Informal Intro to “Gold”-Style Language Learning

With a Sample Target of RAIR-Lab Interest

(or: Human Language Acquisition is Hard!; Sorry Nonhuman Animals)

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Naveen Sundar Govindarajulu
Are Humans Rational?
RPI
11/18/19

How would you formalize/model language learning in the human case?
Informal Intro to Gold-style Learning ...
Formalization of ...

1. *theoretically possible realities;*
Formalization of ...

1. *theoretically possible realities*;

2. *intelligible hypotheses* (each of which imply that a given theoretically possible reality *is* reality);
Formalization of ...

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2. *intelligible hypotheses* (each of which imply that a given theoretically possible reality is reality);

3. *the data about any given theoretically possible reality, were it actual* (the list corresponds to things the language learner perceives in the empirical world);
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4. *the (language, machine, system, …) learner*;
Formalization of ...

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2. *intelligible hypotheses* (each of which imply that a given theoretically possible reality is reality);

3. *the data about any given theoretically possible reality, were it actual* (the list corresponds to things the language learner perceives in the empirical world);

4. the (*language, machine, system, …*) learner;

5. *successful/unsuccessful behavior by a learner trying to learn a given, possible reality.*
Simple Game Setup
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• A set of positive integers is *describable* iff it can be uniquely described by an English expression. Let $D$ be the set of such sets.
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• E.g., “all positive even integers.” I.e., \{2, 4, 6, 8, \ldots\}. These are the aforementioned theoretically possible realities.
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- $C$ is a proper subset of $D$ defined as follows: All sets that contain every positive integer, save for one.
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• $C$ is a proper subset of $D$ defined as follows: All sets that contain every positive integer, save for one.

• E.g., here is a member of $C$: “all positive integers except for 2.” I.e., $\{1, 3, 4, 5, \ldots\}$.
More Setup
Nature: “I’ve selected a member of $C$. You, (language-learning) Child, must discover the set I have in mind. I shall give you clues, as follows. My secret member shall be ordered in a list $L$ that contains all members of my secret set, and I shall then present the members of $L$ one at a time to you. Each time I do so, you offer a hypothesis in the form of an English expression that uniquely describes a set of positive integers. You win the game iff you make only a finite number of conjectures, and the last one is correct.”
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• Nature: “Let’s play!”
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• Nature: “Let’s play!”

• Child: “Okay!”
Some Playing
Some Playing

• \( N: 1. \)
Some Playing

- N: I.
- Child: Silence.
Some Playing

- N: 1.
- Child: Silence.
- N: 3.
Some Playing

• N: 1.

• Child: Silence.

• N: 3.

• Child: “All positive integers except for 2.”
Some Playing

• N: 1.

• Child: Silence.

• N: 3.

• Child: “All positive integers except for 2.”

• N (multiple steps): 4, 5, 6, 7, 8, 9, 10, 11, 12, 13.
Some Playing

- N: 1.
- Child: Silence.
- N: 3.
- Child: “All positive integers except for 2.”
- N (multiple steps): 4, 5, 6, 7, 8, 9, 10, 11, 12, 13.
- Child: Silence.
Some Playing

- N: 1.
- Child: Silence.
- N: 3.
- Child: “All positive integers except for 2.”
- N (multiple steps): 4, 5, 6, 7, 8, 9, 10, 11, 12, 13.
- Child: Silence.
- N: 2.
Some Playing

• N: 1.

• Child: Silence.

• N: 3.

• Child: “All positive integers except for 2.”

• N (multiple steps): 4, 5, 6, 7, 8, 9, 10, 11, 12, 13.

• Child: Silence.

• N: 2.

• N (multiple steps): 15, 16, 17, 18, 19, 20, 21, 22, 23, 24.
Some Playing

• N: 1.

• Child: Silence.

• N: 3.

• Child: “All positive integers except for 2.”

• N (multiple steps): 4, 5, 6, 7, 8, 9, 10, 11, 12, 13.

