Re Monty Hall

AHR?
9/16/19

SB • MG
RAIR Lab

Those who fail are behaving irrationally:

Friedman, D. (1998) “Monty Hall’s Three Doors: Construction and Deconstruction of a Choice Anomaly” *American Economic Review* **88**(4): 933–946.

- http://static.luiss.it/hey/ambiguity/papers/Friedman_1998.pdf

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Sometimes people make decisions that seem inconsistent with rational choice theory. We have a "choice anomaly" when such decisions are systematic and well documented. From a few isolated examples such as the Maurice Allais (1953) paradox and the probability matching puzzle of William K. Estes (1954), the set of anomalies expanded dramatically in the last two decades, especially following the work of Daniel Kahneman and Amos Tversky (e.g., 1979). By now the empirical literature offers dozens of interrelated anomalies documented in hundreds of articles and surveys (e.g., Colin F. Camerer, 1995).

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Anomalies?? You mean irrational decisions?

Don’t Trust the Popular Media!

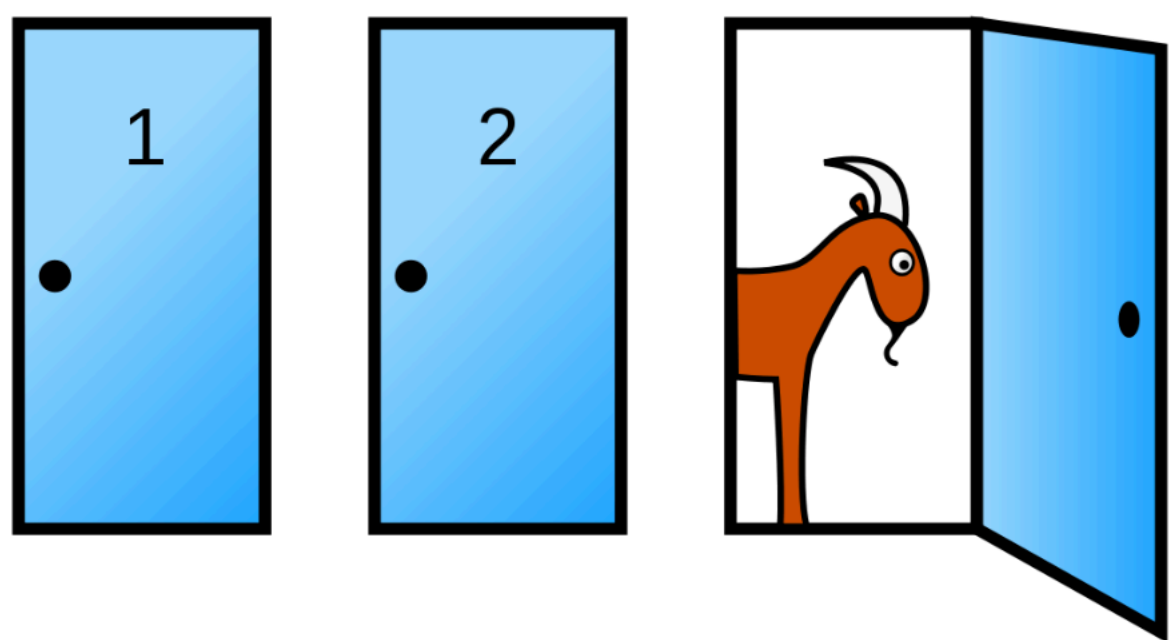
https://www.wired.com/2014/11/monty-hall-erdos-limited-minds/

Don’t Trust the Popular Media!

Monty Hall, Erdos, and Our Limited Minds

SAMUEL ARBESMAN SCIENCE 11.26.14 10:00 AM

Monty Hall, Erdos, and Our Limited Minds



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Monty Hall, Erdos, and Our Limited Minds

THE MONTY HALL problem is a well-known mathematical brainteaser. But I find it intriguing not for how to solve it, but for how widespread having trouble with it is.

Based off of a television game show, the Monty Hall problem begins with a contestant finding herself in front of three doors. She is told that behind one of them is a car, while behind the other two there are goats. Since it is presumed that contestants want to win cars not goats, if nothing else for their resale value, there is a one-third chance of choosing the car and winning.

But now here’s the twist. After the contestant chooses a door, the game show host has another door opened and the contestant is shown a goat. Should she stick with the door she has originally chosen, or switch to the remaining unopened door?

