Addenda to Prop Calc; Selmer’s Monty Fall Problem; The Case of Linda, & Some Probability Formalisms

Selmer Bringsjord

Are Humans Rational?

9/18/19

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Addenda to Prop. Calc. . . .

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T	F
F	T

ϕ	ψ	$\phi \wedge \psi$
T	T	T
T	F	F
F	T	F
F	F	F

ϕ	ψ	$\phi \vee \psi$
T	T	T
T	F	T
F	T	T
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ϕ	ψ	$\phi \rightarrow \psi$
T	T	T
T	F	F
F	T	T
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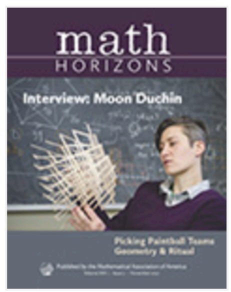
$\vdash \phi$ simply means, then, that ϕ can be proved with only temporary suppositions.

Selmer's Monty *Fall* Problem, and Further Variants …

Interesting Background/Further-Investigation Paper

Math Horizons / Vol. 16, No. 1,... / Monty Hall, Mon...



JOURNAL ARTICLE

Monty Hall, Monty Fall, Monty Crawl

Jeffrey S. Rosenthal
Math Horizons
Vol. 16, No. 1 (September 2008), pp. 5-7

Published by: Taylor & Francis, Ltd. on behalf of the Mathematical Association of America
https://www.jstor.org/stable/25678763
Page Count: 3

Topics: Savants, Children

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Inter ther-

"The Proportionality Principle makes the various Monty Hall variants easy."

Monty Hall, Monty Fall, Monty Crawl

Jeffrey S. Rosenthal
University of Toronto

In 1990, Marilyn vos Savant introduced the infamous *Monty Hall problem* to the general public. Her asserted answer set off a storm of controversy in which she received thousands of letters. Numerous professional mathematicians and others insisted that she was wrong, some using rather strong language ("you are utterly incorrect"; "I am in shock"; "you are the goat"). Vos Savant had the last laugh, when she called upon "math classes all across the country" to estimate the probabilities using pennies and paper cups, and they reported with astonishment that vos Savant was correct.

Despite all the publicity, most people have at best a vague understanding of why vos Savant's answer is correct, and the extent to which it does or does not also apply to variants of the problem. In this paper, we discuss the Proportionality Principle, which allows this and many related problems to be solved easily and confidently.

The Monty Hall Problem and Variants

The original Monty Hall problem may be summarized as follows:

Monty Hall Problem: A car is equally likely to be behind any one of three doors. You select one of the three doors (say, Door #1). The host then reveals one non-selected door (say, Door #3) which does not contain the car. At this point, you choose whether to stick with your original choice (i.e. Door #1), or switch to the remaining door (i.e. Door #2). What are the probabilities that you will win the car if you stick, versus if you switch?

Most people believe, upon first hearing this problem, that the car is equally likely to be behind either of the two unopened doors, so the probability of winning is 1/2 regardless of whether you stick or switch. However, in fact the probabilities of winning are 1/3 if you stick, and 2/3 if you switch. This fact is often justified as follows:

Shaky Solution: When you first selected a door, you had a 1/3 chance of being correct. You knew the host was going to open some other door which did not contain the car, so that doesn't change this probability. Hence, when all is said and done, there is a 1/3 chance that your original selection was correct, and hence a 1/3 chance that you will win by sticking. The remaining probability, 2/3, is the chance you will win by switching.

This solution is actually correct, but I consider it "shaky" because it fails for slight variants of the problem. For example, consider the following:

Monty Fall Problem: In this variant, once you have selected one of the three doors, the host slips on a banana peel and *accidentally* pushes open another door, which just *happens* not to contain the car. *Now* what are the probabilities that you will win the car if you stick with your original selection, versus if you switch to the remaining door?

In this case, it is still true that originally there was just a 1/3 chance that your original selection was correct. And yet, in the Monty Fall problem, the probabilities of winning if you stick or switch are both 1/2, not 1/3 and 2/3. Why the difference? Why doesn't the Shaky Solution apply equally well to the Monty Fall problem?

Another variant is as follows:

Monty Crawl Problem: As in the original problem, once you have selected one of the three doors, the host then reveals one non-selected door which does not contain the car. However, the host is very tired, and *crawls* from his position (near Door #1) to the door he is to open. In particular, if he has a choice of doors to open (i.e., if your original selection happened to be correct), then he opens the *smallest number* available door. (For example, if you selected Door #1 and the car was indeed behind Door #1, then the host would always open Door #2, never Door #3.) What are the probabilities that you will win the car if you stick versus if you switch?

This Monty Crawl problem seems very similar to the original Monty Hall problem; the only difference is the host's actions when he has a *choice* of which door to open. However, the answer now is that if you see the host open the higher-numbered unselected door, then your probability of winning is 0% if you stick, and 100% if you switch. On the other hand, if the

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MHP Defined (original)

Jones has come to a game show, and finds himself thereon selected to play a game on national TV with the show's suave host, Full Monty. Jones is told correctly by Full that hidden behind one of three closed, opaque doors facing the two of them is $1,000,000, while behind each of the other two is a feculent, obstreperous llama whose value on the open market is charitably pegged at $1. Full reminds Jones that this is a game, and a fair one, and that if Jones ends up selecting the door with $1M behind it, all that money will indeed be his. (Jones' net worth has nearly been exhausted by his expenditures in traveling to the show.) Full also reminds Jones that he (= Full) knows what's behind each door, fixed in place until the game ends.

Full asks Jones to select which door he wants the contents of. Jones says, "Door 1." Full then says: "Hm. Okay. Part of this game is my revealing at this point what's behind one of the doors you didn't choose. So ... let me show you what's behind Door 3." Door 3 opens to reveal a very unsavory llama. Full now to Jones: "Do you want to switch to Door 2, or stay with Door 1? You'll get what's behind the door of your choice, and our game will end." Full looks briefly into the camera, directly.

(P1.1) What should Jones do if he's rational?

(P1.2) Prove that your answer is correct. (Diagrammatic proofs are allowed.)

(P1.3) A quantitative hedge fund manager with a PhD in finance from Harvard zipped this email off to Full before Jones made his decision re. switching or not: "Switching would be a royal waste of time (and time is money!). Jones hasn't a doggone clue what's behind Door 1 or Door 2, and it's obviously a 50/50 chance to win whether he stands firm or switches. So the chap shouldn't switch!" Is the fund manager right? Prove that your diagnosis is correct.

(P1.4) Can these answers and proofs be exclusively Bayesian in nature?