• Child: Silence.

• N: 2.

• N (multiple steps): 15, 16, 17, 18, 19, 20, 21, 22, 23, 24.

• Child: 14.
Some Playing

- N: 1.
- Child: Silence.
- N: 3.
- Child: “All positive integers except for 2.”
- N (multiple steps): 4, 5, 6, 7, 8, 9, 10, 11, 12, 13.
- Child: Silence.
- N: 2.
- N (multiple steps): 15, 16, 17, 18, 19, 20, 21, 22, 23, 24.
- Child: 14.
- N: 14.
Some Playing

- N: 1.
- Child: Silence.
- N: 3.
  - Child: “All positive integers except for 2.”
- N (multiple steps): 4, 5, 6, 7, 8, 9, 10, 11, 12, 13.
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  - Child: 14.
- N: 14.
- ...
Some Playing

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• Child: Silence.
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• ...
Guessing Algorithm (G)
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Let $S$ be the set of numbers presented so far. And let $m$ be the smallest positive integer that isn’t a member of $S$. Output the hypothesis: “All positive integers except for $m$.” If this was your last hypothesis, remain silent.
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So, e.g., $G([4, 5, 8, 1]) = 2.$
Theorem 1.
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Regardless of what member of $C$ is chosen by Nature at the start of the game, and no matter what list $L$ is produced from that member, $G$ will win the game for you!
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(Sedulous-and-inquisitive: Prove it!)
One Slight Mod
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We stipulate that $\mathbb{Z}^+$ is in $C$. 
Theorem 2.
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There exists no algorithm that is guaranteed to win this new game. More precisely, for every algorithm $A$ there is a set in the (expanded!) set $C$, and a list $L$ of $C$, such that $A$ fails to produce a last, correct hypothesis.
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Some Issues to Keep in Mind
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1. For this class, and where we are in this class, a language is no doubt more appropriate than a set of integers as a “possible reality,” but the idea is that a set of integers can “code” a language ...
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2. The set of possible realities must on this approach be *countable*. What then about physical quantities whose values are arbitrary real numbers? Isn’t this what we see in physics as carried out by real physicists?
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- Of course, given our Theorems 1 & 2 from above, we can in some sense know that the system Child is using will win. But the Child himself/herself doesn’t know.
CLT-based Model of Language Learning

\[ \sigma \in \text{SEQ} \]

\[ S \]

\[ L \text{ (r.e.)} \]

\[ M_i, i \in \mathbb{N} \] such that \( M_i \) accepts \( L \)
CLT-based Model of Language Learning

\[ S \]

\[ L \text{ (r.e.)} \]

\[ \sigma \in \text{SEQ} \]

\[ i \]

\[ i \]

\[ o \]

\[ M_i, i \in \mathbb{N} \]

such that \( M_i \) accepts \( L \)

Turing machine
CLT-based Model of Language Learning

\[ M_i, i \in \mathbb{N} \text{ such that } M_i \text{ accepts } L \]

\[ \sigma \in \text{SEQ} \]

\[ T = u_0, u_1, \]

Turing machine
CLT-based Model of Language Learning

\[ L \text{ (r.e.)} \]

\[ \sigma \in \text{SEQ} \]

\[ T = u_0, u_1, \]

\[ i \]

\[ M_i, i \in \mathbb{N} \text{ such that } M_i \text{ accepts } L \]

\[ M_1 \]

\[ o \]

Turing machine
CLT-based Model of Language Learning

Let $\sigma \in \text{SEQ}$ and $M_i, i \in \mathbb{N}$ be such that $M_i$ accepts $L$ (r.e.).
CLT-based Model of Language Learning

\[ T = u_0, u_1, \#, \#, \# , \]  

such that \( M_i, i \in \mathbb{N} \) accepts \( L \)
CLT-based Model of Language Learning

\[ T = u_0, u_1, \#, \#, \# \]

such that \( M_i, i \in \mathbb{N} \) accepts \( L \)
CLT-based Model of Language Learning

