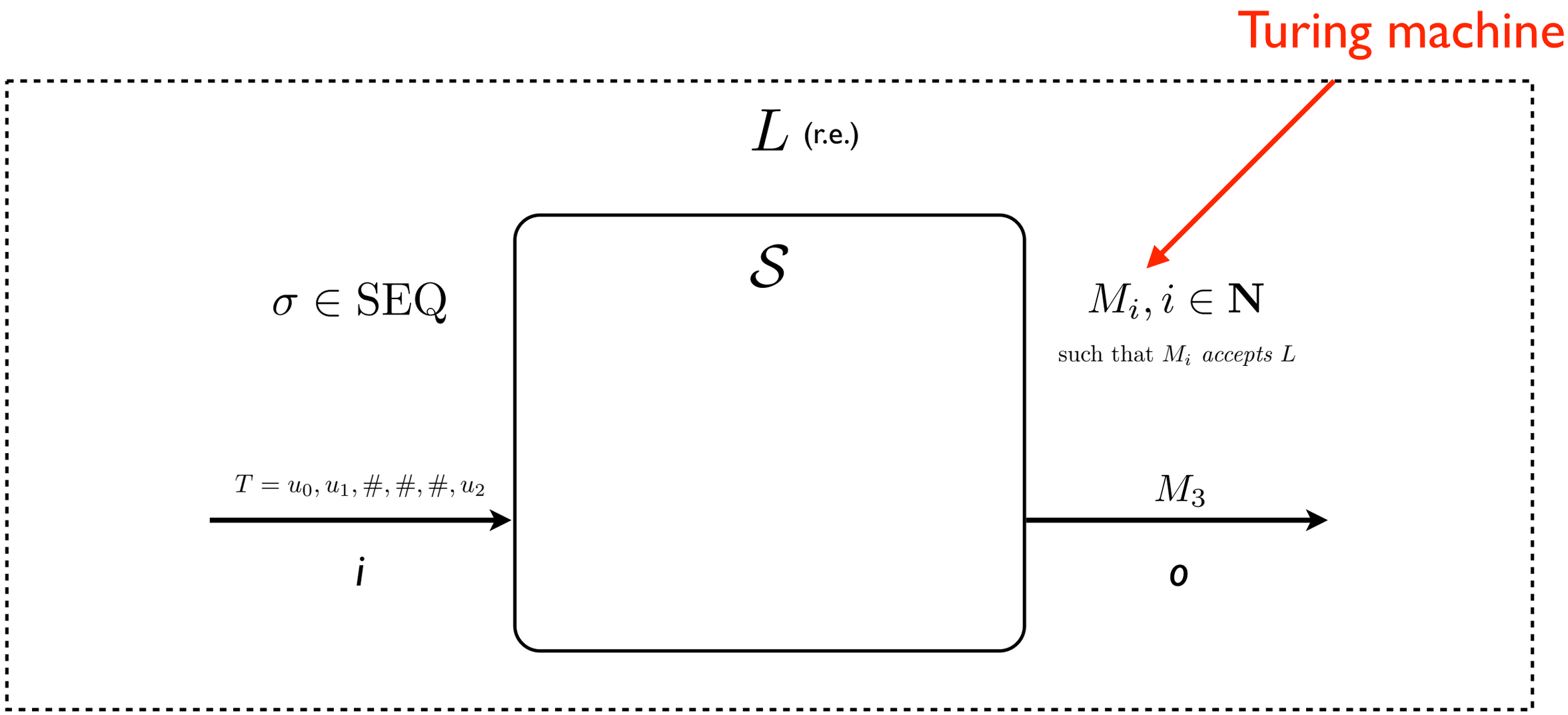

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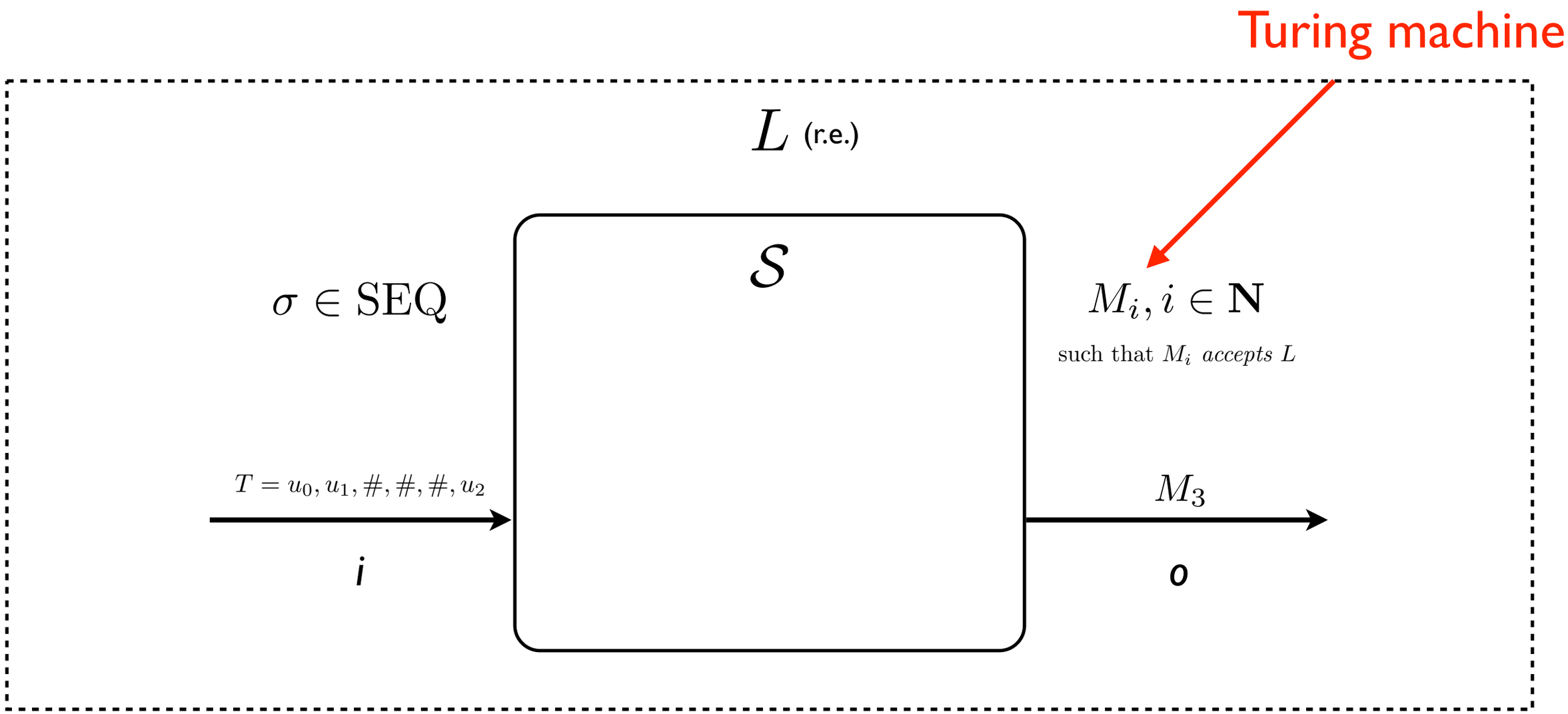

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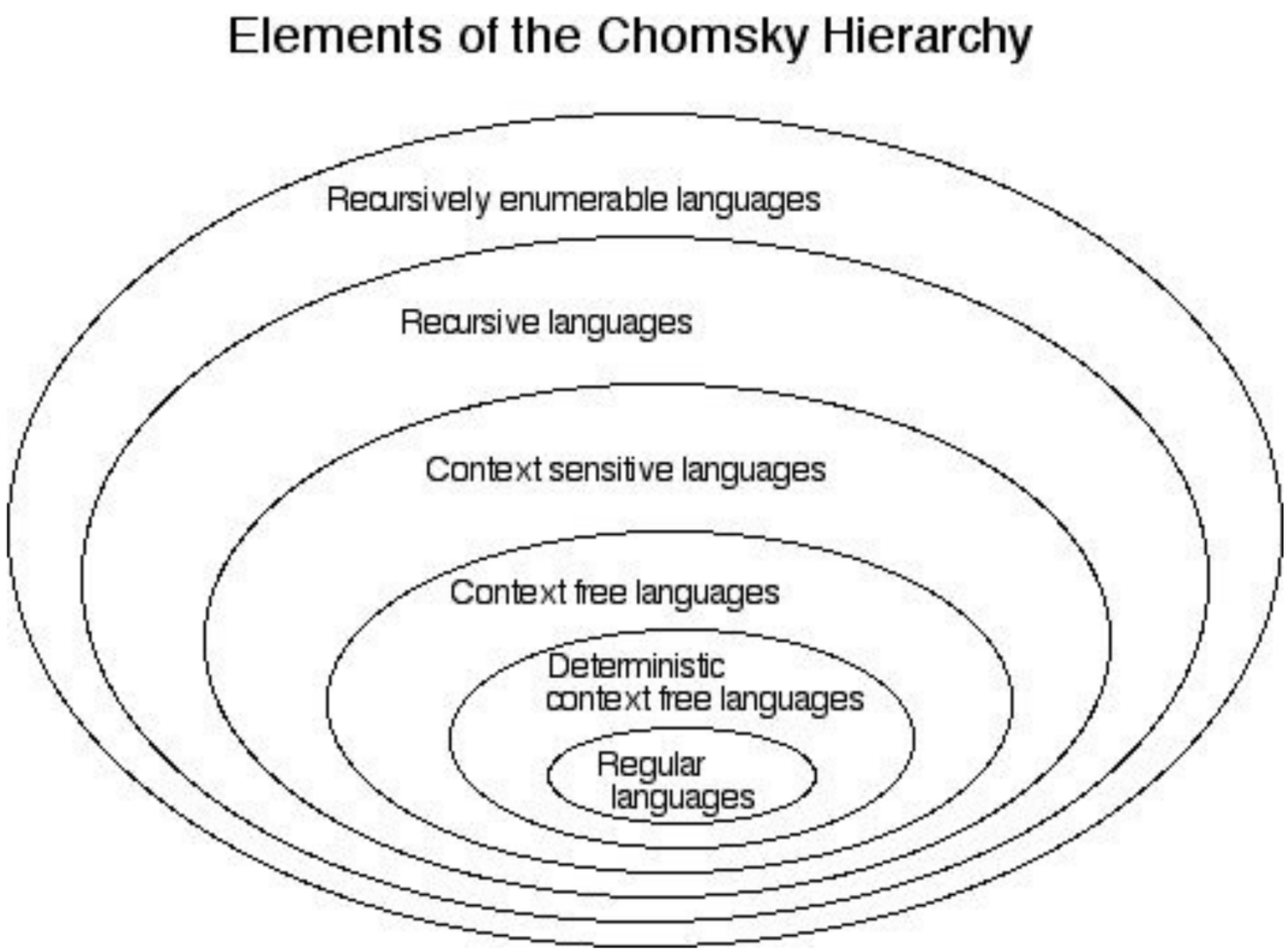
$\mathcal{S}$ identifies L iff $\mathcal{S}$ identifies every text for L

RAIR-Lab-relevant Target for Learning …

Chomsky Hierarchy of Languages



How About Learning the Grammar of the Pure Predicate Calculus?

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\end{array}
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`(Likes sally bill)`

How About Learning the Grammar of the Pure Predicate Calculus?

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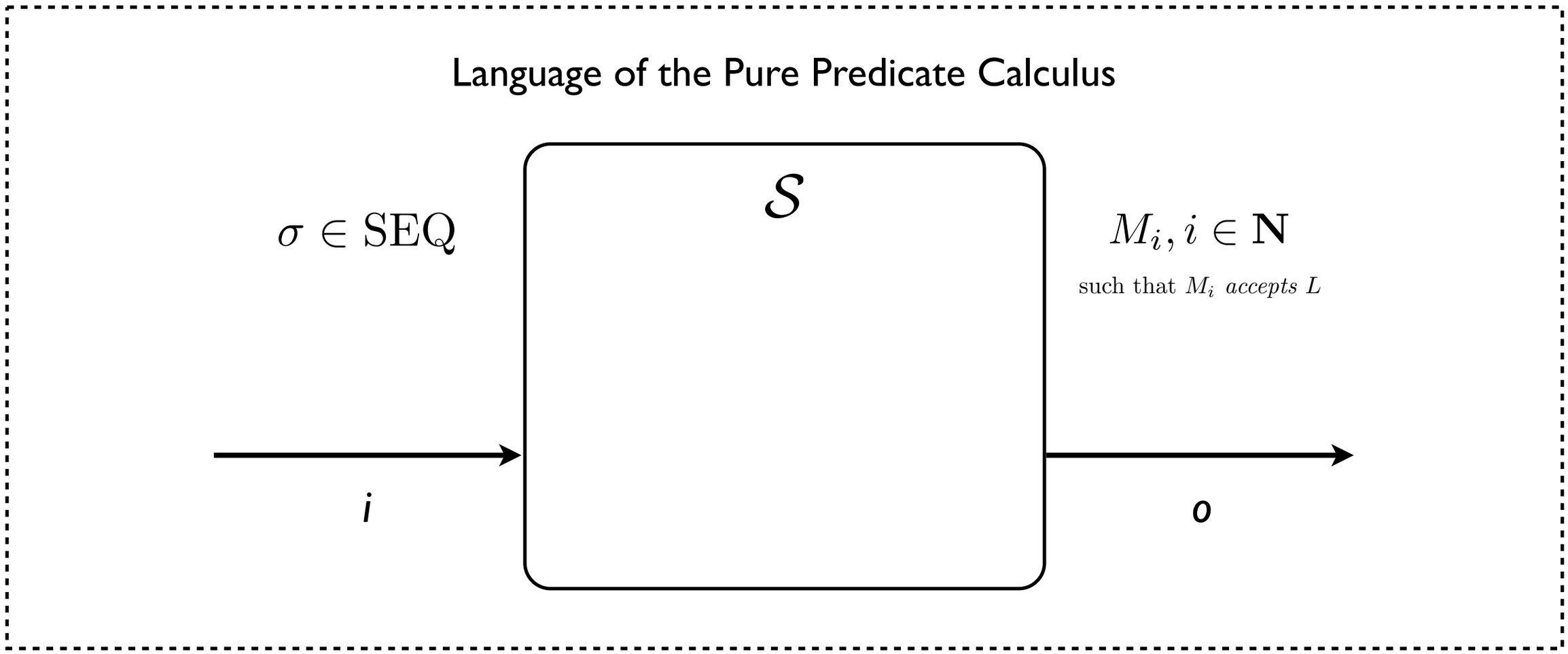
Make sure you can simulate a machine that says "Yes that sentence is okay!" whenever it's conforms to this grammar!