There are many ways to examine this, but it turns out that it is always better to switch. Many people assume that the probability remains the same—it’s fifty-fifty so switching doesn’t matter—but they are wrong. There is a higher probability of the car being behind the door when you switch (here is a detailed discussion but I like to think about it based on an extreme version, one with 100 doors. One has a car and the others all have goats. You choose a door. The host opens 98 other doors, showing all goats. Should you switch? Of course! The host has done the work of almost certainly finding of the car for you.)

Anyway, I’m not concerned with the particulars of the problem but rather with how people respond to it. Namely, many listeners, even highly-trained mathematicians, are initially confused by the probabilities. In fact, until I learned of the extreme version with 100 doors, I didn't really understand why switching is better either.

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Painful!

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Don’t Trust Lazy Mathematicians!

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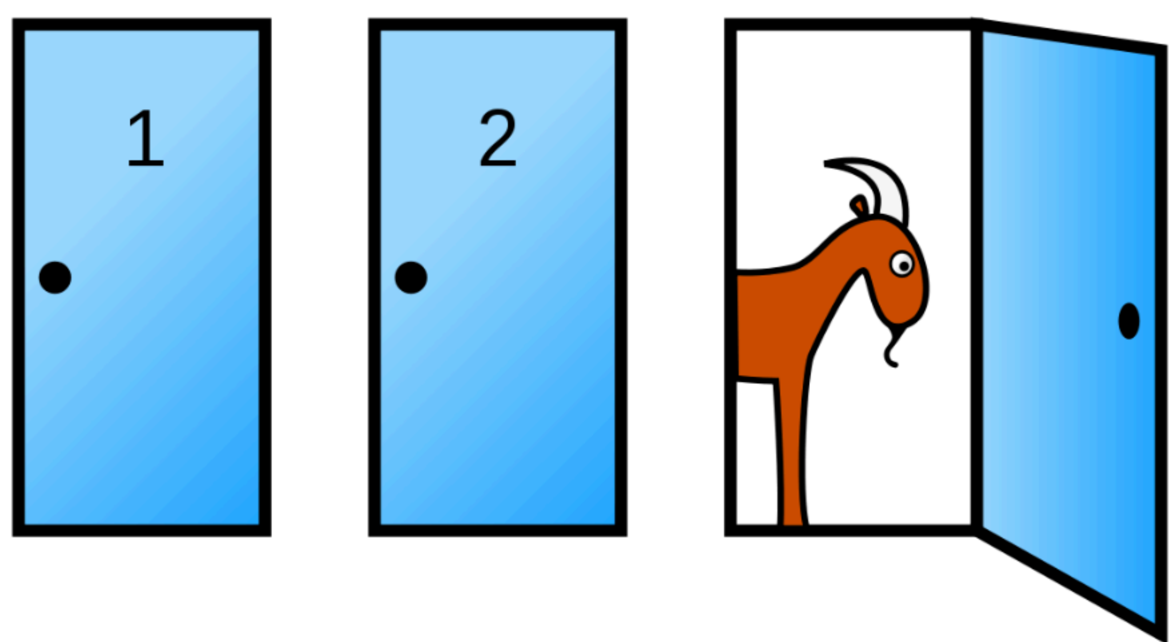