$\sigma \in \text{SEQ}$

$T = u_0, u_1, \#, \#, \#$

$i$

$L \ (\text{r.e.})$

$S$

$M_i, i \in \mathbb{N}$

such that $M_i$ accepts $L$

$M_2$

$o$

Turing machine
CLT-based Model of Language Learning

\[
\sigma \in \text{SEQ} \\
\begin{array}{c}
S \\
L \text{ (r.e.)}
\end{array} \\
M_i, i \in \mathbb{N} \\
\text{such that } M_i \text{ accepts } L
\]

Turing machine
CLT-based Model of Language Learning

\[ L \text{ (r.e.)} \]

\[ M_i, i \in N \]

such that \( M_i \) accepts \( L \)

\[ \sigma \in \text{SEQ} \]

\[ T = u_0, u_1, \#, \#, \#, u_2 \]

\[ i \]

\[ M_2 \]

\[ 0 \]
CLT-based Model of Language Learning

\[ \sigma \in \text{SEQ} \]

\[ T = u_0, u_1, \#, \#, \#, u_2 \]

\[ i \rightarrow S \]

\[ L \text{ (r.e.)} \]

\[ M_i, i \in \mathbb{N} \]

such that \( M_i \) accepts \( L \)

Turing machine
CLT-based Model of Language Learning

\[ \sigma \in \text{SEQ} \]

\[ T = u_0, u_1, \#, \#, \#, u_2 \]

\[ i \]

\[ S \]

\[ L \text{ (r.e.)} \]

\[ M_i, i \in \mathbb{N} \]

such that \( M_i \) accepts \( L \)

Turing machine
CLT-based Model of Language Learning

And when is identification of $L$ achieved?

$T = u_0, u_1, \#, \#, \#, u_2$

$\sigma \in \text{SEQ}$

$M_i, i \in \mathbb{N}$

such that $M_i$ accepts $L$

$M_3$

$\emptyset$

Turing machine
CLT-based Model of Language Learning

\[ S \]

\[ M_i, i \in \mathbb{N} \]

such that \( M_i \) accepts \( L \)

And when is identification of \( L \) achieved?

\[ \forall \infty n \in \mathbb{N} \quad S(T[n]) = M_k \]
CLT-based Model of Language Learning

$\sigma \in \text{SEQ}$

$T = u_0, u_1, \#, \#, \#, u_2$

$i$

$L \ (r.e.)$

$M_i, i \in \mathbb{N}$

such that $M_i$ accepts $L$

$S$

$M_3$

$0$

And when is identification of $L$ achieved?

$\forall^\infty n \in \mathbb{N} \ S(T[n]) = M_k$

$S$ identifies $L$ iff $S$ identifies every text for $L$
RAIR-Lab-relevant Target for Learning …
Chomsky Hierarchy of Languages

Elements of the Chomsky Hierarchy

- Recursively enumerable languages
- Recursive languages
- Context sensitive languages
- Context free languages
- Deterministic context free languages
- Regular languages
How About Learning the Grammar of the Pure Predicate Calculus?

\[
\begin{align*}
\text{Formula} & \Rightarrow \text{AtomicFormula} \\
& \quad | \quad (\text{Formula Connective Formula}) \\
& \quad | \quad \neg \text{Formula} \\
\text{AtomicFormula} & \Rightarrow (\text{Predicate Term}_1 \ldots \text{Term}_k) \\
& \quad | \quad (\text{Term} = \text{Term}) \\
\text{Term} & \Rightarrow (\text{Function Term}_1 \ldots \text{Term}_k) \\
& \quad | \quad \text{Constant} \\
\text{Connective} & \Rightarrow \land | \lor | \rightarrow | \leftrightarrow \\
\text{Predicate} & \Rightarrow P_1 | P_2 | P_3 \ldots \\
\text{Constant} & \Rightarrow c_1 | c_2 | c_3 \ldots \\
\text{Function} & \Rightarrow f_1 | f_2 | f_3 \ldots
\end{align*}
\]
How About Learning the Grammar of the Pure Predicate Calculus?