[ ] Need modify because now the language is recursive!

CLT-based Model Instantiated



And when is identification of *L* achieved?

$$\forall^{\infty} n \in \mathbf{N}\ \mathcal{S}(T[n]) = M_k$$

$\mathcal{S}$ identifies L iff $\mathcal{S}$ identifies every text for L

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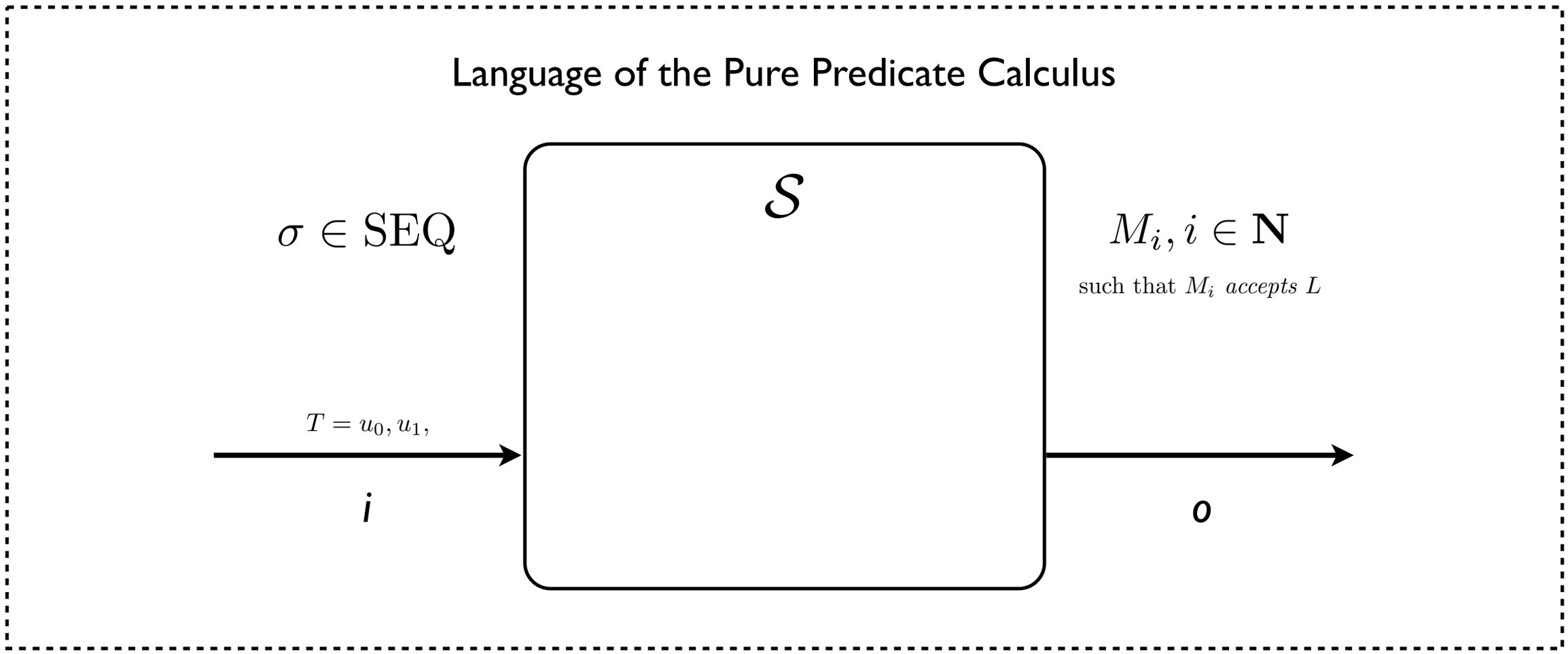

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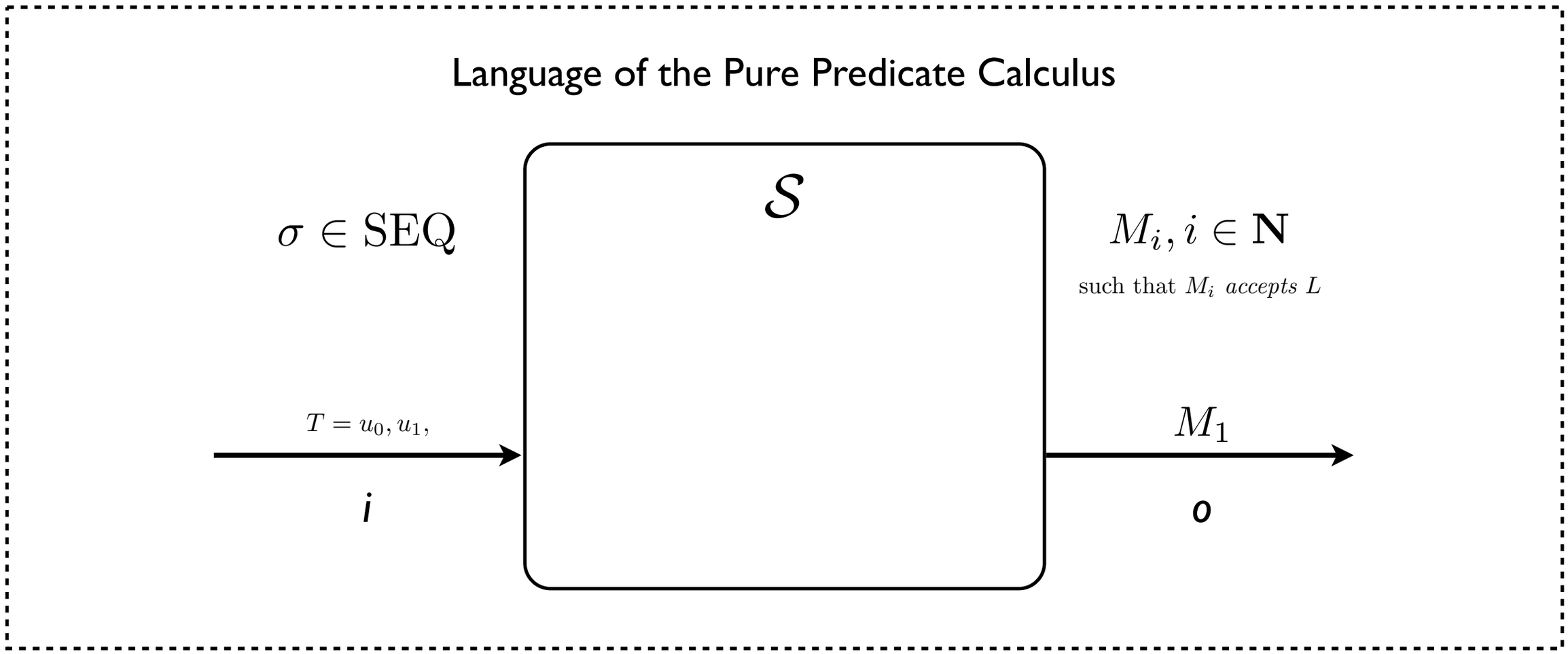

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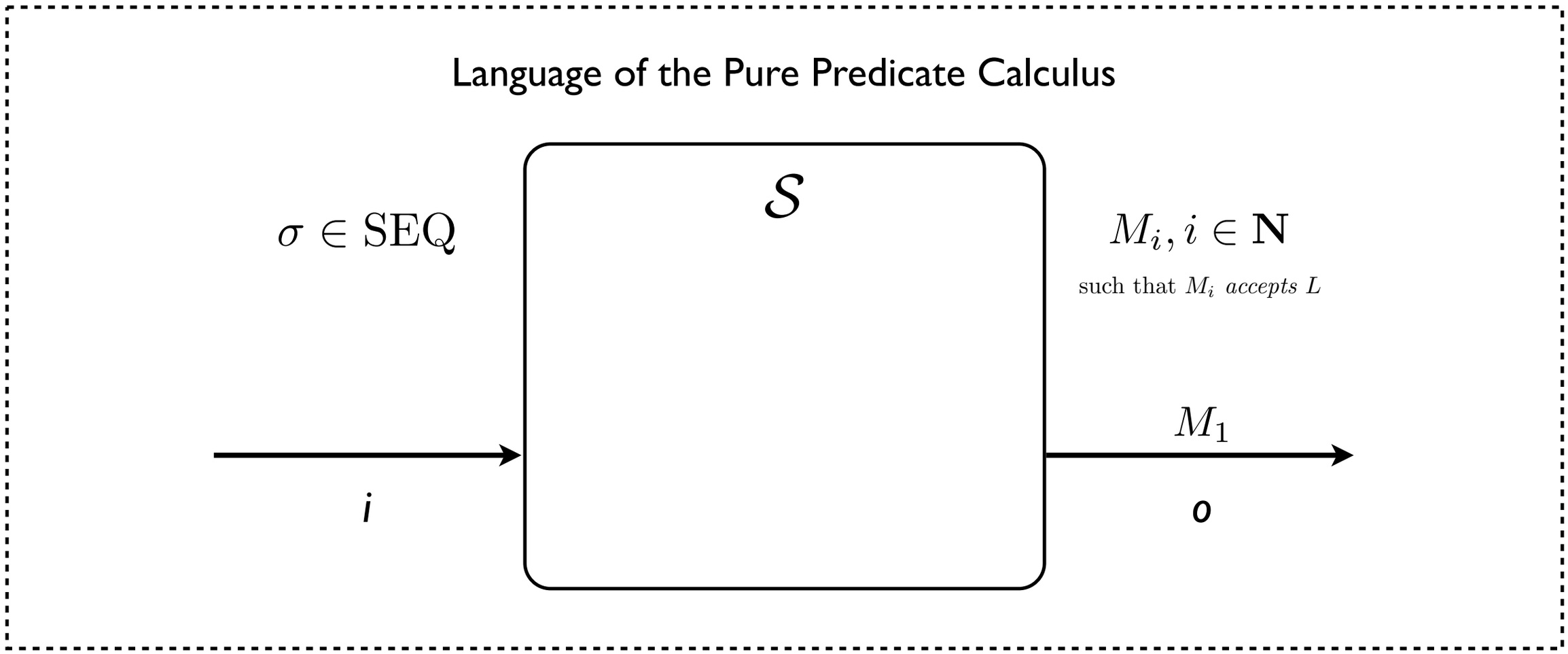

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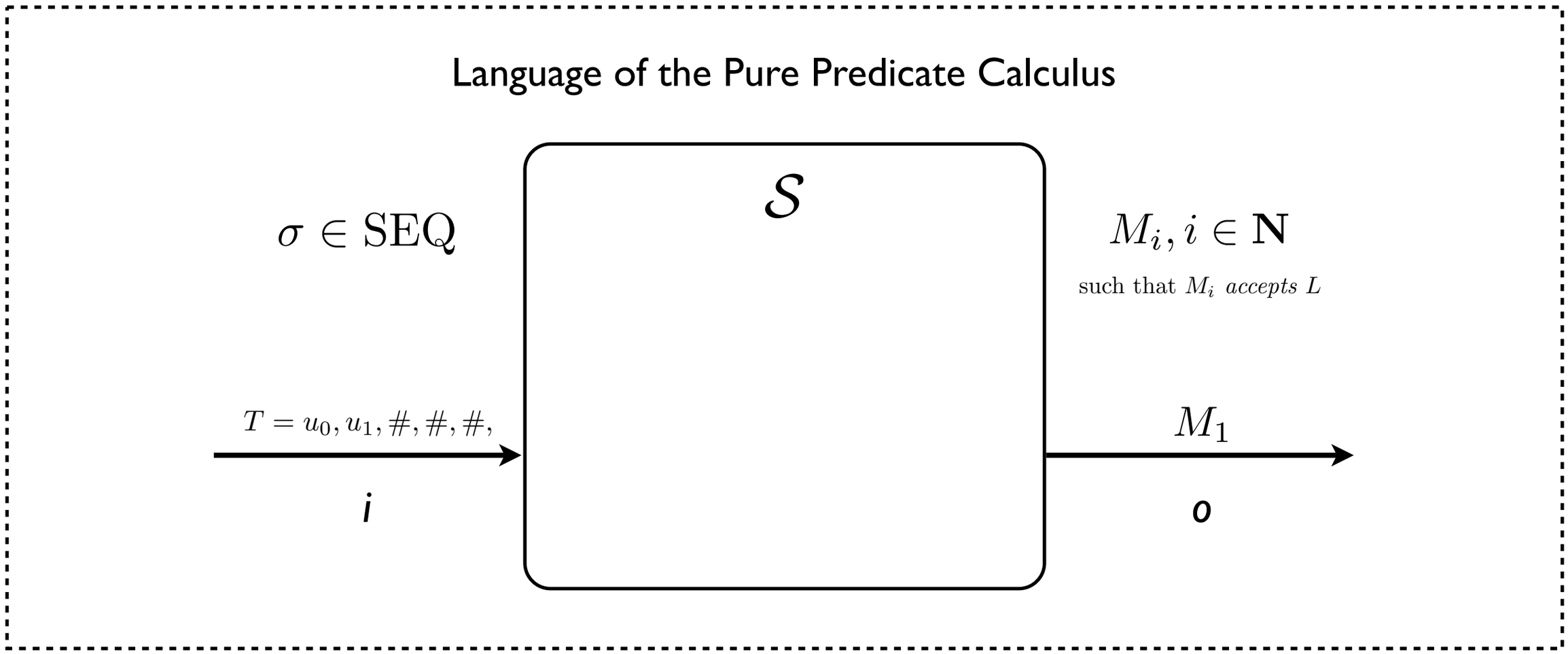