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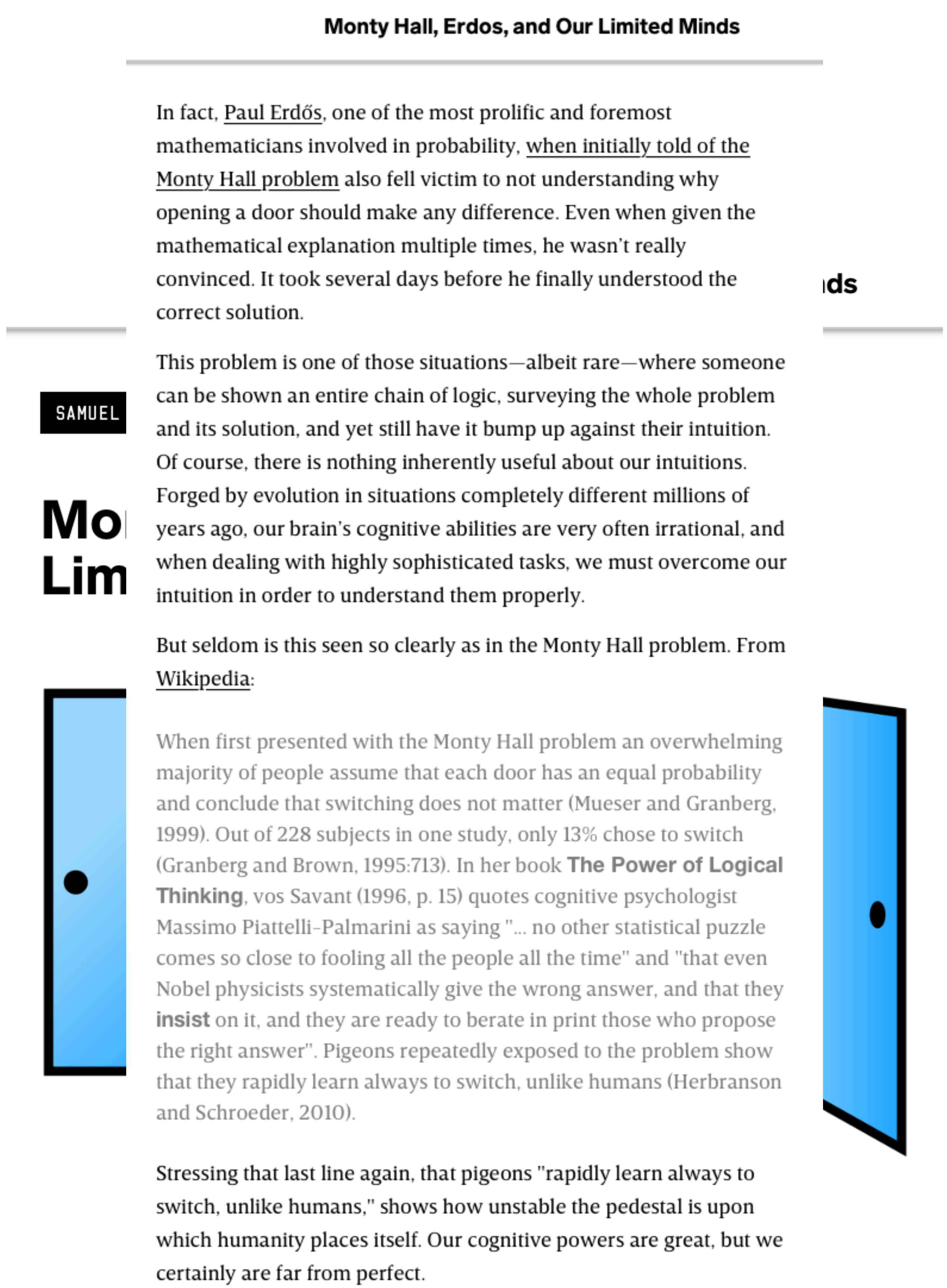

Monty Hall, Erdos, and Our Limited Minds

In fact, Paul Erdős, one of the most prolific and foremost mathematicians involved in probability, when initially told of the Monty Hall problem also fell victim to not understanding why opening a door should make any difference. Even when given the mathematical explanation multiple times, he wasn’t really convinced. It took several days before he finally understood the correct solution.

This problem is one of those situations—albeit rare—where someone can be shown an entire chain of logic, surveying the whole problem and its solution, and yet still have it bump up against their intuition. Of course, there is nothing inherently useful about our intuitions. Forged by evolution in situations completely different millions of years ago, our brain’s cognitive abilities are very often irrational, and when dealing with highly sophisticated tasks, we must overcome our intuition in order to understand them properly.

But seldom is this seen so clearly as in the Monty Hall problem. From Wikipedia:

> When first presented with the Monty Hall problem an overwhelming majority of people assume that each door has an equal probability and conclude that switching does not matter (Mueser and Granberg, 1999). Out of 228 subjects in one study, only 13% chose to switch (Granberg and Brown, 1995:713). In her book **The Power of Logical Thinking**, vos Savant (1996, p. 15) quotes cognitive psychologist Massimo Piattelli-Palmarini as saying "... no other statistical puzzle comes so close to fooling all the people all the time" and "that even Nobel physicists systematically give the wrong answer, and that they **insist** on it, and they are ready to berate in print those who propose the right answer". Pigeons repeatedly exposed to the problem show that they rapidly learn always to switch, unlike humans (Herbranson and Schroeder, 2010).

Stressing that last line again, that pigeons "rapidly learn always to switch, unlike humans," shows how unstable the pedestal is upon which humanity places itself. Our cognitive powers are great, but we certainly are far from perfect.