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<td>$\Rightarrow$ Constant</td>
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| Connective | $\Rightarrow$ $\land$ | $\lor$ | $\rightarrow$ | $\leftrightarrow$ |

| Predicate | $\Rightarrow$ $P_1$ | $P_2$ | $P_3$ | $\ldots$ |
| Constant  | $\Rightarrow$ $c_1$ | $c_2$ | $c_3$ | $\ldots$ |
| Function  | $\Rightarrow$ $f_1$ | $f_2$ | $f_3$ | $\ldots$ |

Sally likes Bill.

(Likes sally bill)
How About Learning the Grammar of the Pure Predicate Calculus?

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| AtomicFormula         | $\Rightarrow$ | (Predicate Term$_1$ ... Term$_k$) |
|                       |               | (Term $=$ Term) |

| Term                  | $\Rightarrow$ | (Function Term$_1$ ... Term$_k$) |
|                       |               | Constant |

| Connective            | $\Rightarrow$ | $\land$ | $\lor$ | $\rightarrow$ | $\leftrightarrow$ |

| Predicate             | $\Rightarrow$ | $P_1$ | $P_2$ | $P_3$ | ... |
| Constant              | $\Rightarrow$ | $c_1$ | $c_2$ | $c_3$ | ... |
| Function              | $\Rightarrow$ | $f_1$ | $f_2$ | $f_3$ | ... | Lexicon |
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| Predicate | $\Rightarrow$ | $P_1$ | $P_2$ | $P_3$ | … |
| Constant  | $\Rightarrow$ | $c_1$ | $c_2$ | $c_3$ | … |
| Function  | $\Rightarrow$ | $f_1$ | $f_2$ | $f_3$ | … |

Lexicon

Sally likes Bill.
(Likes sally bill)
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  - $\Rightarrow$ AtomicFormula
  - $(\text{Formula Connective Formula})$
  - $\neg \text{Formula}$

- **AtomicFormula**
  - $(\text{Predicate Term}_1 \ldots \text{Term}_k)$
  - $(\text{Term} = \text{Term})$

- **Term**
  - $(\text{Function Term}_1 \ldots \text{Term}_k)$
  - Constant

- **Connective**
  - $\Rightarrow \land | \lor | \rightarrow | \leftarrow$

- **Predicate**
  - $\Rightarrow P_1 | P_2 | P_3 \ldots$

- **Constant**
  - $\Rightarrow c_1 | c_2 | c_3 \ldots$

- **Function**
  - $\Rightarrow f_1 | f_2 | f_3 \ldots$

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Formula $\Rightarrow$ AtomicFormula
| (Formula Connective Formula)
| $\neg$ Formula

AtomicFormula $\Rightarrow$ (Predicate Term$_1$ ... Term$_k$)
| (Term = Term)

Term $\Rightarrow$ (Function Term$_1$ ... Term$_k$)
| Constant

Connective $\Rightarrow$ $\wedge | \vee | \rightarrow | \leftrightarrow$

Predicate $\Rightarrow$ $P_1 | P_2 | P_3$ ...

Constant $\Rightarrow$ $c_1 | c_2 | c_3$ ...

Function $\Rightarrow$ $f_1 | f_2 | f_3$ ...

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| Connective               | $\Rightarrow$ $\land$ | $\lor$ | $\rightarrow$ | $\leftrightarrow$ |

Lexicon:
- **Likes**
  - Predicate $\Rightarrow P_1 | P_2 | P_3 \ldots$
  - Constant $\Rightarrow c_1 | c_2 | c_3 \ldots$
  - Function $\Rightarrow f_1 | f_2 | f_3 \ldots$
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<td>$f_3$ ...</td>
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Sally likes Bill.