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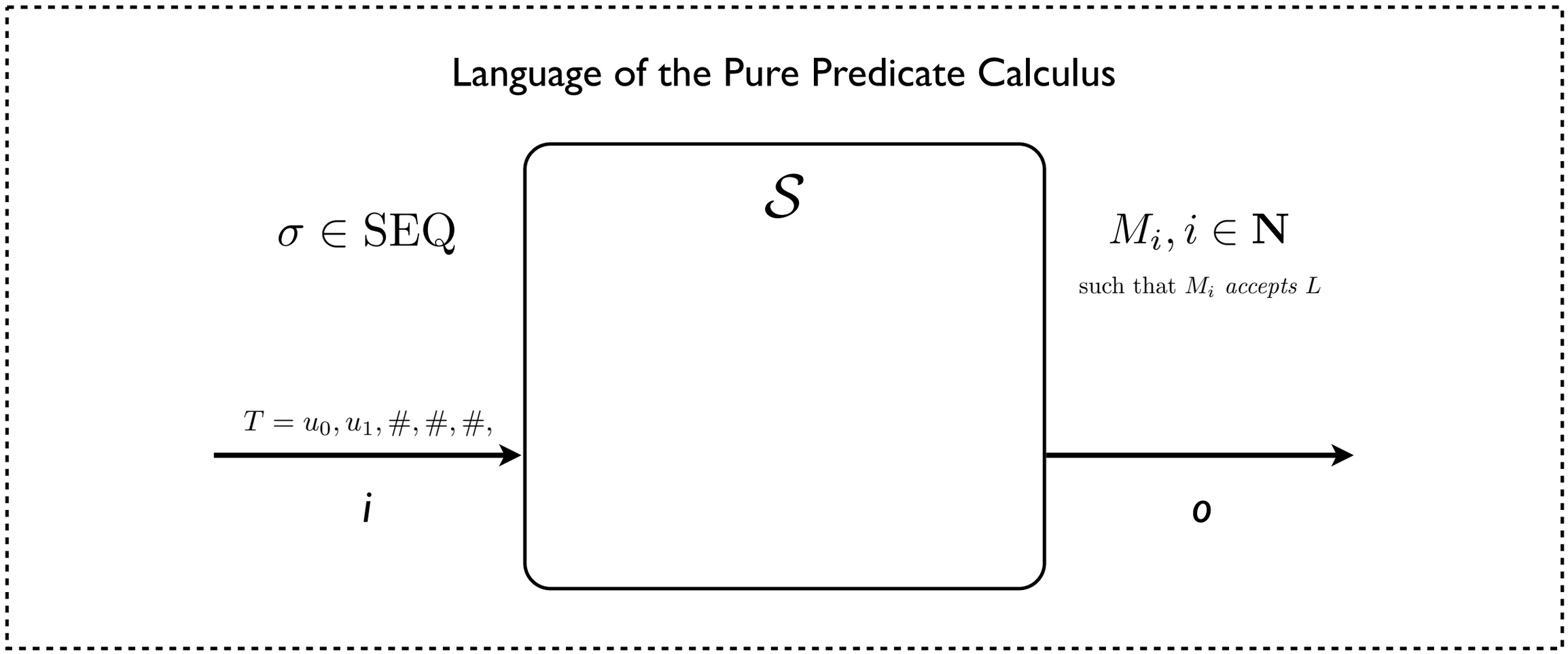

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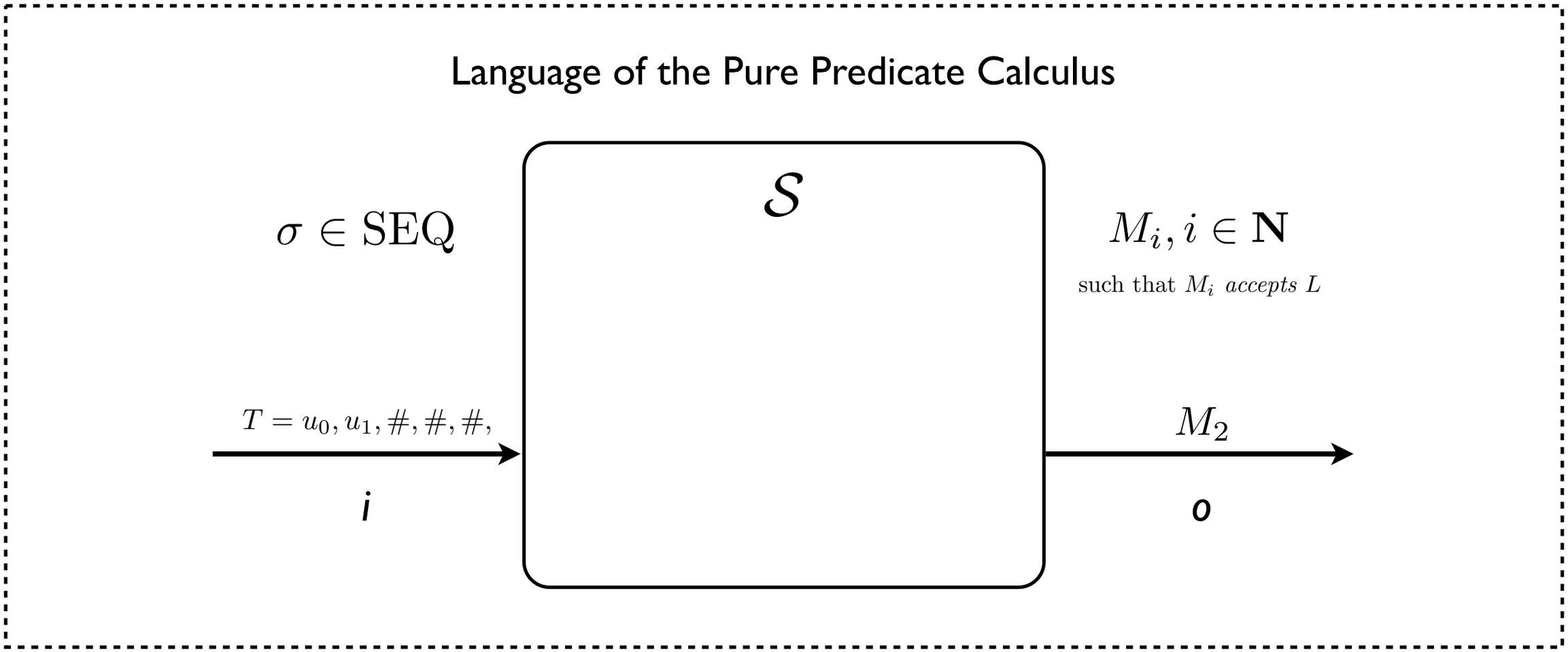

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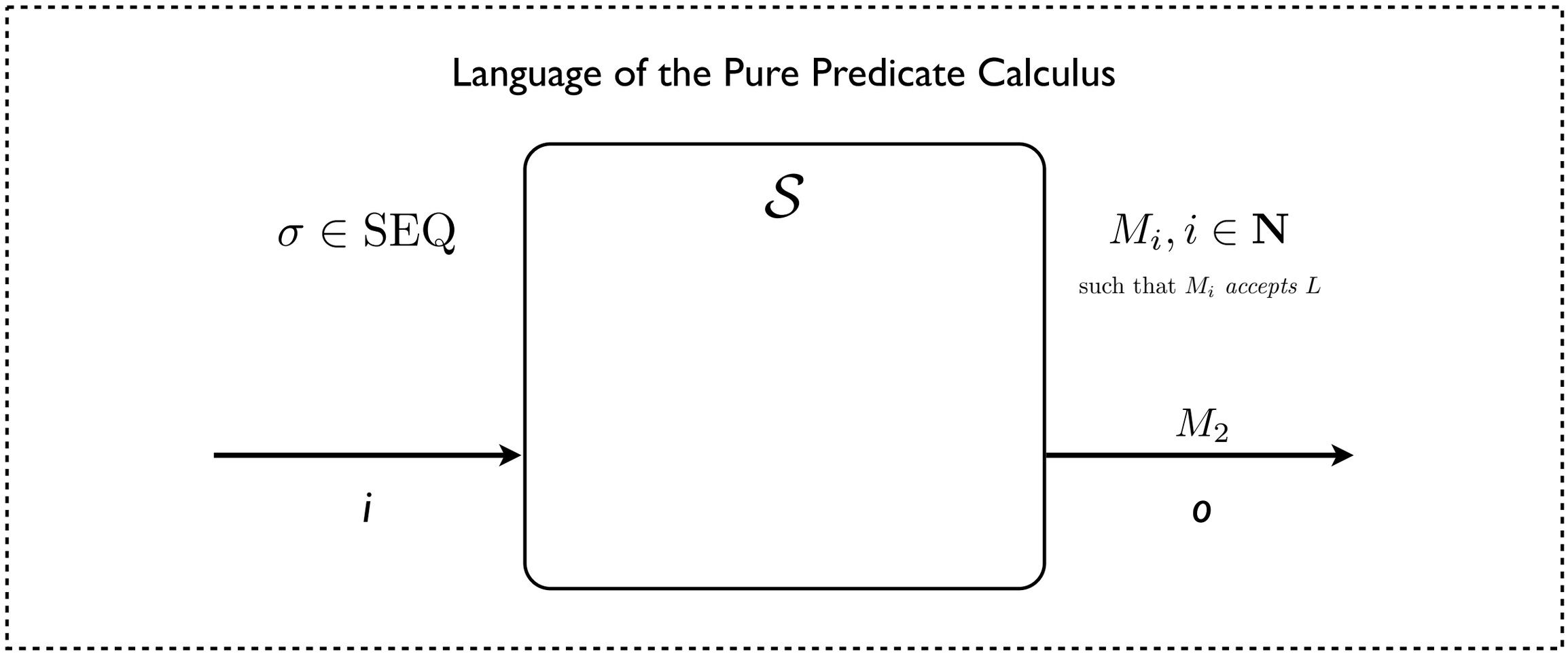

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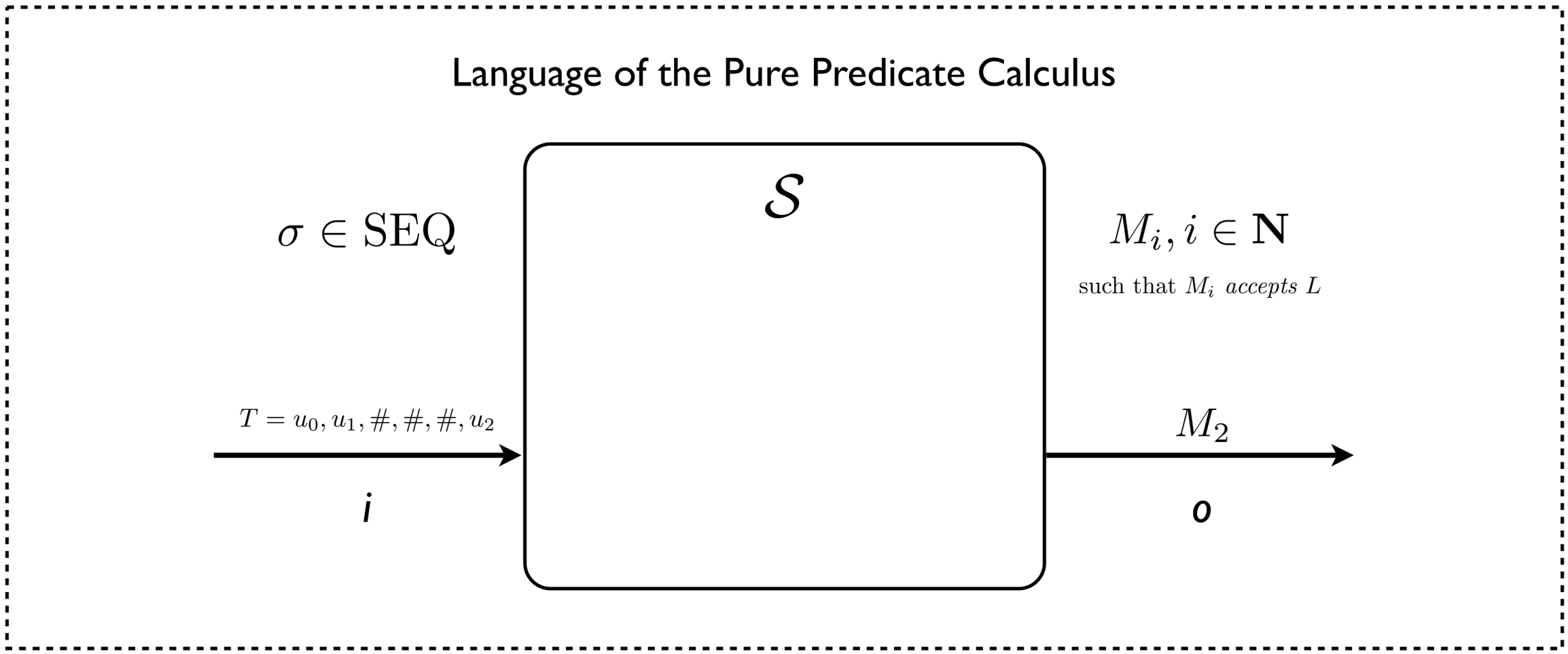