The Switching Policy Rational!

Proof: Our overarching technique will be proof by cases.

We denote the possible cases for initial distribution using a simple notation, according to which for example 'LLM' means that, there is a lama behind Door 1, a llama behind Door 2, and the million dollars behind Door 3. With this notation in hand, our three starting cases are: Case 1: MLL; Case 2: LML; Case 3: LLM. There are only three top-level cases for distribution. The odds of picking at the start the million-dollar door is 1/3, obviously — for each case. Hence we know that the odds of a HOLD policy winning is 1/3.

Now we proceed in a proof by sub-cases under the three cases above, to show that the overall odds of a SWITCH policy is greater than 1/3. Each sub-case is simply based on what the initial choice by Jones is, under one of the three main cases. Here we go:

Suppose Case 3, LLM, holds, and that [this (Case 3.1) is the first of three sub-cases under Case 3] Jones picks Door 1. Then FM must reveal Door 2 to reveal a llama. Switching to Door 3 wins, guaranteed. In sub-case 3.2 suppose that J's choice Door 2. Then FM will reveal Door 1. Again, switching to Door 3 wins, guaranteed. In the final sub-case, J initially selects Door 3 under Case 3; this is sub-case 3.3. Here, FM shows either Door 1 or Door 2 (as itself a random choice). This time switching loses, guaranteed. Hence, in two of the sub-cases out of three (2/3), winning is guaranteed (*prob* of 1). An exactly parallel result can be deduced for Case 2 and Case 1; i.e., in each of these two, in two of the three (2/3) sub-cases winning is 1. Hence the odds of winning by following the switching policy is 2/3, which is greater than 1/3. Hence it's rational to be a switcher. **QED**

MFP Defined

Jones has come to a game show, and finds himself thereon selected to play a game on national TV with the show's suave host, Full Monty. Jones is familiar with the game from watching prior shows, and has had plenty of time to develop a strategy for the game, to be applied if he's lucky enough to get the chance to play — and he *has* been lucky. Jones is told correctly by Full that hidden behind one of three closed, opaque doors facing the two of them is $1,000,000, while behind each of the other two is a feculent, obstreperous llama whose value on the open market is charitably pegged at $1. Full reminds Jones that this is a game, and a fair one, and that if Jones ends up selecting the door with $1M behind it, all that money will indeed be his. (Jones' net worth has nearly been exhausted by his expenditures in traveling to the show.)

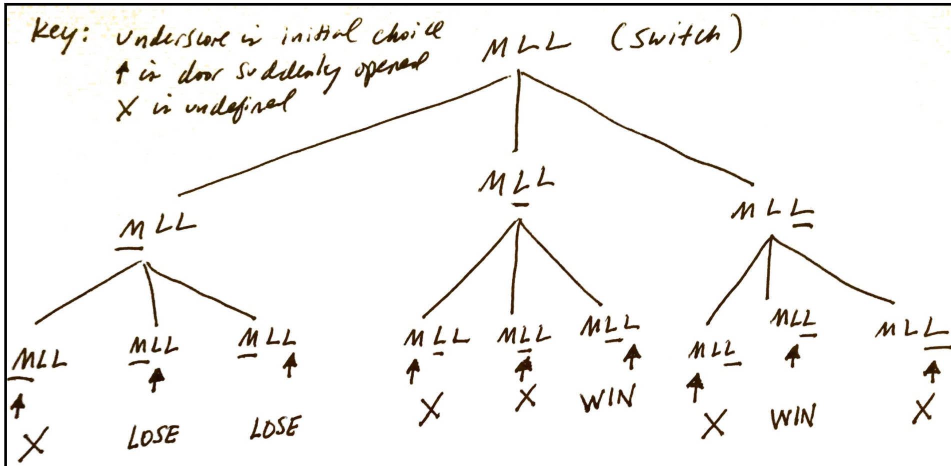
But, sometimes a disguised mad professor of probability is in the audience, and when he's present, he jumps up and shouts out which number, 1, 2, or 3, his (genuine) random number generator has just generated, and the door number he shouts out immediately thereafter opens. If the door the contestant has initially picked springs open as a result of this, the result is declared UNDEFINED, and the game must start over after the professor has been escorted out. *Also*, if the door that springs open reveals the $1M, the result is UNDEFINED and everything must be reset after the prof is removed. But, if one of the other two doors opens, the contestant is allowed to switch by Full Monty. Jones can of course also stay with his initial selection. All of this was explained to Jones before he came on the show, so the challenge to Jones is to have a two-part strategy: one for when things go smoothly and normally, and one just in case the prof snuck in and does his disruption.

Full asks Jones to select which door he wants the contents of. Jones says, "Door 1." Full then says: "Okay. Now let's op—"

Suddenly the mad professor jumps up and shouts out "2!" and immediately thereafter Door 2 opens to reveal a llama.

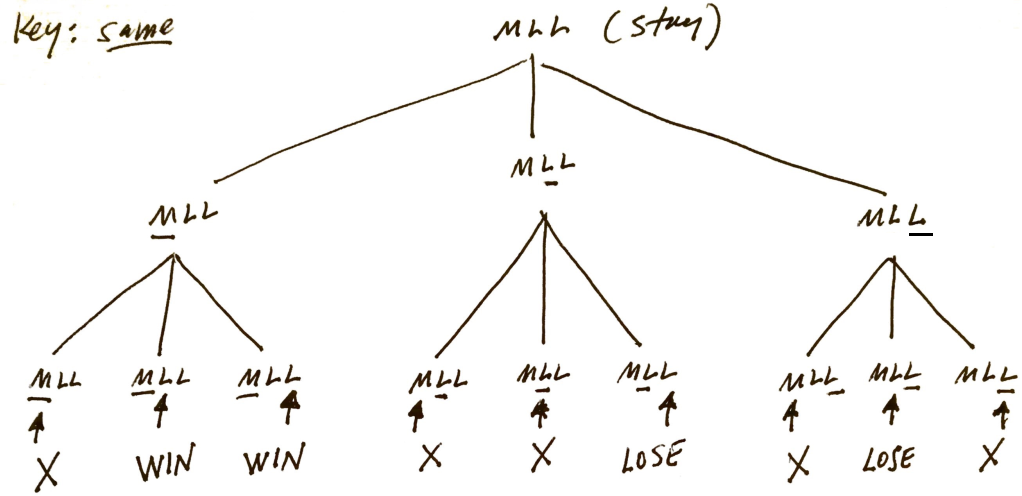
(P2.1) What should Jones's policy be, and, following it, what should he now do, assuming he's rational?