(Likes sally bill)

Sally likes Bill and Bill likes Sally.
How About Learning the Grammar of the Pure Predicate Calculus?

- **Formula**
  - $\Rightarrow$ **AtomicFormula**
  - $(\text{Formula Connective Formula})$
  - $\neg$ **Formula**

- **AtomicFormula**
  - $(\text{Predicate Term}_1 \ldots \text{Term}_k)$
  - $(\text{Term} = \text{Term})$

- **Term**
  - $(\text{Function Term}_1 \ldots \text{Term}_k)$
  - **Constant**

- **Connective**
  - $\Rightarrow \land | \lor | \rightarrow | \leftrightarrow$

- **Likes**
  - **Predicate**
    - $\Rightarrow P_1 | P_2 | P_3 \ldots$
  - **Constant**
    - $\Rightarrow c_1 | c_2 | c_3 \ldots$
  - **Function**
    - $\Rightarrow f_1 | f_2 | f_3 \ldots$

Sally likes Bill.

\[(\text{Likes sally bill})\]

Sally likes Bill and Bill likes Sally.

Sally likes Bill’s mother.
How About Learning the Grammar of the Pure Predicate Calculus?

Formula $\Rightarrow$ AtomicFormula  
$\quad \mid$ (Formula Connective Formula)  
$\quad \mid$ ¬ Formula

AtomicFormula $\Rightarrow$ (Predicate Term$_1$ ... Term$_k$)  
$\quad \mid$ (Term = Term)

Term $\Rightarrow$ (Function Term$_1$ ... Term$_k$)  
$\quad \mid$ Constant

Connective $\Rightarrow$ $\wedge$ $\mid$ $\vee$ $\mid$ $\rightarrow$ $\mid$ $\leftrightarrow$

Lexicon

Sally likes Bill.  
(Likes sally bill)

Sally likes Bill and Bill likes Sally.  
Sally likes Bill’s mother.  
Sally likes Bill only if Bill’s mother is tall.
How About Learning the Grammar of the Pure Predicate Calculus?

How About Learning the Grammar of the Pure Predicate Calculus?

Sally likes Bill.
(Likes sally bill)

Sally likes Bill and Bill likes Sally.
Sally likes Bill’s mother.
Sally likes Bill only if Bill’s mother is tall.
Matilda is Bill’s super-smart mother.

Lexicon
How About Learning the Grammar of the Pure Predicate Calculus?

Formula $\Rightarrow$ AtomicFormula
\| (Formula Connective Formula)
\| $\neg$ Formula

AtomicFormula $\Rightarrow$ (Predicate Term$_1$ ... Term$_k$)
\| (Term = Term)

Term $\Rightarrow$ (Function Term$_1$ ... Term$_k$)
\| Constant

Connective $\Rightarrow$ $\land$ | $\lor$ | $\rightarrow$ | $\leftrightarrow$

Likes
Predicate $\Rightarrow$ $P_1$ | $P_2$ | $P_3$ ... 
Constant $\Rightarrow$ $c_1$ | $c_2$ | $c_3$ ... 
Function $\Rightarrow$ $f_1$ | $f_2$ | $f_3$ ...

Lexicon

Sally likes Bill.
(Likes sally bill)

Sally likes Bill and Bill likes Sally.
Sally likes Bill’s mother.
Sally likes Bill only if Bill’s mother is tall.
Matilda is Bill’s super-smart mother.
5 plus 5 equals the number 10.
How About Learning the Grammar of the Pure Predicate Calculus?

<table>
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<td>Constant</td>
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</table>

| Connective       | $\Rightarrow$ | $\wedge$ | $\vee$ | $\rightarrow$ | $\leftrightarrow$ |

| Likes            | $\Rightarrow$ | $P_1$ | $P_2$ | $P_3$ | ... |
| Predicate        |               | $c_1$ | $c_2$ | $c_3$ | ... |
| Constant         |               | $f_1$ | $f_2$ | $f_3$ | ... |

Lexicon

Sally likes Bill.
(Likes sally bill)

Sally likes Bill and Bill likes Sally.