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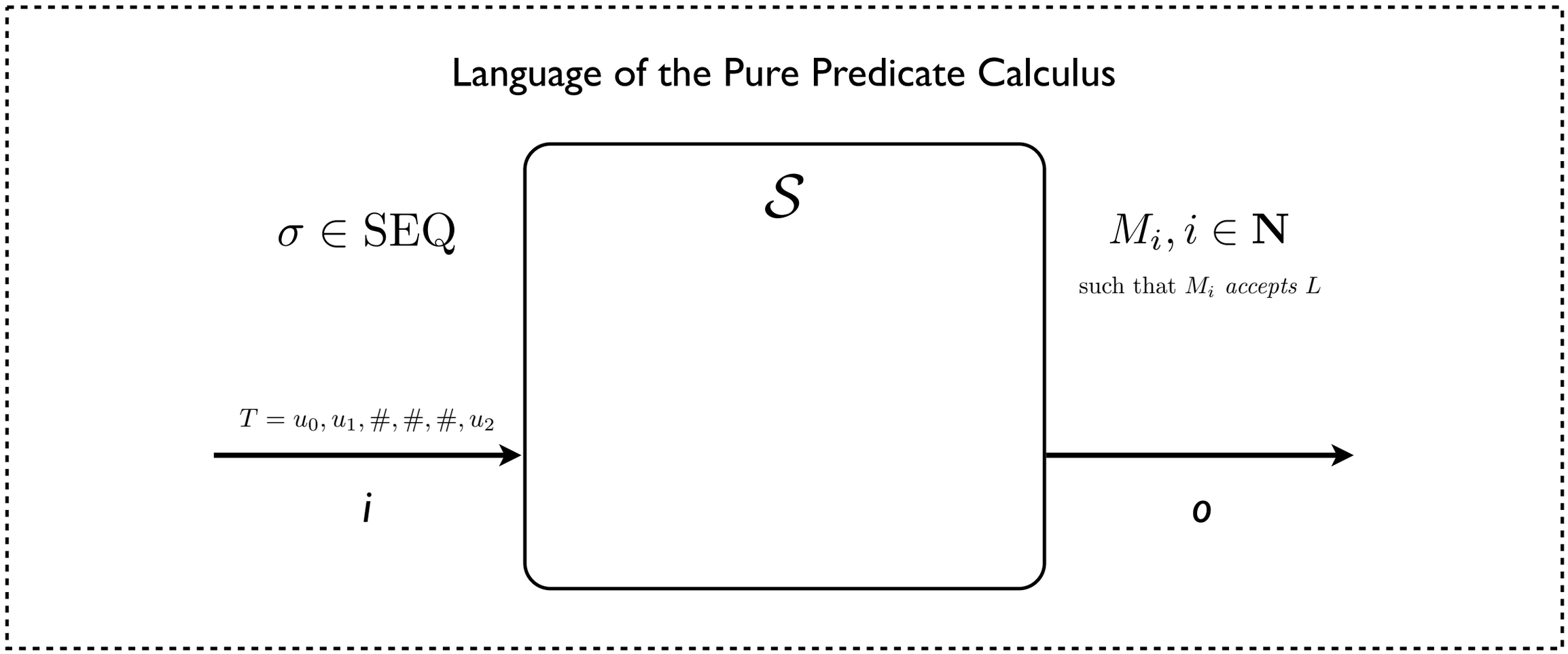

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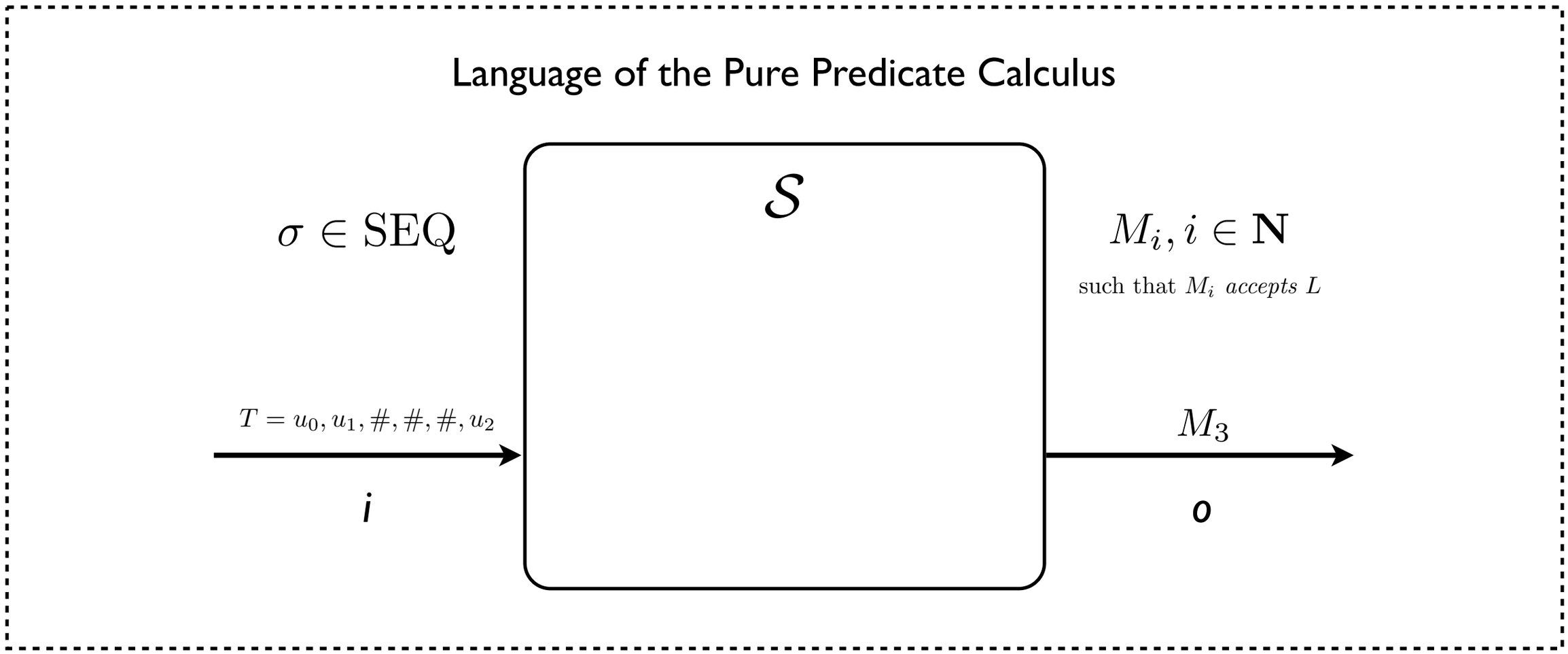

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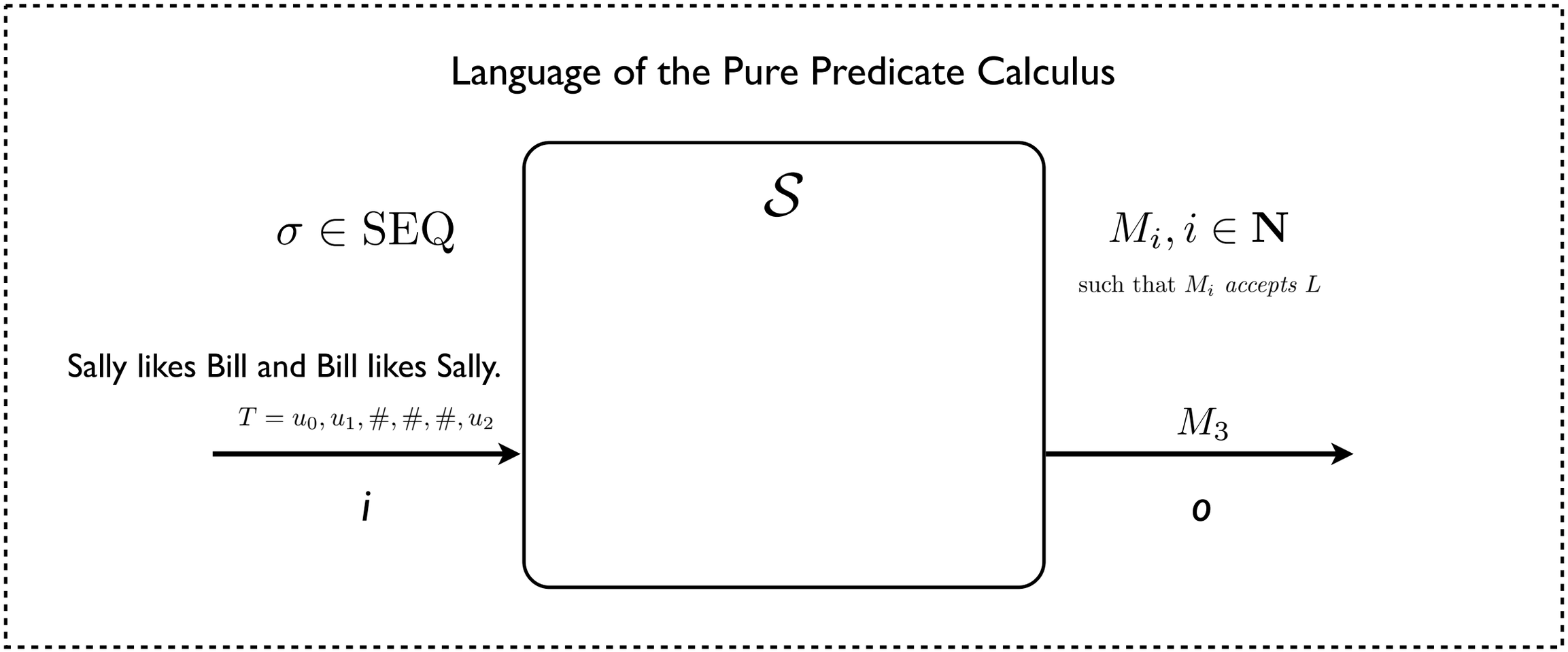

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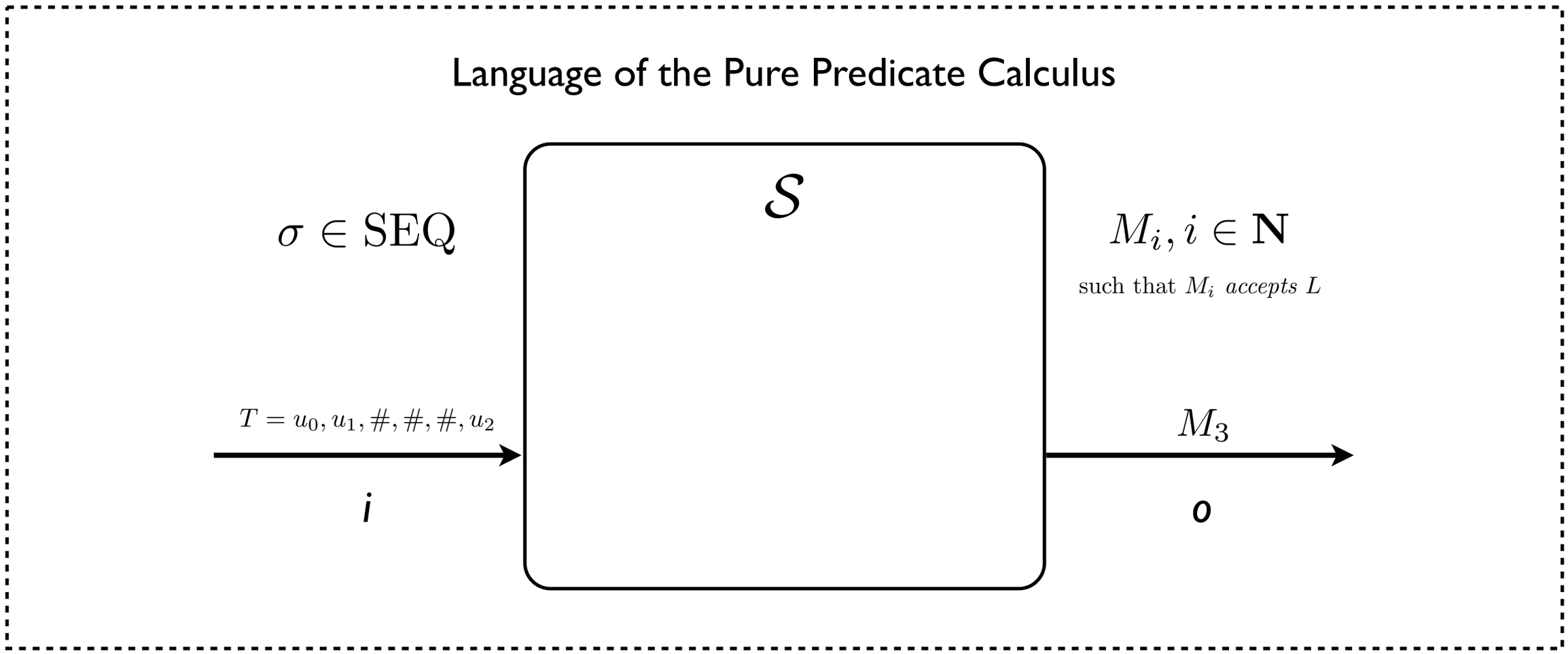