(P2.2) Prove that your answer is correct. (Diagrammatic proofs are allowed.)



Proposition[1]: The SWITCH policy will result in wins at a probability of $.5 = \frac{2}{4}$.

Proof: We proceed, as in MHP, using proof by cases. The initial case considered above is MLL, but this loses no generality, since the result will be the same for LML and LLM. Three sub-cases arise, one for each initial selection. Next, three sub-sub-cases arise, one for each door that springs open. Hence we have 9 leaves in a tree (see above) that lays out the space. Five of these nine are undefined. One WIN in 2; i.e. $\frac{2}{4} = .5$ is the overall probability of winning. QED

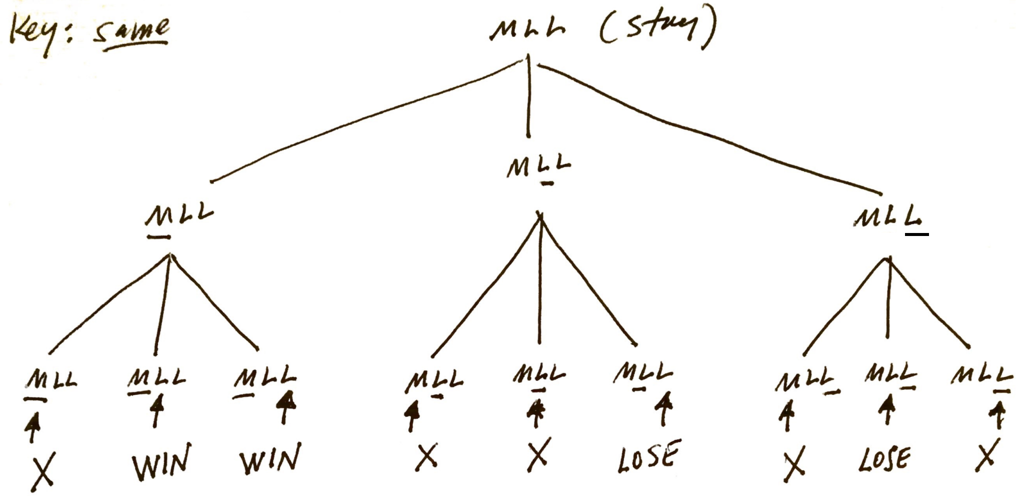


<u>Proposition 2</u>: The STAY policy will result in wins at a probability of $.5 = \frac{2}{4}$.

<u>Proof</u>: Trivial: a parallel of the proof of <u>Proposition 1</u>. <u>QED</u>

<u>Proposition 3</u>: A rational agent should adopt a policy of STAY.

<u>Proof</u>: .5 for SWITCH = .5 for STAY, but since time is money, STAY is preferable. <u>QED</u>



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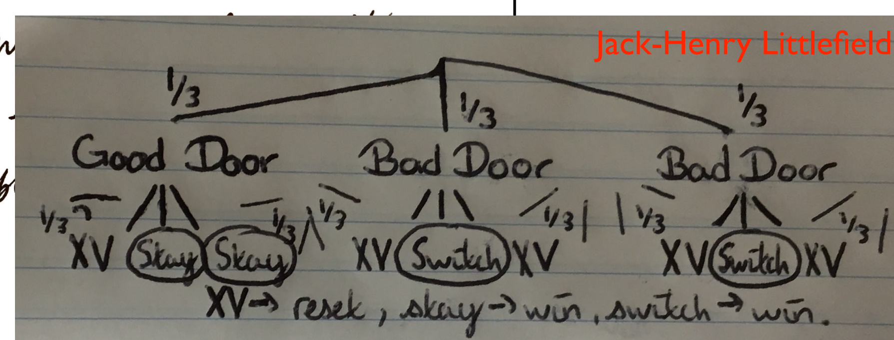

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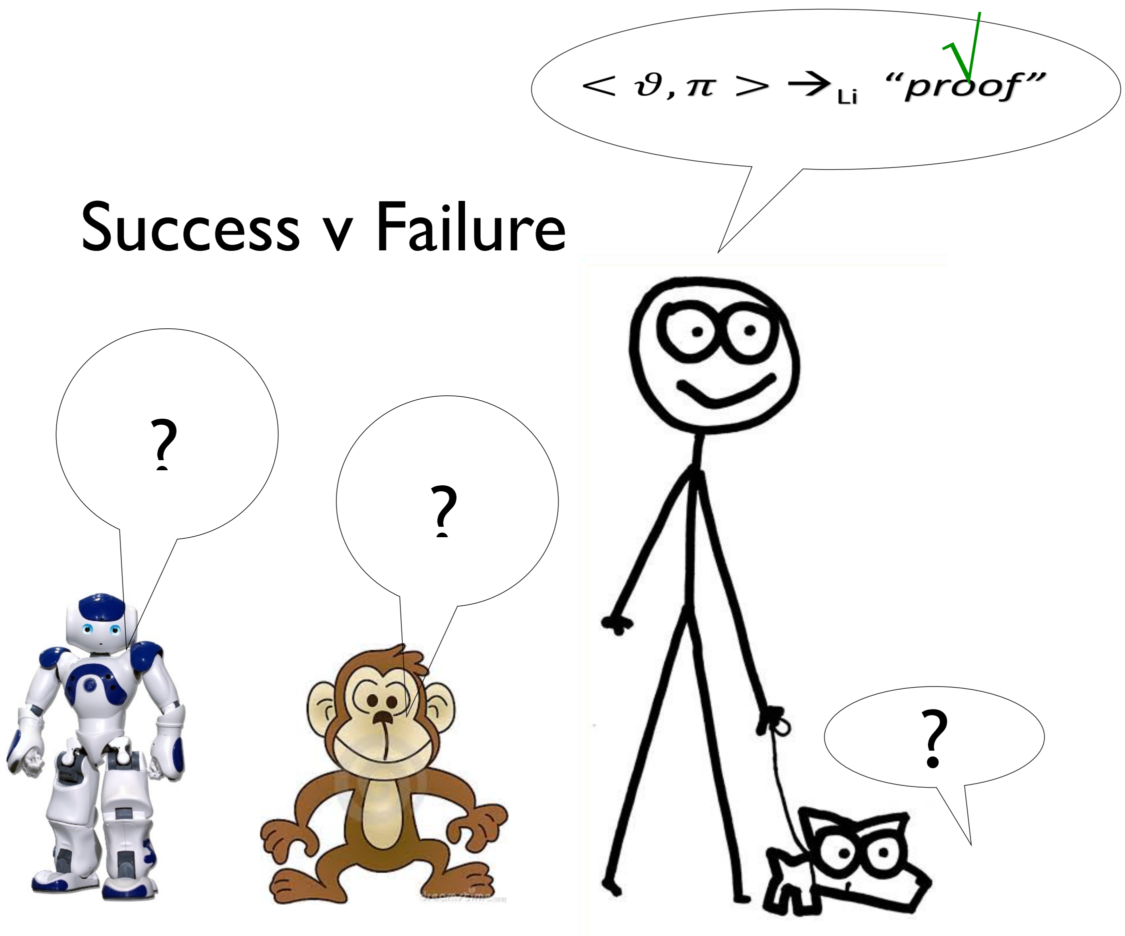