Sally likes Bill’s mother.

Sally likes Bill only if Bill’s mother is tall.

Matilda is Bill’s super-smart mother.

5 plus 5 equals the number 10.

...
How About Learning the Grammar of the Pure Predicate Calculus?

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<td>Constant</td>
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| Connective | ⇒ ∧ | ∨ | → | ↔ |

| Likes Predicate | ⇒ P₁ | P₂ | P₃ ... |
| Constant | ⇒ c₁ | c₂ | c₃ ... |
| Function | ⇒ f₁ | f₂ | f₃ ... |

Make sure you can simulate a machine that says “Yes that sentence is okay!” whenever it’s conforms to this grammar!
CLT-based Model Instantiated

Language of the Pure Predicate Calculus

\[ \forall \infty n \in \mathbb{N} \ S(T[n]) = M_k \]

\( S \) identifies \( L \) iff \( S \) identifies every text for \( L \)

[ ] Need modify because now the language is recursive!
CLT-based Model Instantiated

And when is identification of $L$ achieved?

$$\forall \infty n \in \mathbb{N} \ S(T[n]) = M_k$$

$S$ identifies $L$ iff $S$ identifies every text for $L$
CLT-based Model Instantiated

Language of the Pure Predicate Calculus

\[ \sigma \in \text{SEQ} \]

\[ T = u_0, u_1, \]

And when is identification of \( L \) achieved?

\[ \forall \infty n \in \mathbb{N} \ S(T[n]) = M_k \]

\( S \) identifies \( L \) iff \( S \) identifies every text for \( L \)

**Need modify because now the language is recursive!**
CLT-based Model Instantiated

Language of the Pure Predicate Calculus

\[ \sigma \in \text{SEQ} \]

\[ S \]

\[ M_i, i \in \mathbb{N} \]

such that \( M_i \) accepts \( L \)

And when is identification of \( L \) achieved?

\[ \forall \infty n \in \mathbb{N} \quad S(T[n]) = M_k \]

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CLT-based Model Instantiated

Language of the Pure Predicate Calculus

\[ \sigma \in \text{SEQ} \]

\[ T = u_0, u_1, \#, \#, \# \]

\[ i \]

\[ S \]

\[ M_i, i \in \mathbb{N} \]

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And when is identification of \( L \) achieved?

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CLT-based Model Instantiated

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Language of the Pure Predicate Calculus

\[ \sigma \in \text{SEQ} \]

\[ T = u_0, u_1, \#, \#, \#, \]  

\[ i \rightarrow S \]

\[ M_i, i \in \mathbb{N} \]

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CLT-based Model Instantiated

Language of the Pure Predicate Calculus

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\[ \forall n \in \mathbb{N} \quad S(T[n]) = M_k \]

\( S \) identifies \( L \) iff \( S \) identifies every text for \( L \)
CLT-based Model Instantiated

Language of the Pure Predicate Calculus

σ ∈ SEQ

S

M_i, i ∈ N

such that M_i accepts L

M_2

0

And when is identification of L achieved?

∀∞ n ∈ N  S(T[n]) = M_k

S identifies L iff S identifies every text for L

[ ] Need modify because now the language is recursive!
CLT-based Model Instantiated

Language of the Pure Predicate Calculus

$\sigma \in \text{SEQ}$

$T = u_0, u_1, \#, \#, \#, u_2$

$i \rightarrow S$

$M_i, i \in \mathbb{N}$

such that $M_i$ accepts $L$

$M_2 \rightarrow 0$

And when is identification of $L$ achieved?