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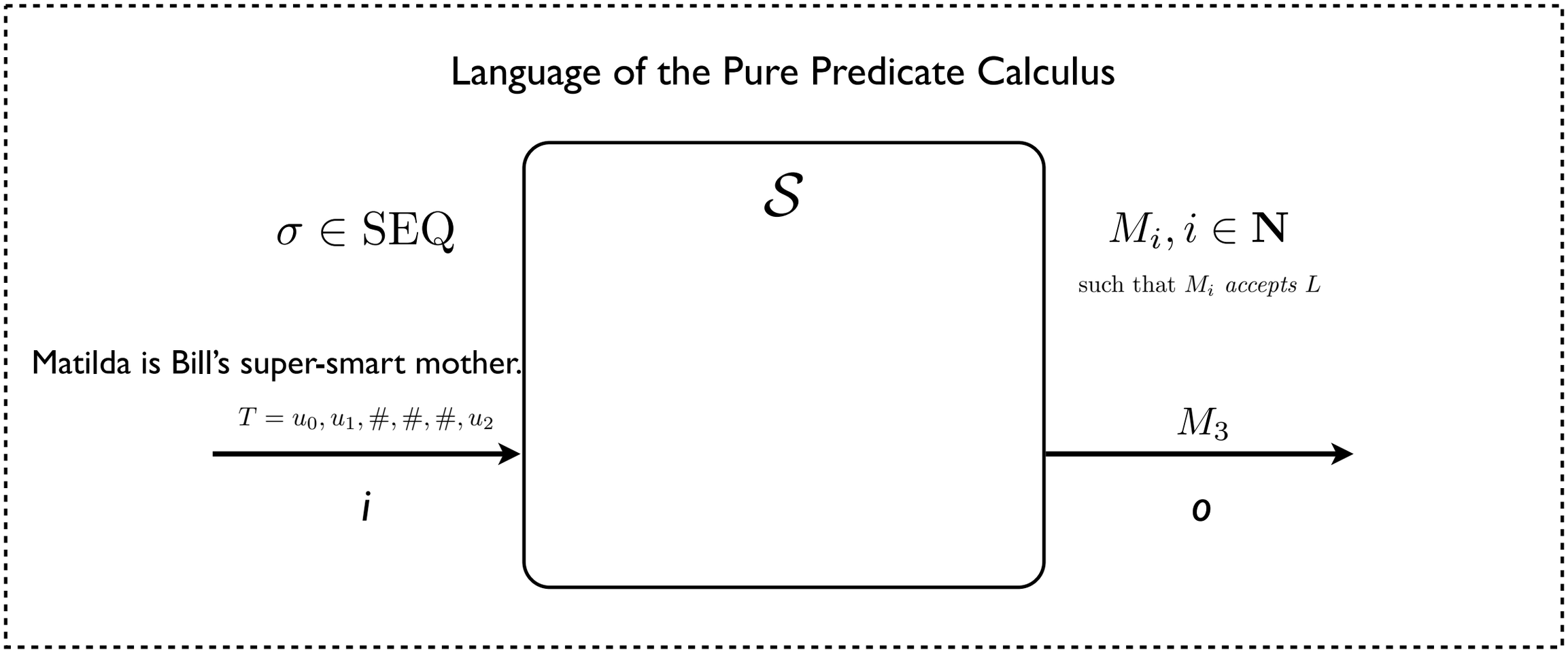

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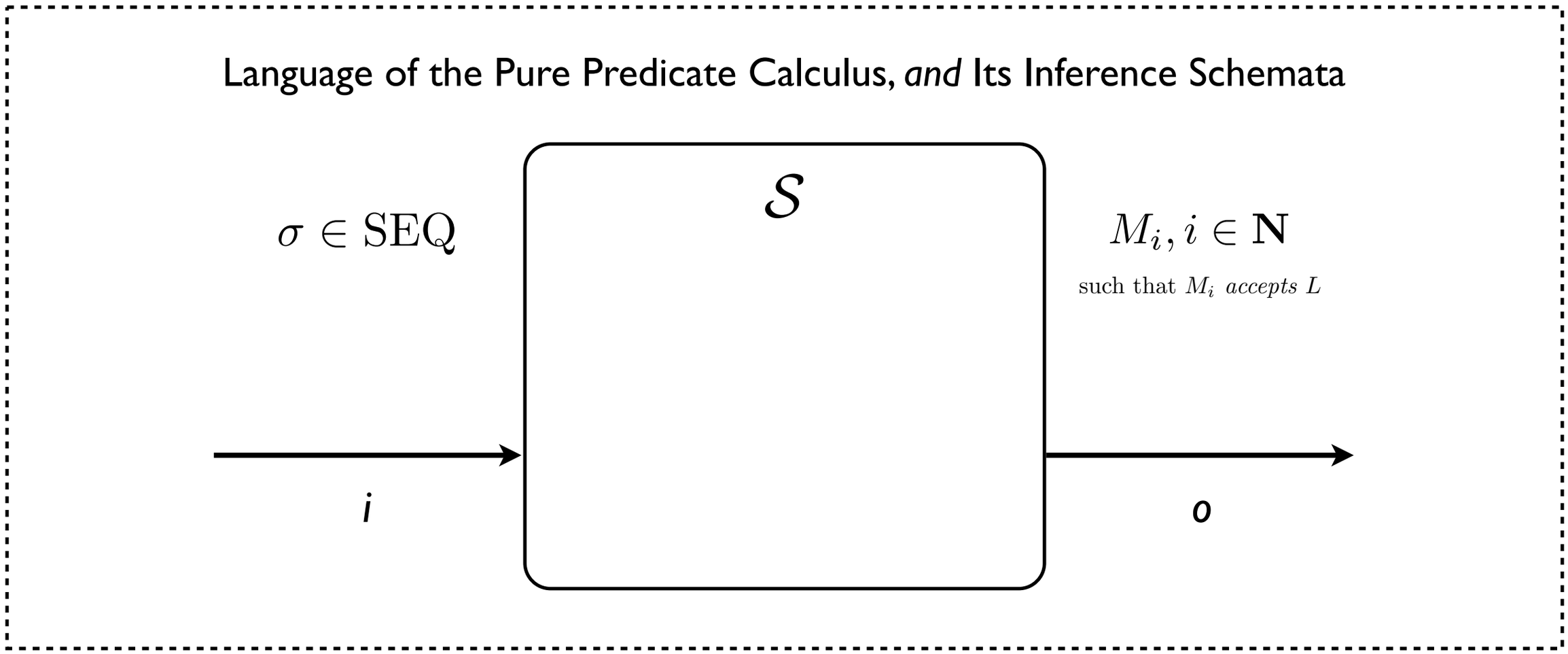

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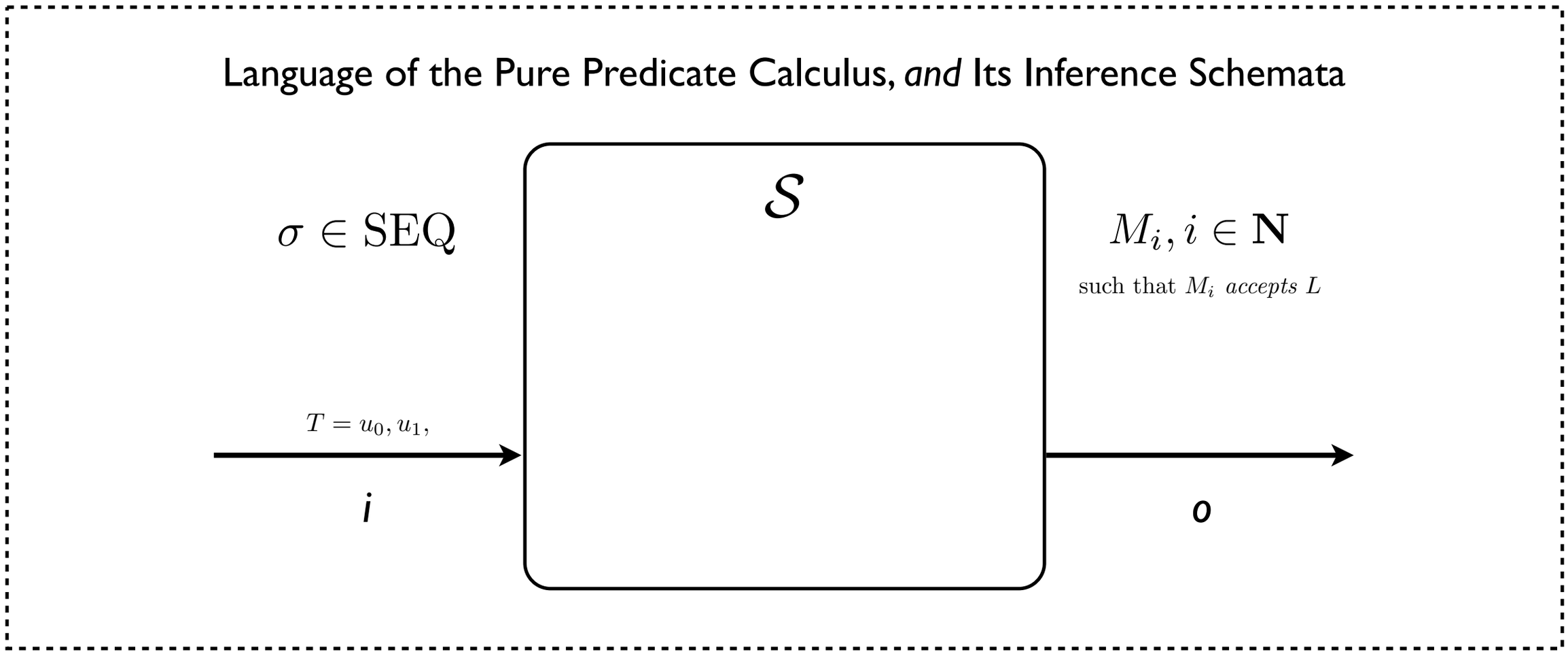

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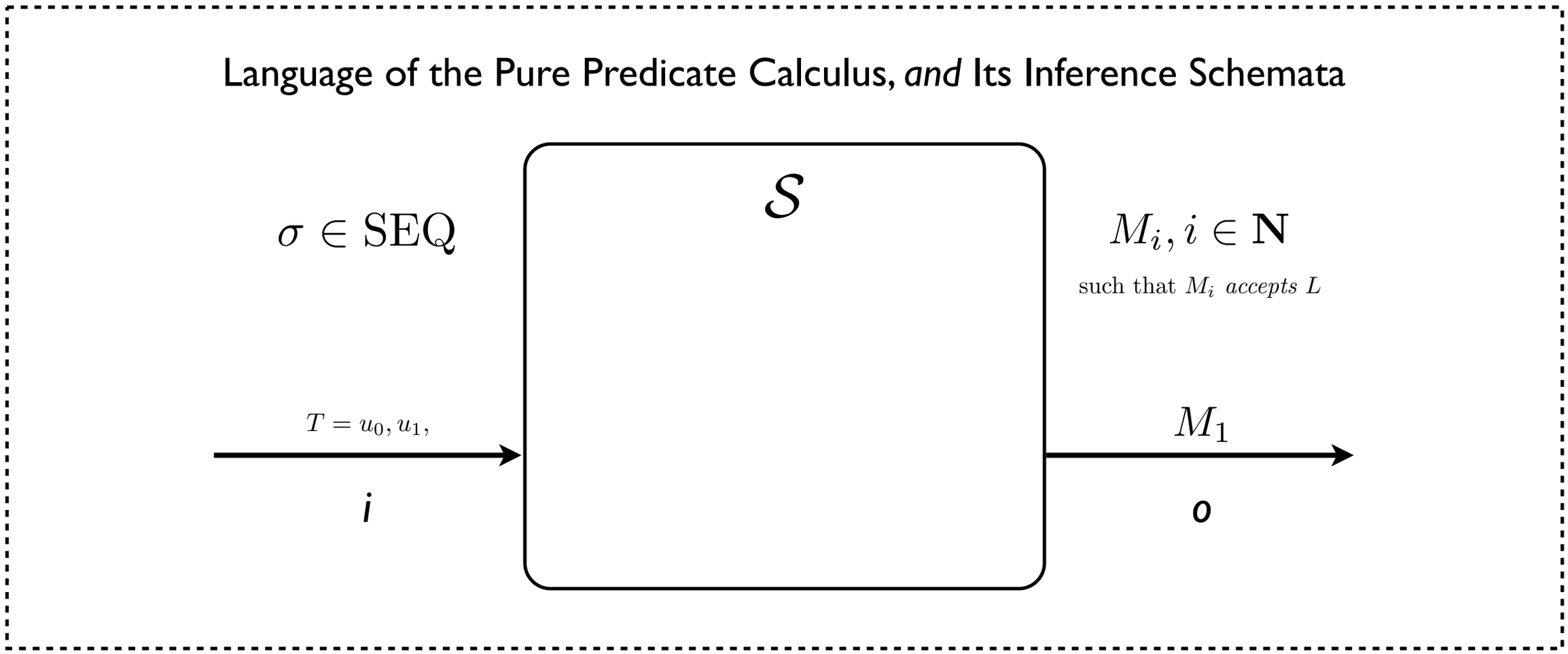