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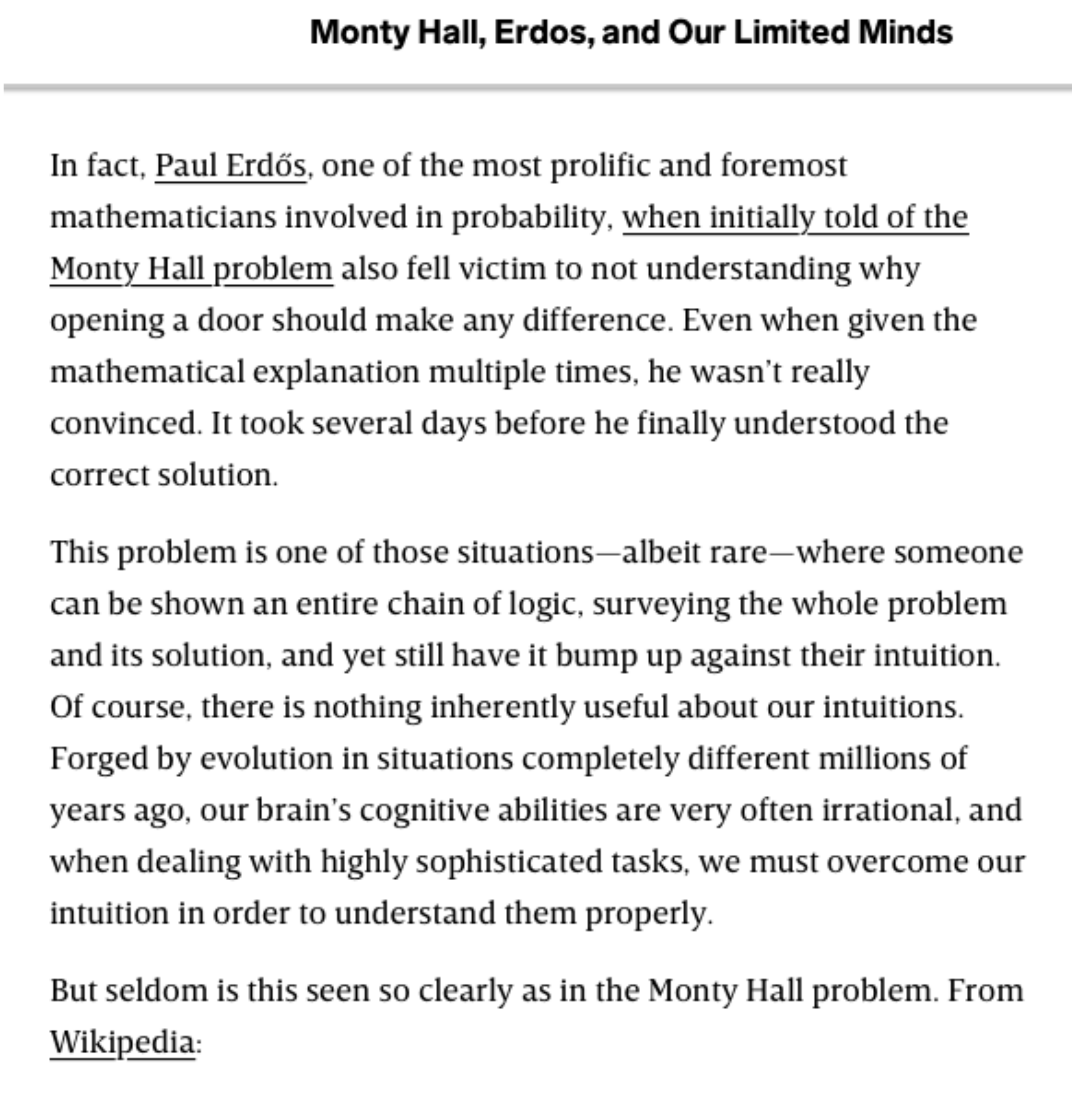
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Gotta prove it, Paul!

ty-hall-erdos-limited-minds/

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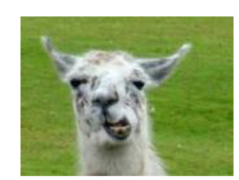

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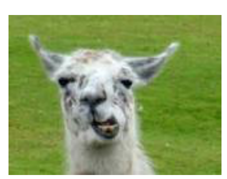

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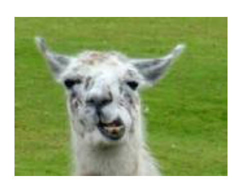