$\forall \infty n \in \mathbb{N} \ S(T[n]) = M_k$

$S$ identifies $L$ iff $S$ identifies every text for $L$
CLT-based Model Instantiated

Language of the Pure Predicate Calculus

\[ \sigma \in \text{SEQ} \]

\[ T = u_0, u_1, \#, \#, \#, u_2 \]

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Need modify because now the language is recursive!
CLT-based Model Instantiated

Sally likes Bill and Bill likes Sally.

Language of the Pure Predicate Calculus

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CLT-based Model Instantiated

Language of the Pure Predicate Calculus

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Need modify because now the language is recursive!

CLT-based Model Instantiated

Language of the Pure Predicate Calculus

\( \sigma \in \text{SEQ} \)

Matilda is Bill's super-smart mother:

\( T = u_0, u_1, \#, \#, \#, u_2 \)

And when is identification of \( L \) achieved?

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\( S \) identifies \( L \) iff \( S \) identifies every text for \( L \)
CLT-based Model Instantiated to Target Deductive Calculi

Language of the Pure Predicate Calculus, \textit{and} Its Inference Schemata

\[ \sigma \in \text{SEQ} \]

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CLT-based Model Instantiated to Target Deductive Calculi

Language of the Pure Predicate Calculus, and Its Inference Schemata

\[ \sigma \in \text{SEQ} \]

\[ T = u_0, u_1, \]

\[ i \rightarrow S \]

\[ S \]

\[ o \rightarrow M_i, i \in \mathbb{N} \]

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Language of the Pure Predicate Calculus, and Its Inference Schemata

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\[ T = u_0, u_1, \quad i \]

\[ M_i, i \in \mathbb{N} \text{ such that } M_i \text{ accepts } L \]

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CLT-based Model Instantiated to Target Deductive Calculi

Language of the Pure Predicate Calculus, and Its Inference Schemata

\[ \sigma \in \text{SEQ} \]

\[ T = u_0, u_1, \#, \#, \# \]

\[ \forall i \in \mathbb{N} \quad S(T[n]) = M_i \]

And when is identification of \( L \) achieved?

\[ S \text{ identifies } L \iff S \text{ identifies every text for } L \]
CLT-based Model Instantiated to Target Deductive Calculi

Language of the Pure Predicate Calculus, and Its Inference Schemata

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Language of the Pure Predicate Calculus, and Its Inference Schemata

\[ \sigma \in \text{SEQ} \]

\[ T = u_0, u_1, \#, \#, \#, u_2 \]

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\[ M_i, i \in \mathbb{N} \text{ such that } M_i \text{ accepts } L \]

\[ M_2 \]

\[ o \]

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\( S \) identifies \( L \) iff \( S \) identifies every text for \( L \).
CLT-based Model Instantiated to Target Deductive Calculi

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CLT-based Model Instantiated to Target Deductive Calculi

Language of the Pure Predicate Calculus, and Its Inference Schemata

\( \sigma \in \text{SEQ} \)

Sally likes Bill and Bill likes Sally.

\( T = u_0, u_1, \# , \# , \# , u_2 \)

And when is identification of \( L \) achieved?

\[ \forall \infty n \in \mathbb{N} \ S(T[n]) = M_k \]

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CLT-based Model Instantiated to Target Deductive Calculi

Language of the Pure Predicate Calculus, *and* Its Inference Schemata

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Language of the Pure Predicate Calculus, and Its Inference Schemata

\[ \sigma \in \text{SEQ} \]

Matilda is Bill's super-smart mother:

\[ T = u_0, u_1, \#, \#, \#, u_2 \]

And when is identification of \( L \) achieved?

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\( S \) identifies \( L \) iff \( S \) identifies every text for \( L \)

Sally likes Bill and Bill likes Sally.
Bill likes Sally.

\[ \frac{Lsb \land Lbs}{Lbs} \]
RAIR Lab needs to work out the details and engineer an AI that can learn the language of the pure predicate calculus!
Future Work

• What about learning the language(s) of mathematics?

• What about learning to be self-conscious?