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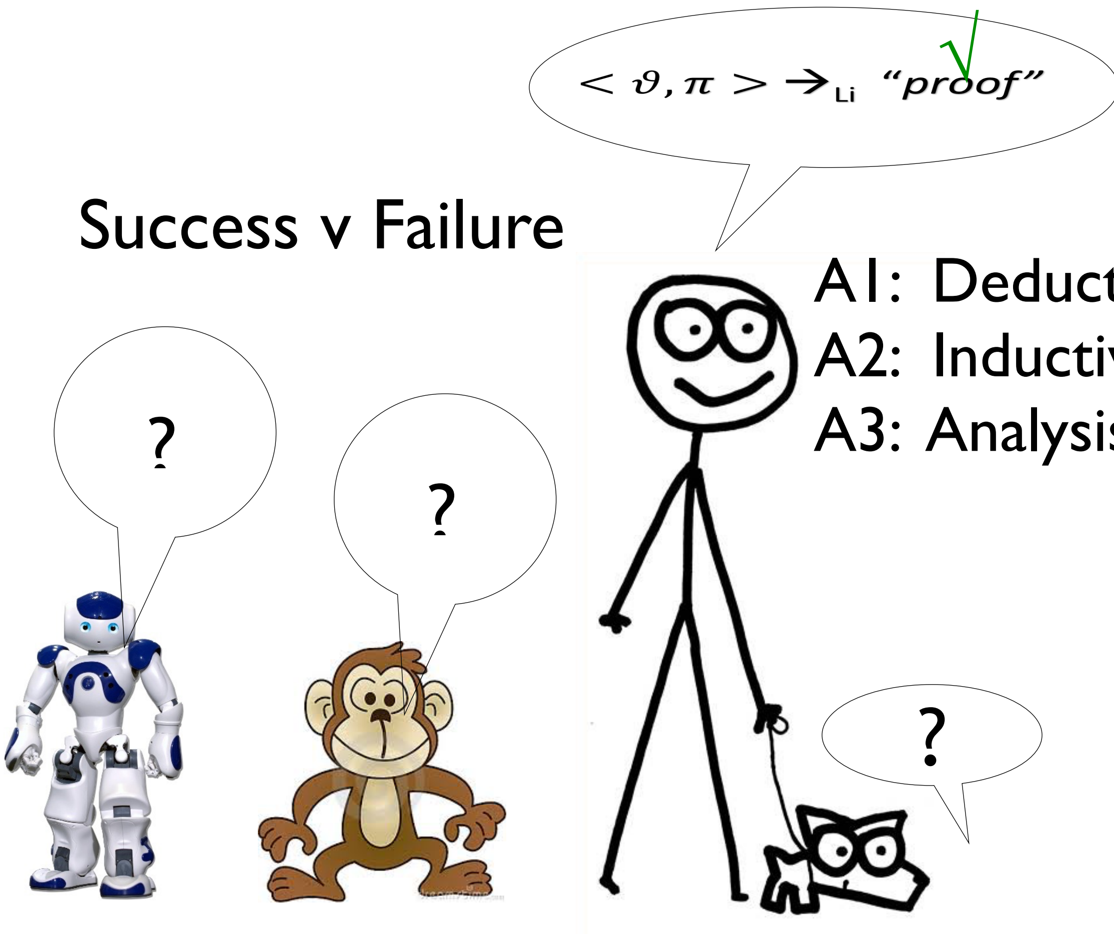

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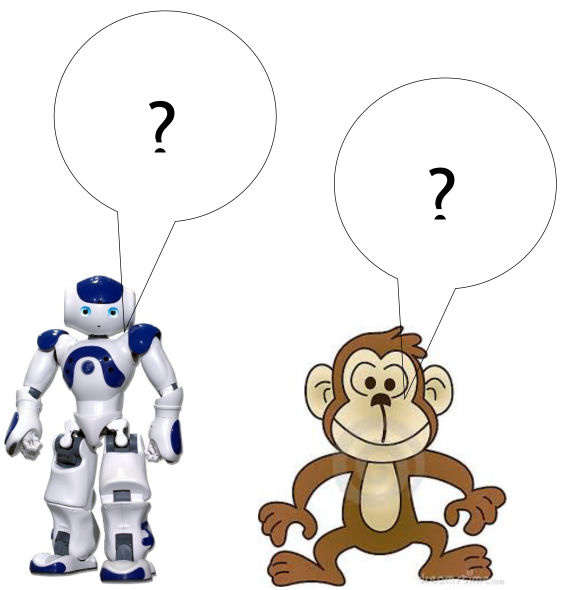