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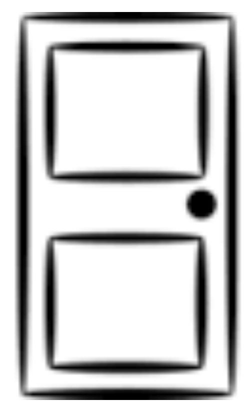

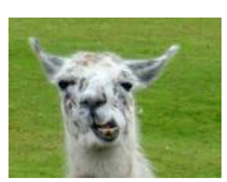
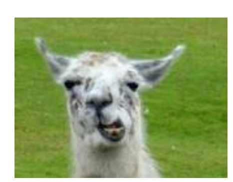
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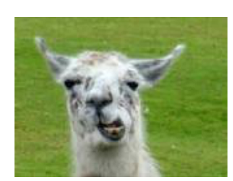
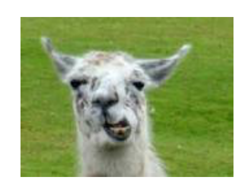
$1M  

MHP Defined

Jones has come to a game show, and finds himself thereon selected to play a game on national TV with the show's suave host, Full Monty. Jones is told correctly by Full that hidden behind one of three closed, opaque doors facing the two of them is \$1,000,000, while behind each of the other two is a feculent, obstreperous llama whose value on the open market is charitably pegged at \$1. Full reminds Jones that this is a game, and a fair one, and that if Jones ends up selecting the door with \$1M behind it, all that money will indeed be his. (Jones' net worth has nearly been exhausted by his expenditures in traveling to the show.) Full also reminds Jones that he (= Full) knows what's behind each door, fixed in place until the game ends.

Full asks Jones to select which door he wants the contents of. Jones says, "Door 1." Full then says: "Hm. Okay. Part of this game is my revealing at this point what's behind one of the doors you didn't choose. So ... let me show you what's behind Door 3." Door 3 opens to reveal a very unsavory llama. Full now to Jones: "Do you want to switch to Door 2, or stay with Door 1? You'll get what's behind the door of your choice, and our game will end." Full looks briefly into the camera, directly.

(P1.1) What should Jones do if he's rational?

(P1.2) Prove that your answer is correct. (Diagrammatic proofs are allowed.)

(P1.3) A quantitative hedge fund manager with a PhD in finance from Harvard zipped this email off to Full before Jones made his decision re. switching or not: "Switching would be a royal waste of time (and time is money!). Jones hasn't a doggone clue what's behind Door 1 or Door 2, and it's obviously a 50/50 chance to win whether he stands firm or switches. So the chap shouldn't switch!" Is the fund manager right? Prove that your diagnosis is correct.

(P1.4) Can these answers and proofs be exclusively Bayesian in nature?

Any questions about how the game is played?

The Switching Policy Rational!

Proof: Our overarching technique will be proof by cases.

We denote the possible cases for initial distribution using a simple notation, according to which for example 'LLM' means that, there is a lama behind Door 1, a llama behind Door 2, and the million dollars behind Door 3. With this notation in hand, our three starting cases are: Case 1: MLL; Case 2: LML; Case 3: LLM. There are only three top-level cases for distribution. The odds of picking at the start the million-dollar door is 1/3, obviously — for each case. Hence we know that the odds of a HOLD policy winning is 1/3.

Now we proceed in a proof by sub-cases under the three cases above, to show that the overall odds of a SWITCH policy is greater than 1/3. Each sub-case is simply based on what the initial choice by Jones is, under one of the three main cases. Here we go:

Suppose Case 3, LLM, holds, and that [this (Case 3.1) is the first of three sub-cases under Case 3] Jones picks Door 1. Then FM must reveal Door 2 to reveal a llama. Switching to Door 3 wins, guaranteed. In sub-case 3.2 suppose that J's choice Door 2. Then FM will reveal Door 1. Again, switching to Door 3 wins, guaranteed. In the final sub-case, J initially selects Door 3 under Case 3; this is sub-case 3.3. Here, FM shows either Door 1 or Door 2 (as itself a random choice). This time switching loses, guaranteed. Hence, in two of the sub-cases out of three (2/3), winning is guaranteed (*prob* of 1). An exactly parallel result can be deduced for Case 2 and Case 1; i.e., in each of these two, in two of the three (2/3) sub-cases winning is 1. Hence the odds of winning by following the switching policy is 2/3, which is greater than 1/3. Hence it's rational to be a switcher. **QED**