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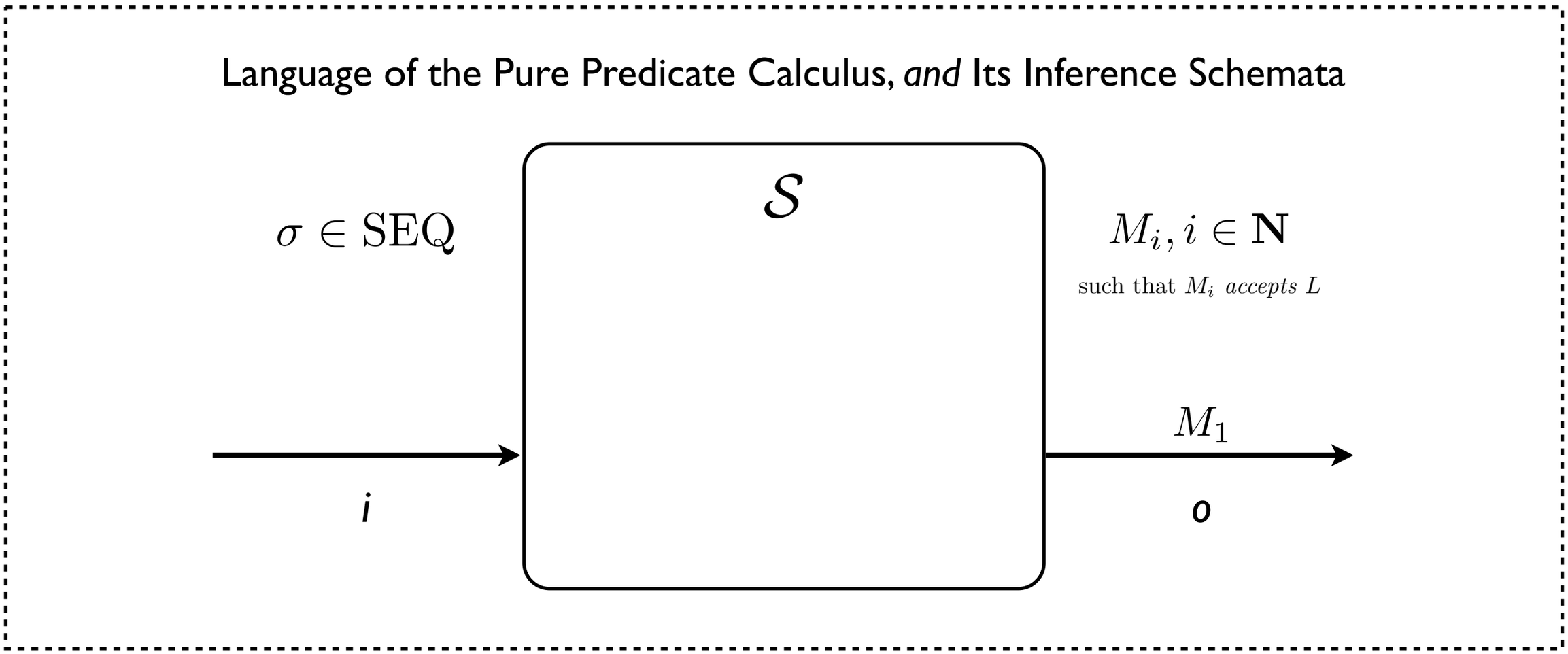

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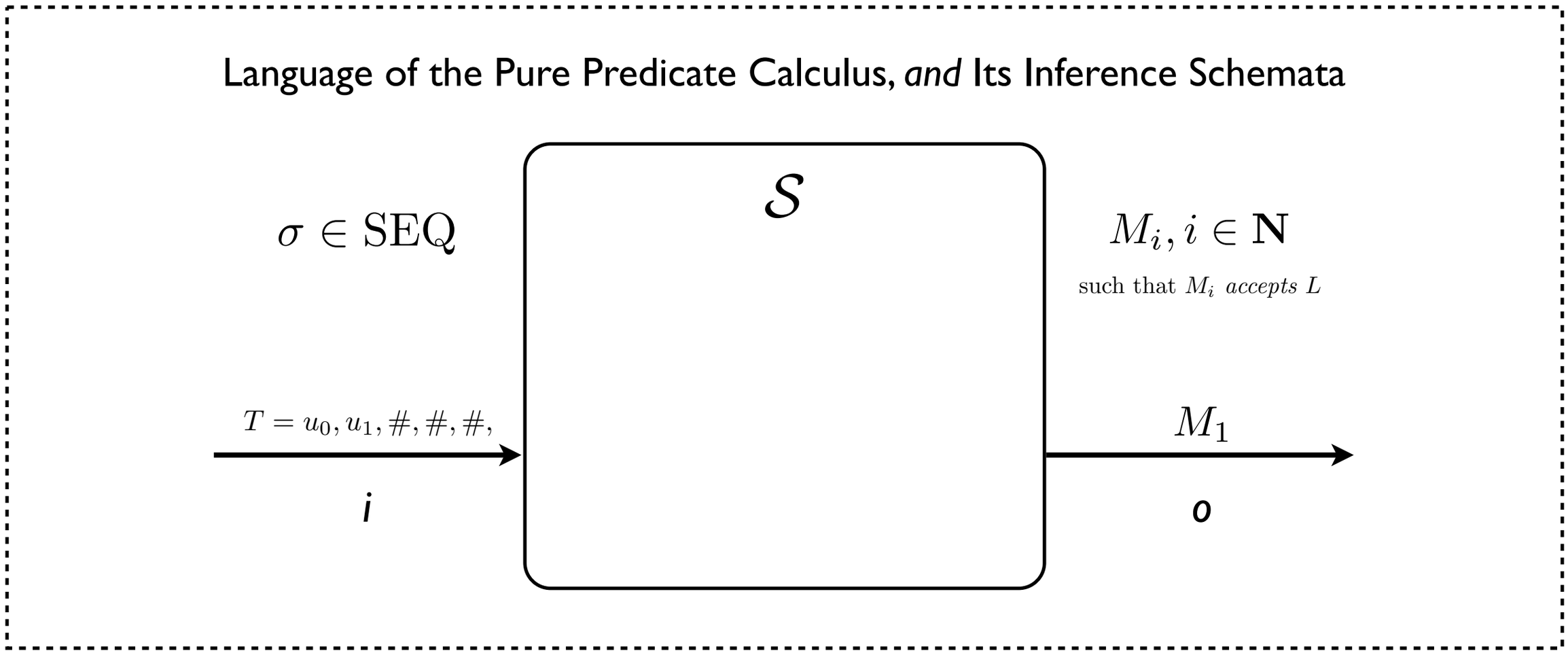

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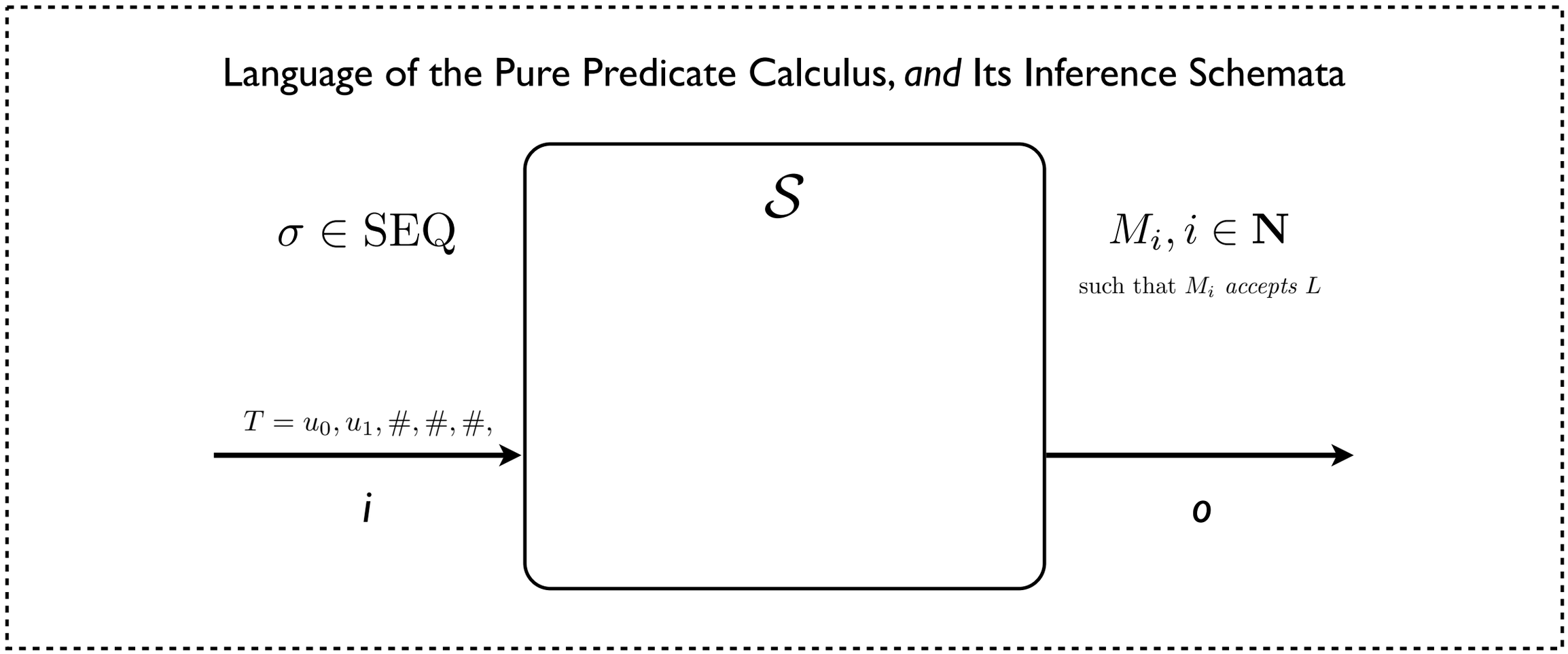

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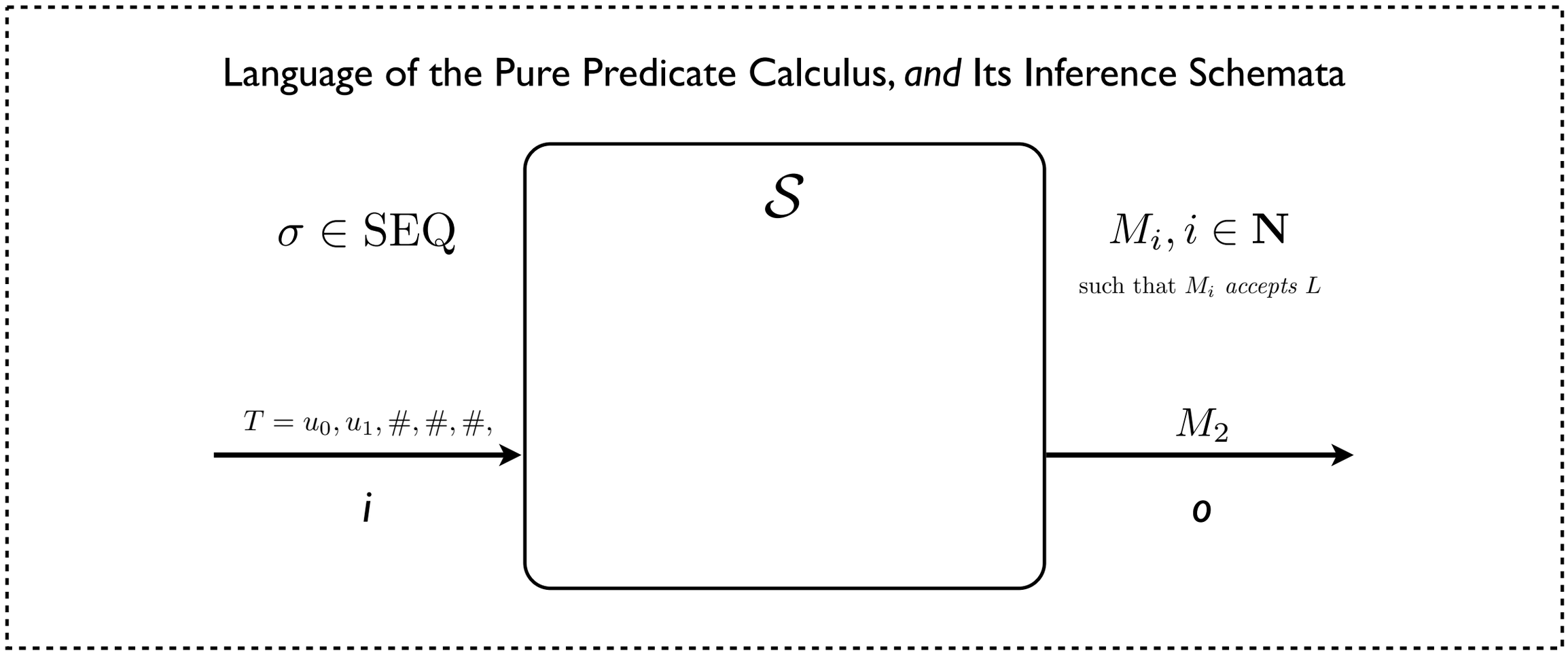