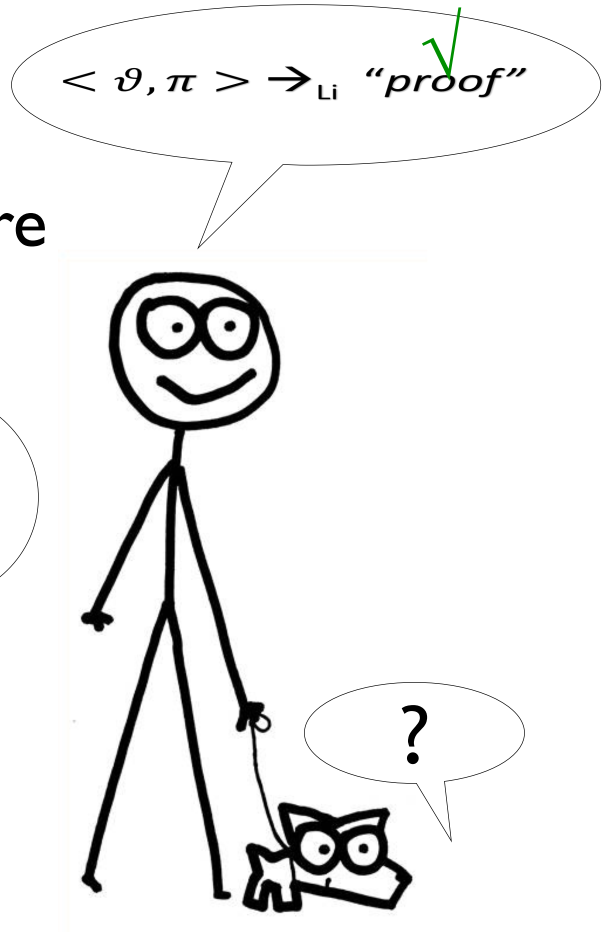

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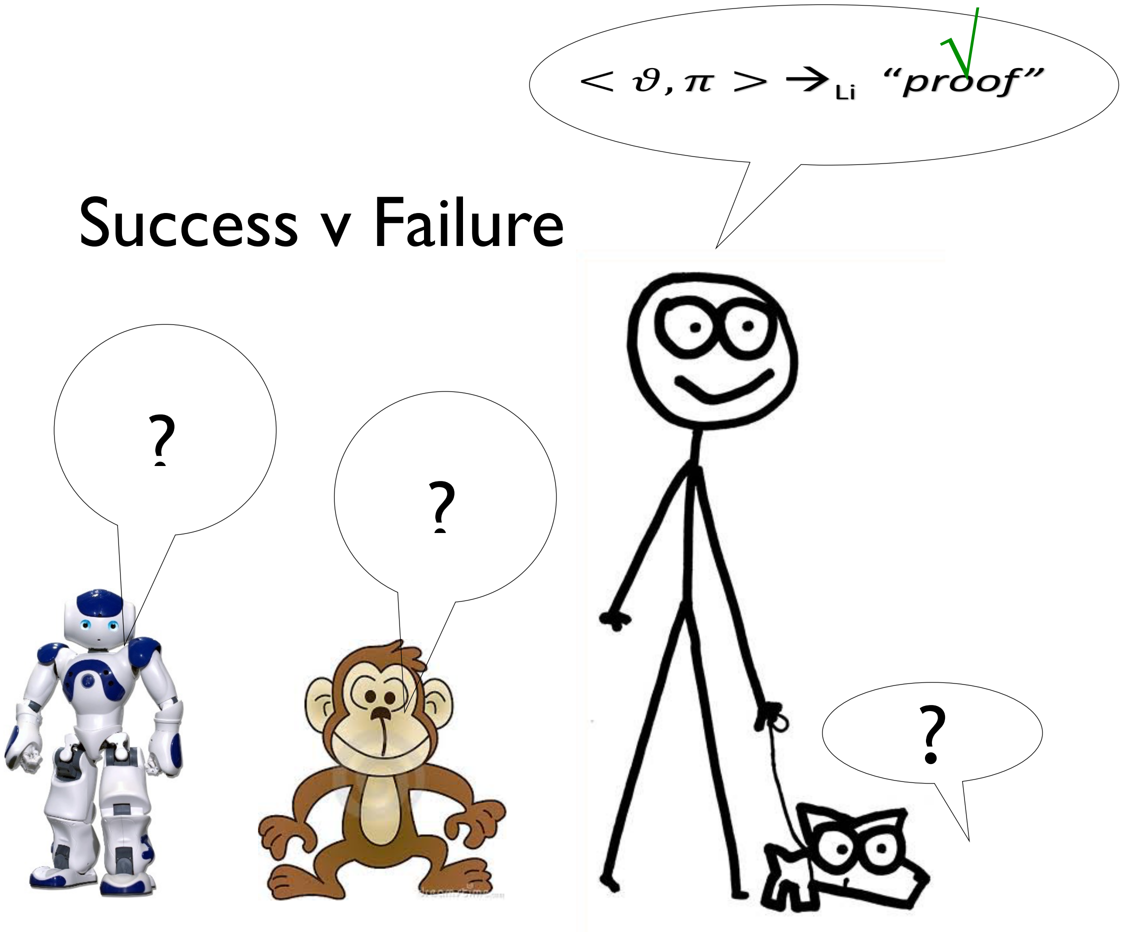

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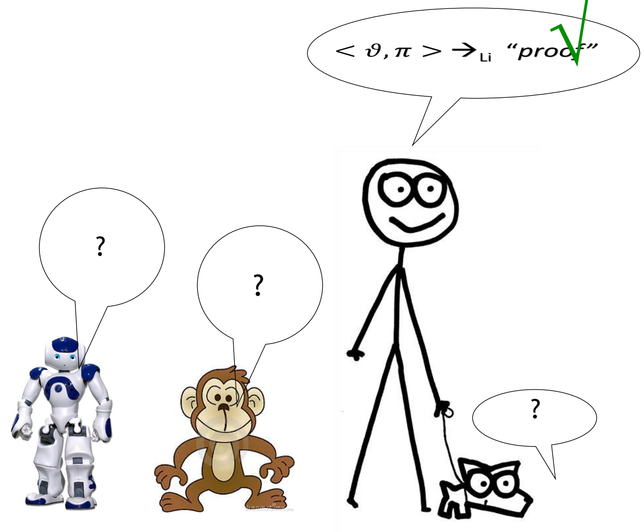

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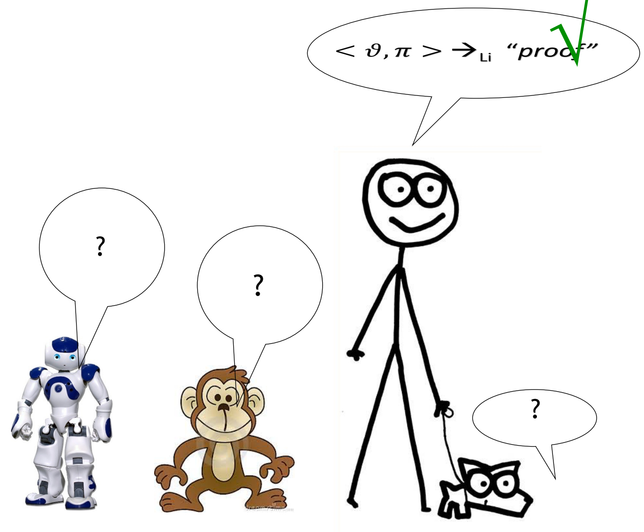

