Re Monty Hall
(& a harder probability problem for takehome!)

(version 2)

AHR?
9/18/16

RP • SB
The Monty Hall Problem
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Jones has come to a game show, and finds himself thereon selected to play a game on national TV with the show’s suave host, Full Monty. Jones is told correctly by Full that hidden behind one of three closed, opaque doors facing the two of them is $1,000,000, while behind each of the other two is a feculent, obstreperous llama whose value on the open market is charitably pegged at $1. Full reminds Jones that this is a game, and a fair one, and that if Jones ends up selecting the door with $1M behind it, all that money will indeed be his. (Jones’ net worth has nearly been exhausted by his expenditures in traveling to the show.) Full also reminds Jones that he (= Full) knows what’s behind each door, fixed in place until the game ends.

Full asks Jones to select which door he wants the contents of. Jones says, "Door 1." Full then says: "Hm. Okay. Part of this game is my revealing at this point what's behind one of the doors you didn’t choose. so ... let me show you what's behind Door 3." Door 3 opens to reveal a very unsavory llama. Full now to Jones: "Do you want to switch to Door 2, or stay with Door 1? You'll get what's behind the door of your choice, and our game will end." Full looks briefly into the camera, directly.

(P1.1) What should Jones do if he's rational?

(P1.2) Prove that your answer is correct. (Diagrammatic proofs are allowed.)

(P1.3) A quantitative hedge fund manager with a PhD in finance from Harvard zipped this email off to Full before Jones made his decision re. switching or not: "Switching would be a royal waste of time (and time is money!). Jones hasn’t a doggone clue what's behind Door 1 or Door 2, and it's obviously a 50/50 chance to win whether he stands firm or switches. So the chap shouldn’t switch!" Is the fund manager right? Prove that your diagnosis is correct.
Any questions about how the game is played?
The Switching Policy Rational!

**Proof:** We’ll denote the permutations this way: Permutation (1) is a donkey behind Door 1, a donkey behind Door 2, and the million behind Door 3; denoted as: ‘ddm.’ (2) is then dmd, and (3) is mdd. There are only three permutations, so the odds of picking at the start the million-dollar one is 1/3, obviously.

Now we proceed in a proof by cases: Suppose Jones picked Door 1. And suppose he does this under (1). Then FM will reveal Door 2 to reveal a donkey. Switching to Door 3 wins. Next suppose that J’s choice of Door 1 is under (2). Then FM will reveal Door 3. Again, switching to Door 2 wins. In the final possibility, J has initially selected Door 1 under permutation (3). Here, FM shows either Door 2 or Door 3 (as itself a random choice). This time switching loses. But, switching won in 2 our of 3 cases. Hence the odds of winning by following the switching policy is 2/3, which is greater than 1/3. Hence it’s rational to be a switcher. Finally, the reasoning is exactly parallel if the original supposition is that J has selected Door 2 initially, or if J has selected Door 3 initially. QED
If one of the following assertions is true then so is the other:

(1) If there is an apple in the bucket then there is a battery in the bucket; and, if there is a battery in the bucket then there is an apple in the bucket.

(2) There is an apple in the bucket.

Which is more likely to be in the bucket, if either: the apple or the battery?
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Now class, here’s a bucket. You can’t see in to it, but it’s either empty, has an apple in it and no other object, or has a battery in it and no other object. Knowing the answer to this problem, I placed only an apple in the bucket if the apple is more likely to be in there. If the battery is more likely to be in there, I put only a battery in. And if neither is more likely to be in the bucket, I left it empty. So: Tell me also what’s in the bucket! If you’re right, and can prove that you are, here’s a $20 for you, on the spot.