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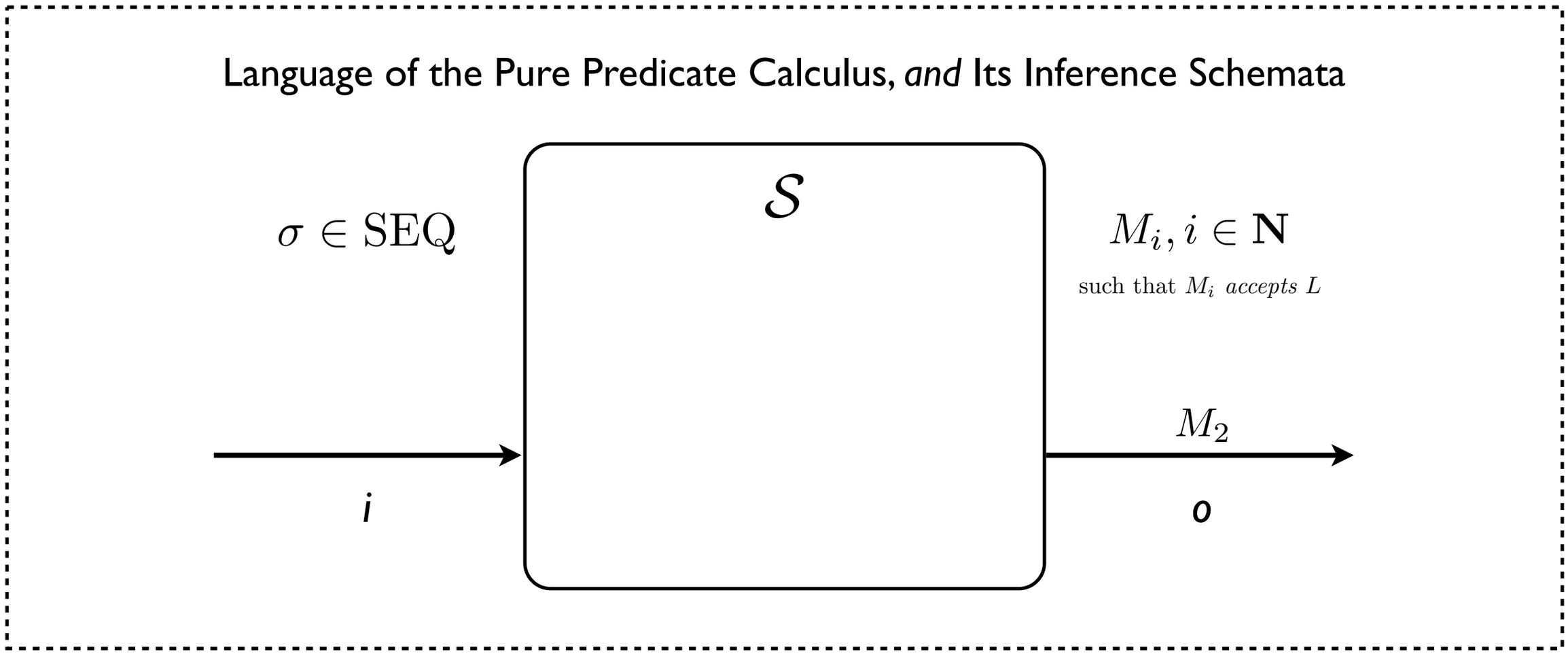

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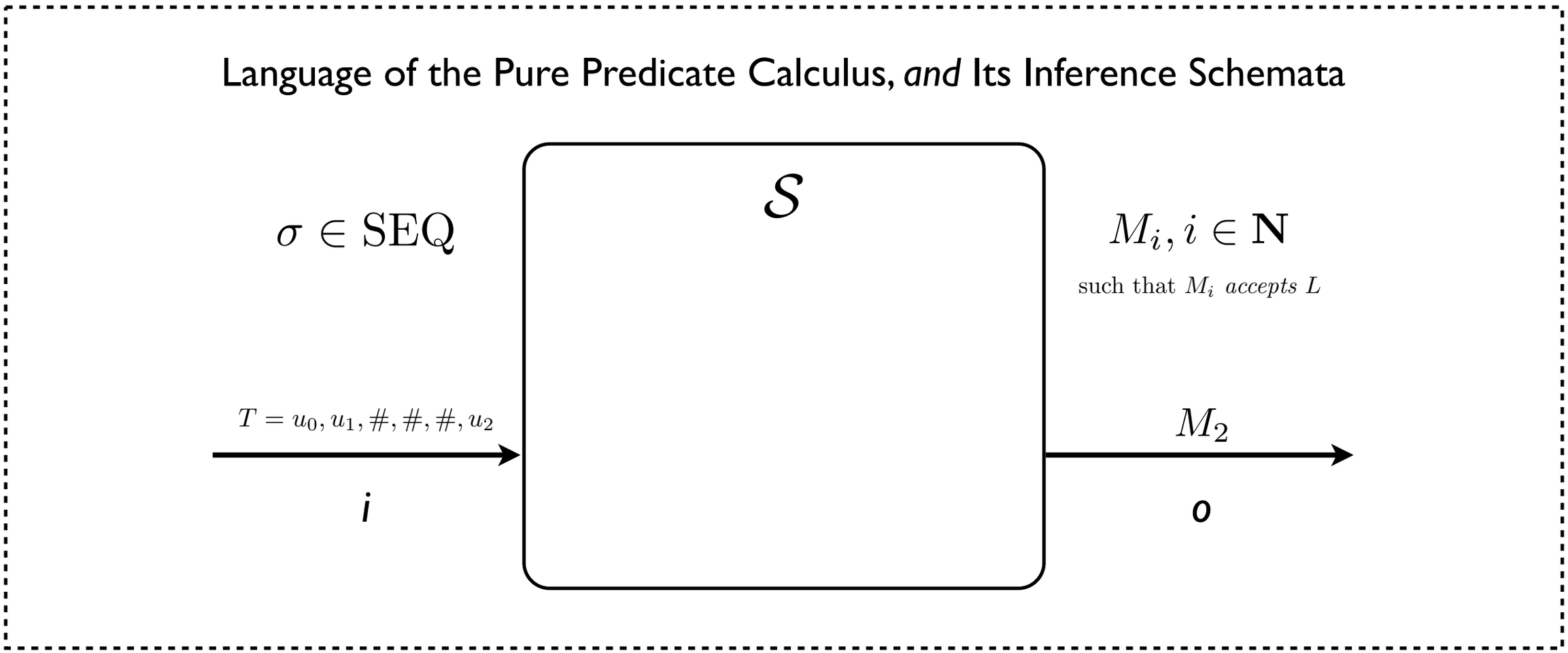

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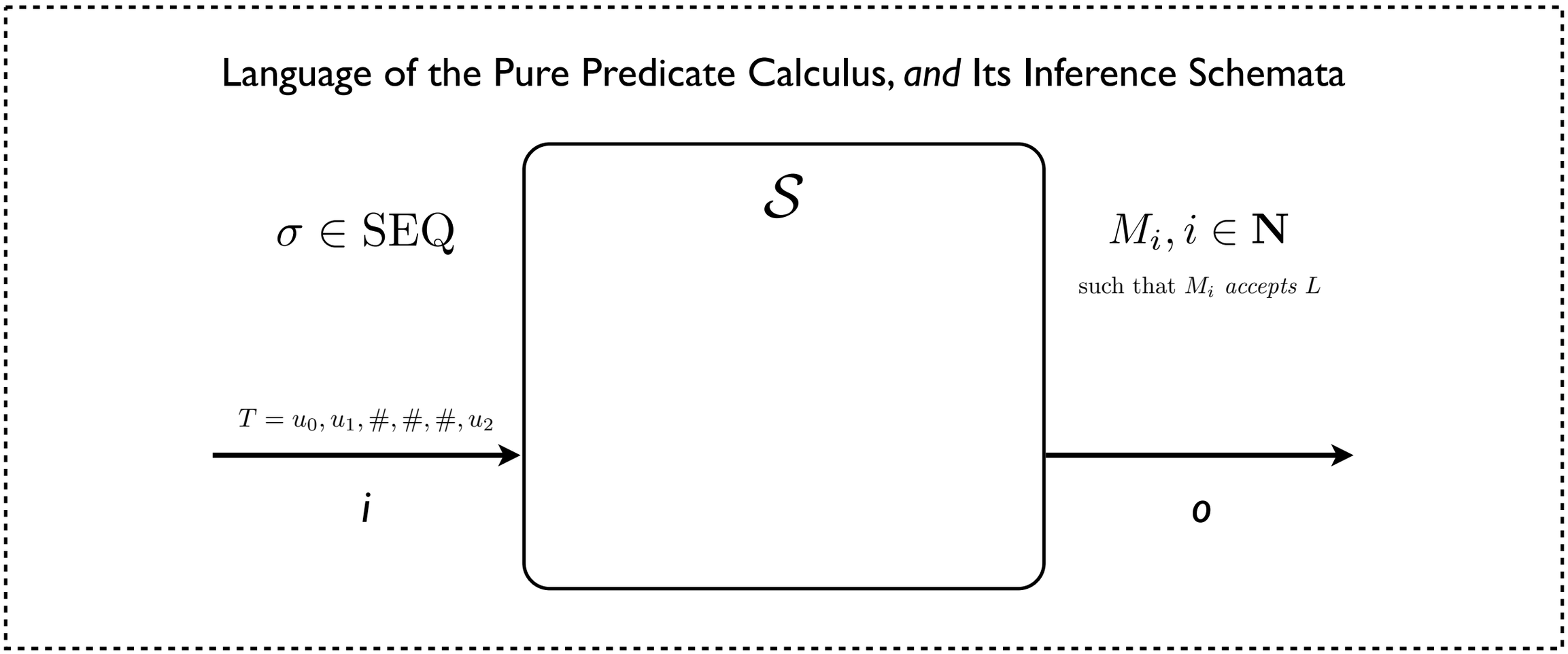

And when is identification of *L* achieved?

$$\forall^{\infty} n \in \mathbf{N}\ \mathcal{S}(T[n]) = M_k$$

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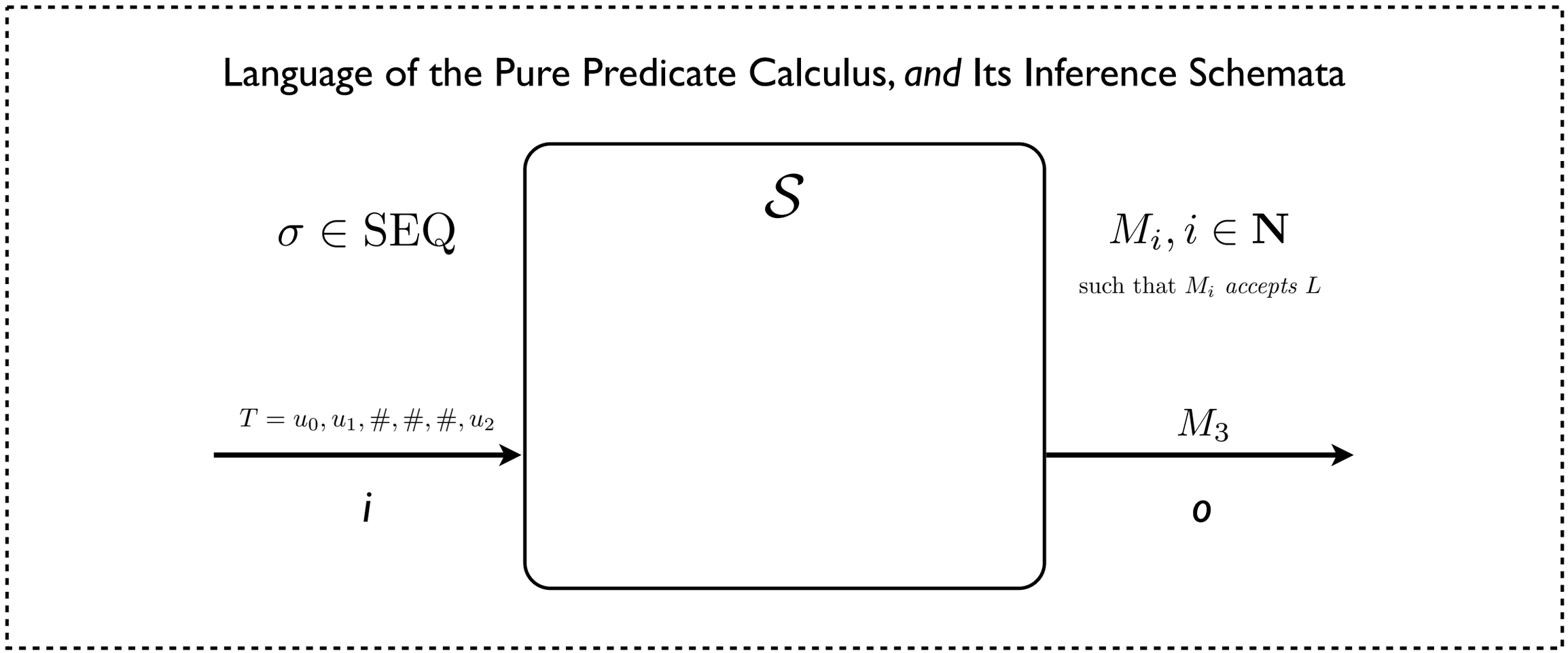