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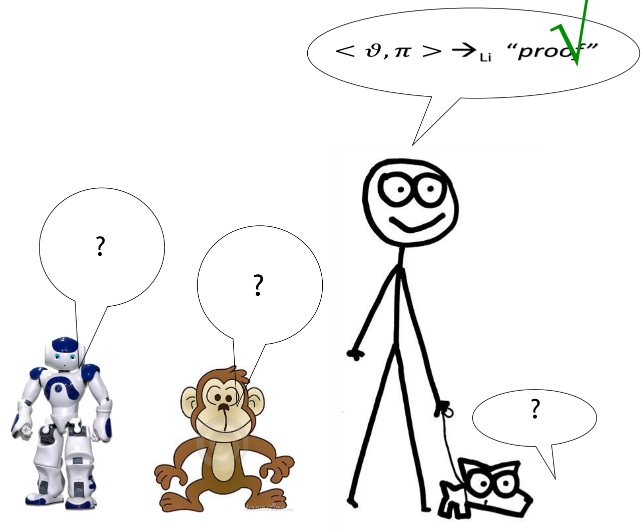

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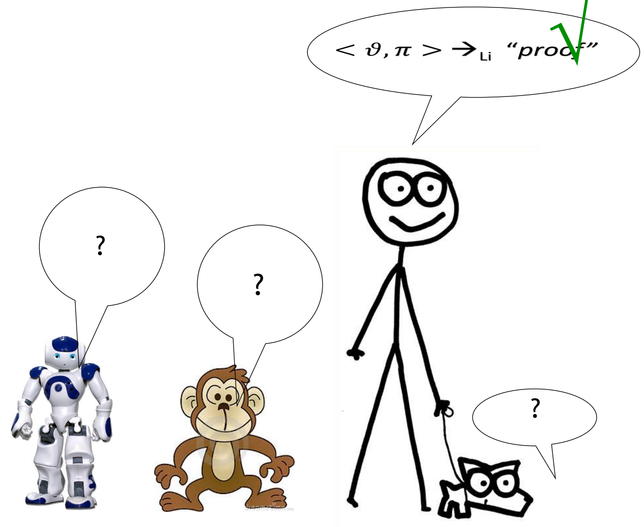

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(The "beauty" of Monty-X problems is that they call for weaving together A1 & A2 I used simple deduction woven together with urn-based reasoning.)

15

Linda: Less Is More

The best-known and most controversial of our experiments involved a fictitious lady called Linda. Amos and I made up the Linda problem to provide conclusive evidence of the role of heuristics in judgment and of their incompatibility with logic. This is how we described Linda:

> Linda is thirty-one years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in antinuclear demonstrations.

The audiences who heard this description in the

By the way, as mentioned, the “crisis” roiling around priming:

https://replicationindex.wordpress.com/2017/02/02/reconstruction-of-a-train-wreck-how-priming-research-went-of-the-rails/#comment-1454

Kahneman: “… I placed too much faith in underpowered studies …”

1980s always laughed because they immediately knew that Linda had attended the University of California at Berkeley, which was famous at the time for its radical, politically engaged students. In one of our experiments we presented participants with a list of eight possible scenarios for Linda. As in the Tom W problem, some ranked the scenarios by representativeness, others by probability. The Linda problem is similar, but with a twist.

Linda is a teacher in elementary school.
Linda works in a bookstore and takes yoga classes.
Linda is active in the feminist movement.
Linda is a psychiatric social worker.
Linda is a member of the League of Women Voters.
Linda is a bank teller.
Linda is an insurance salesperson.
Linda is a bank teller and is active in the feminist movement.

The problem shows its age in several ways. The League of Women Voters is no longer as prominent as it was, and the idea of a feminist "movement" sounds quaint, a testimonial to the change in the status of women over the last thirty years. Even in the Facebook era, however, it is still easy to guess the almost perfect consensus of judgments: Linda is a very good fit for an active feminist, a fairly good fit for someone who works in a bookstore and takes yoga classes—and a very poor fit for a bank teller or an insurance salesperson.

Now focus on the critical items in the list: Does Linda look more like a bank teller, or more like a bank teller who is active in the feminist movement? Everyone agrees that Linda fits the idea of a "feminist bank teller" better than she fits the stereotype of bank tellers. The stereotypical bank teller is not a feminist activist, and adding that detail to the description makes for a more coherent story.

The twist comes in the judgments of likelihood, because there is a logical relation between the two scenarios. Think in terms of Venn diagrams. The set of feminist bank tellers is wholly included in the set of bank tellers, as every feminist bank teller is a bank teller. Therefore the probability that Linda is a feminist bank teller *must* be lower than the probability of her being a bank teller. When you specify a possible event in greater detail you can only lower its probability. The problem therefore sets up a conflict between the intuition of representativeness and the logic of probability.

Our initial experiment was between-subjects. Each participant saw a set of seven outcomes that included only one of the critical items ("bank teller" or "feminist bank teller"). Some ranked the outcomes by resemblance, others by likelihood. As in the case of Tom W, the average rankings by resemblance and by likelihood were identical; "feminist bank teller" ranked higher than "bank teller" in both.

Then we took the experiment further, using a within-subject design. We made up the questionnaire as you saw it, with "bank teller" in the sixth position in the list and "feminist bank teller" as the last item. We were convinced that subjects would notice the relation between the two outcomes, and that their rankings would be consistent with logic. Indeed, we were so certain of this that we did not think it worthwhile to conduct a special experiment. My assistant was running another experiment in the lab, and she asked the subjects to complete the new Linda questionnaire while signing out, just before they got paid.

About ten questionnaires had accumulated in a tray on my assistant's desk before I casually glanced at them and found that all the subjects had ranked "feminist bank teller" as more probable than "bank teller." I was so surprised that I still retain a "flashbulb memory" of the gray color of the metal desk and of where everyone was when I made that discovery. I quickly called Amos in great excitement to tell him what we had found: we had pitted logic against representativeness, and representativeness had won!

In the language of this book, we had observed a failure of System 2: our participants had a fair opportunity to detect the relevance of the logical rule, since both outcomes were included in the same ranking. They did not take advantage of that opportunity. When we extended the experiment, we found that 89% of the undergraduates in our sample violated the logic of probability. We were convinced that statistically sophisticated respondents would do better, so we administered the same questionnaire to doctoral students in the decision-science program of the Stanford Graduate School of Business, all of whom had taken several advanced courses in probability, statistics, and decision theory. We were surprised again: 85% of these respondents also ranked "feminist bank teller" as more likely than "bank teller."

In what we later described as "increasingly desperate" attempts to eliminate the error, we introduced large groups of people to Linda and asked them this simple question:

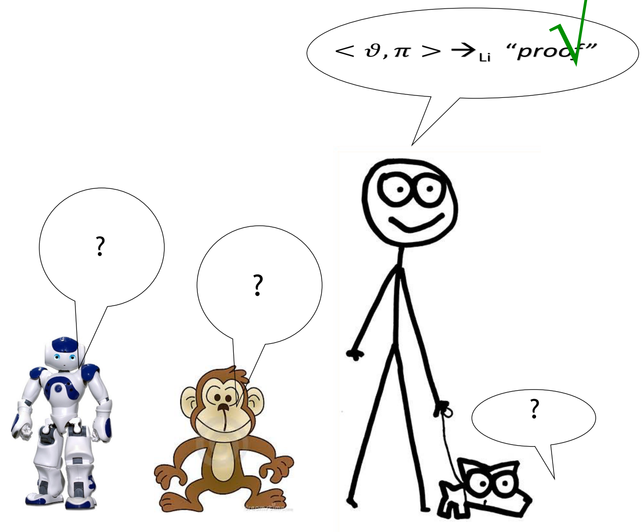
> Which alternative is more probable?
> Linda is a bank teller.
> Linda is a bank teller and is active in the feminist movement.

This stark version of the problem made Linda famous in some circles, and it earned us years of controversy. About 85% to 90% of undergraduates at several major universities chose the second option, contrary to logic. Remarkably, the sinners seemed to have no shame. When I asked my large undergraduate class in some indignation, "Do you realize that you have violated an elementary logical rule?" someone in the back row shouted, "So what?" and a graduate student who made the same error explained herself by saying, "I thought you just asked for my opinion."

The word *fallacy* is used, in general, when people fail to apply a logical rule that is obviously relevant. Amos and I introduced the idea of a *conjunction fallacy*, which people commit when they judge a conjunction of two events (here, bank teller and feminist) to be more probable than one of the events (bank teller) in a direct comparison.

As in the Müller-Lyer illusion, the fallacy remains attractive even when you recognize it for what it is. The naturalist Stephen Jay Gould described his own struggle with the Linda problem. He knew the correct answer, of course, and yet, he wrote, "a little homunculus in my head continues to jump up and down, shouting at me—'but she can't just be a bank teller; read the description.'" The little homunculus is of course Gould's System 1 speaking to him in insistent tones. (The two-system terminology had not yet been introduced when he wrote.)

The correct answer to the short version of the Linda problem was the majority response in only one of our studies: 64% of a group of graduate students in the social sciences at Stanford and at Berkeley correctly judged "feminist bank teller" to be less probable than "bank teller." In the original version with eight outcomes (shown above), only 15% of a similar group of graduate students had



A1: Deductive Formalisms

A2: Inductive Formalisms

A3: Analysis/Cont. Formalisms

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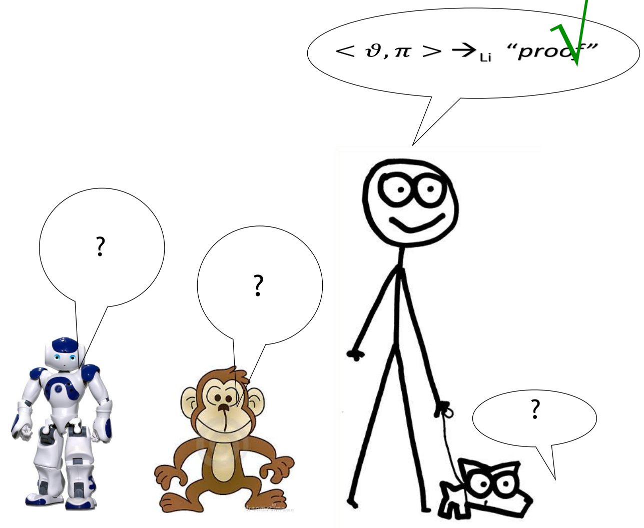

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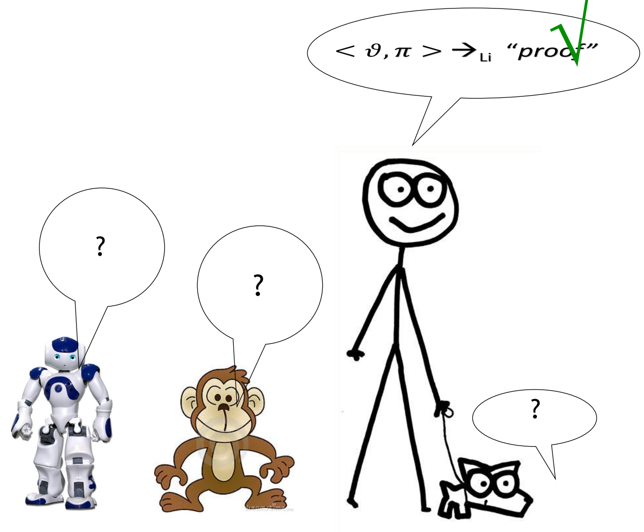

Urn-based Reasoning

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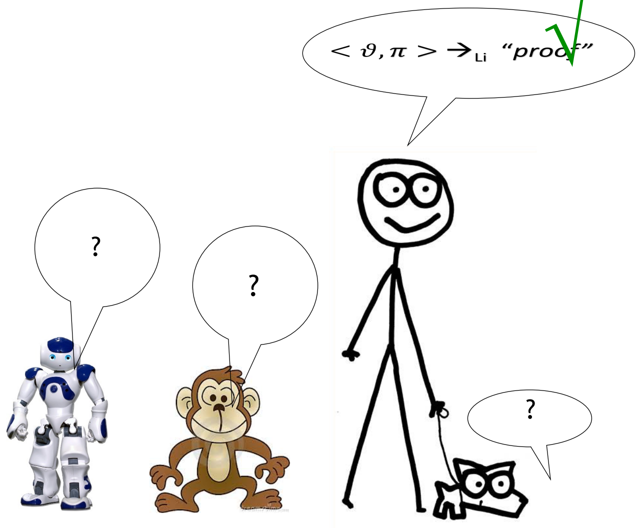

Urn-based Reasoning
Kolmogorov's Axioms

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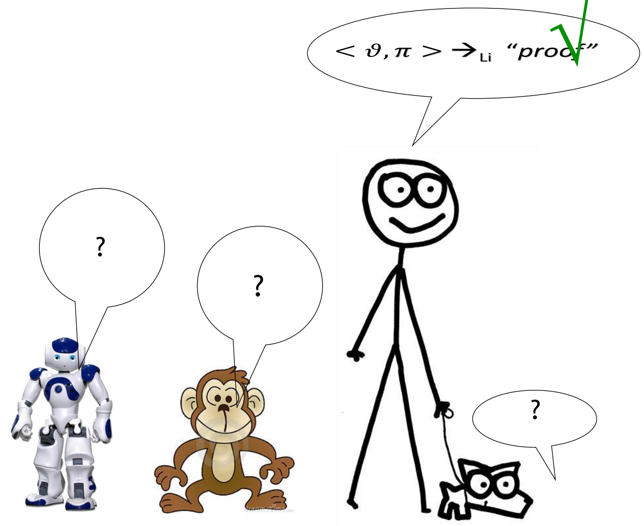