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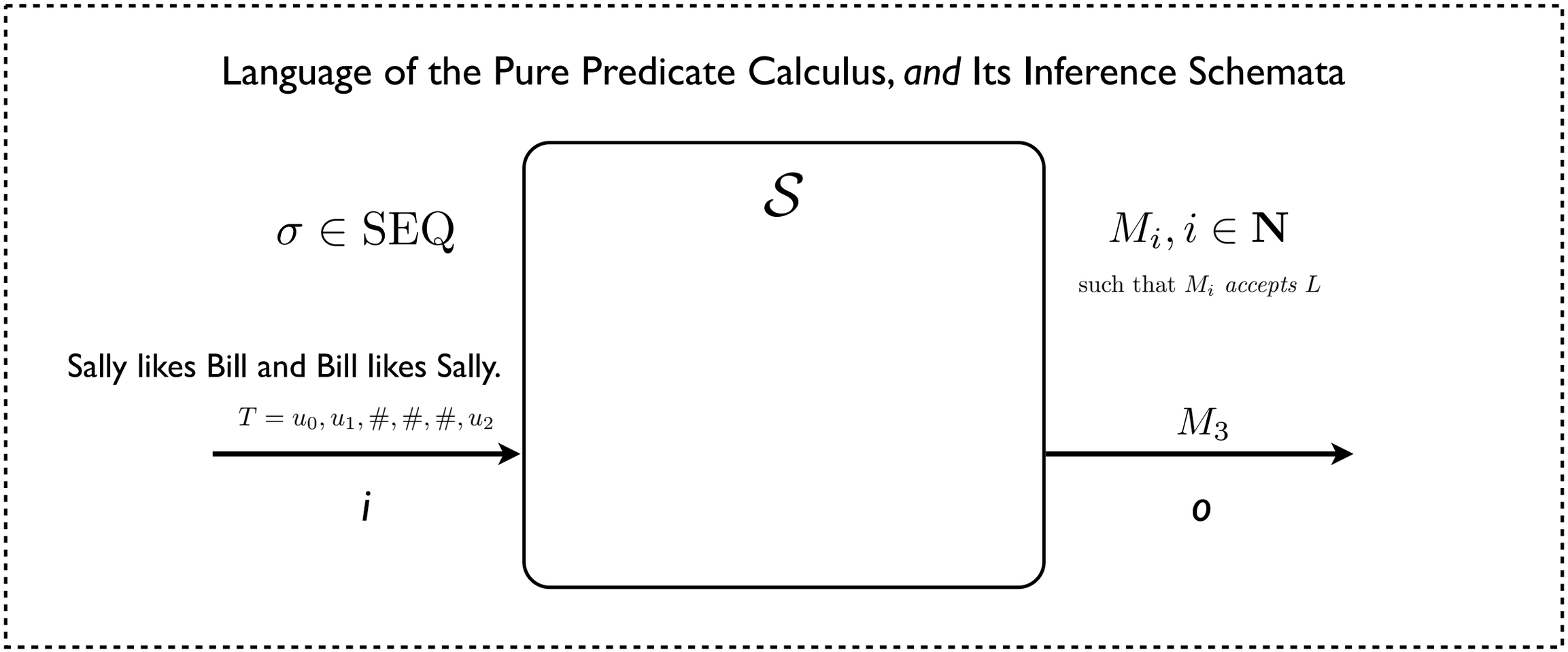

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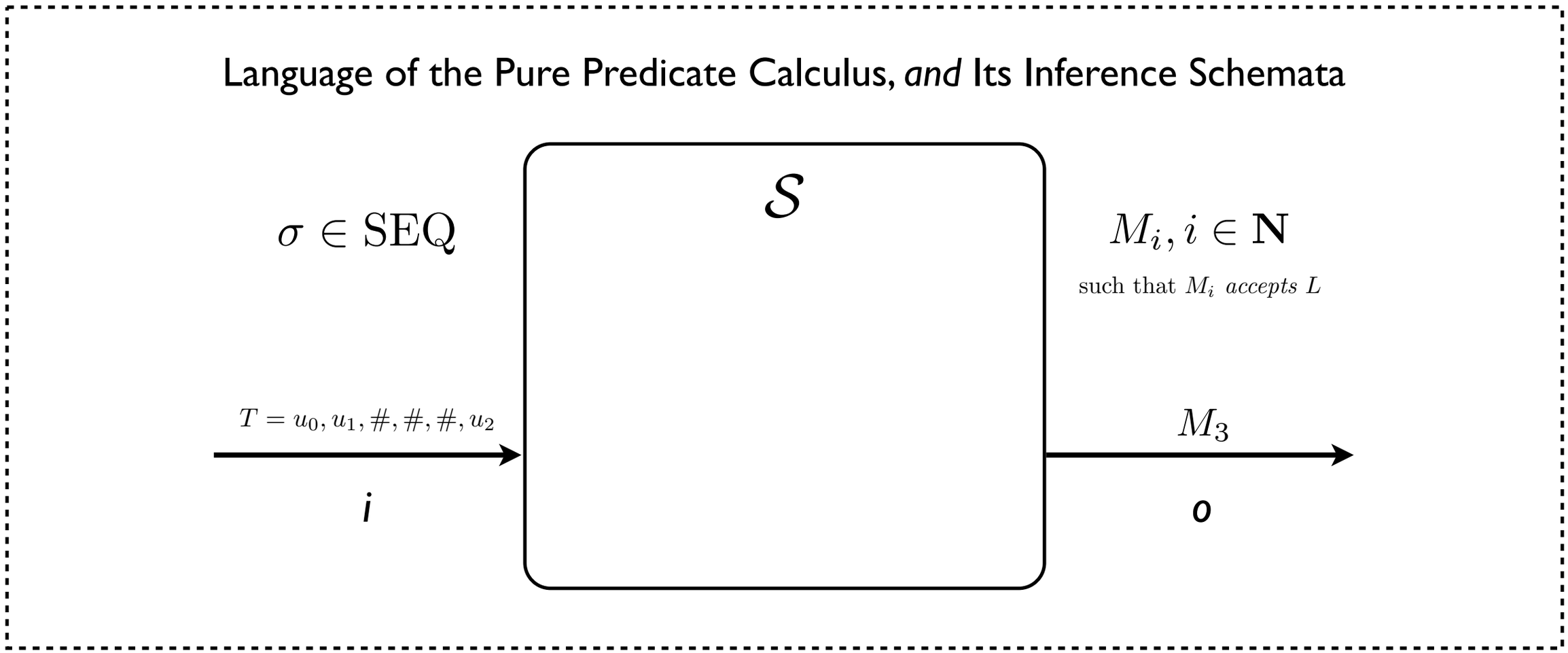

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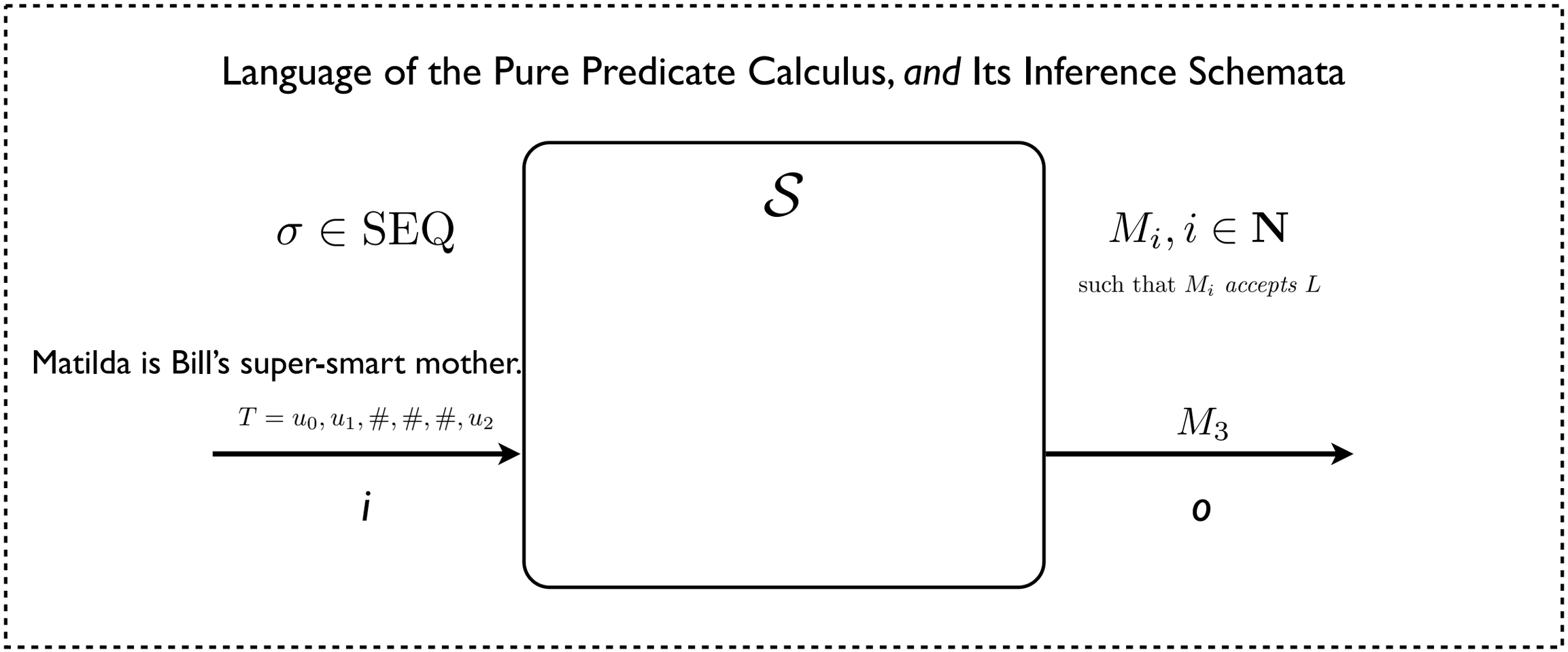

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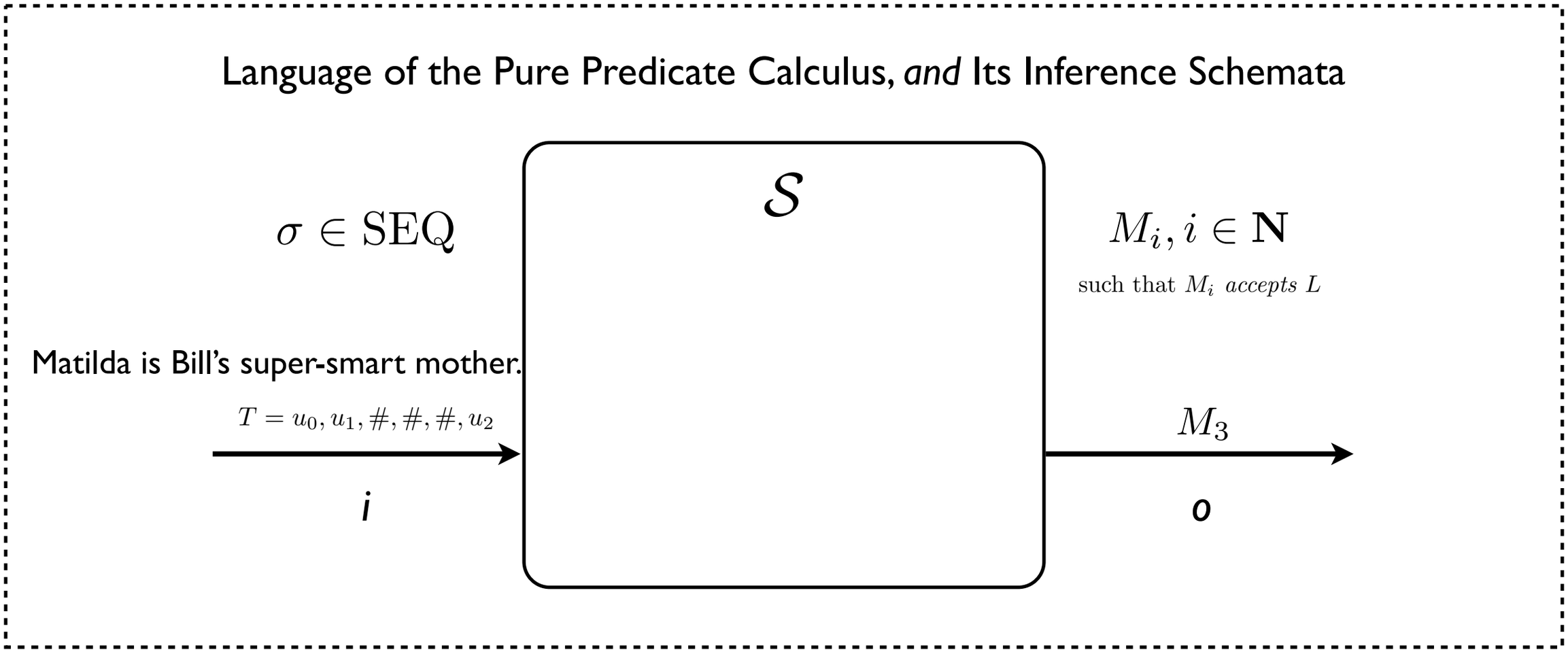

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Sally likes Bill and Bill likes Sally.
Bill likes Sally.

$$\frac{Lsb \wedge Lbs}{Lbs}$$

RAIR Lab needs to work out the details and engineer an AI that can learn the language of the pure predicate calculus!

Future Work

- What about learning the language(s) of mathematics?
- What about learning to be self-conscious?