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Urn-based Reasoning
Kolmogorov’s Axioms
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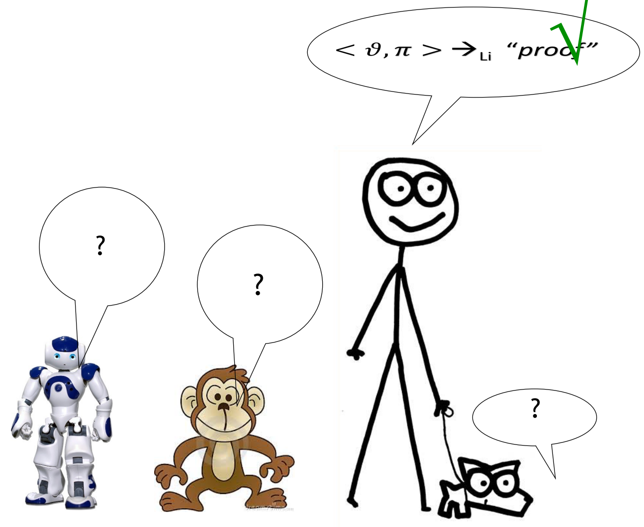

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"Narratological" Probability
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Urn-Based Reasoning …

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Probability that ball b_n is a black ball?

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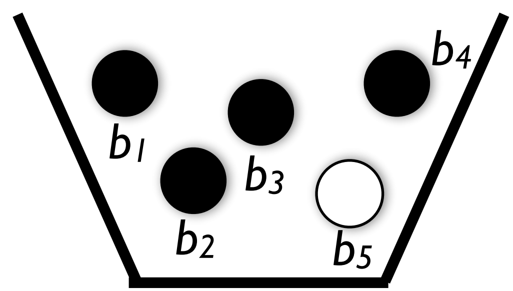

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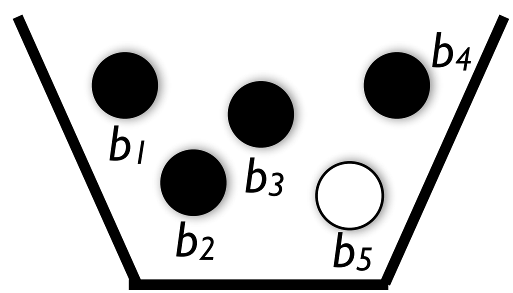

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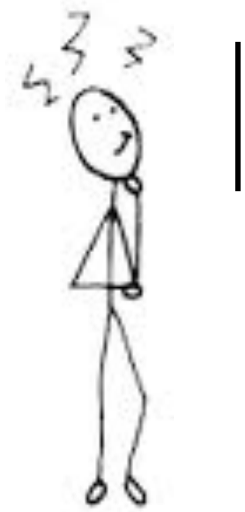

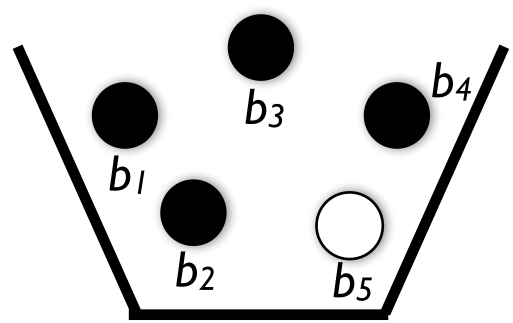

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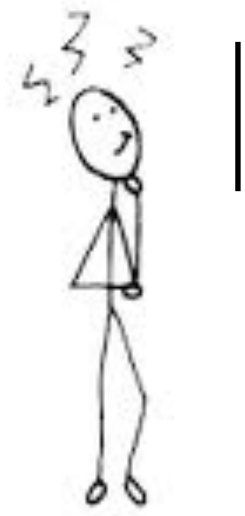

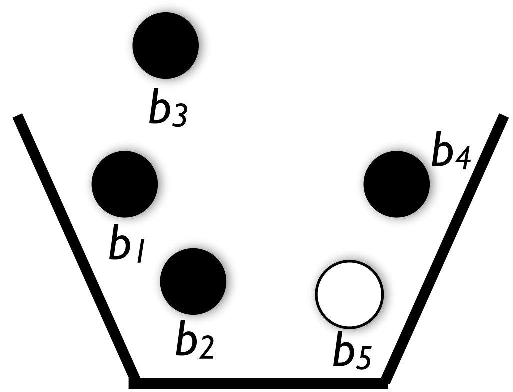

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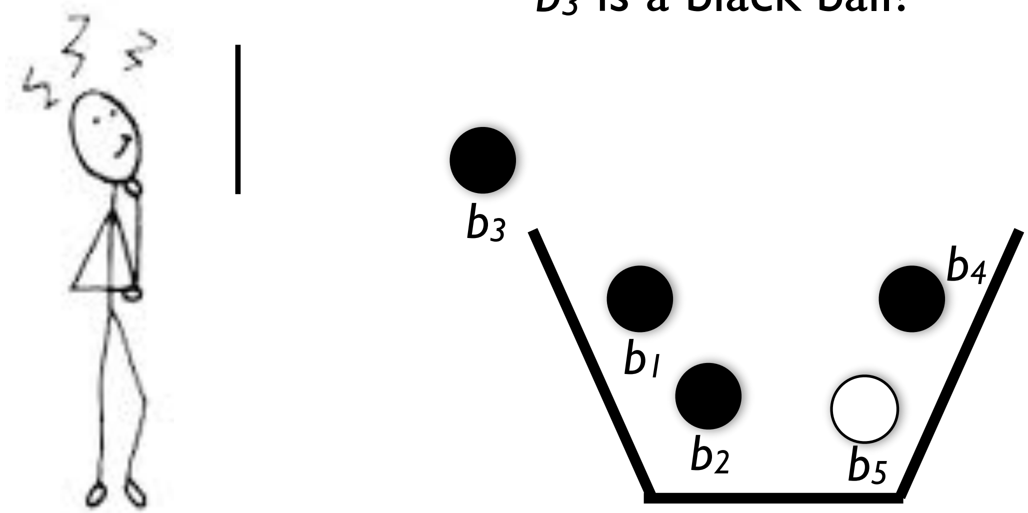

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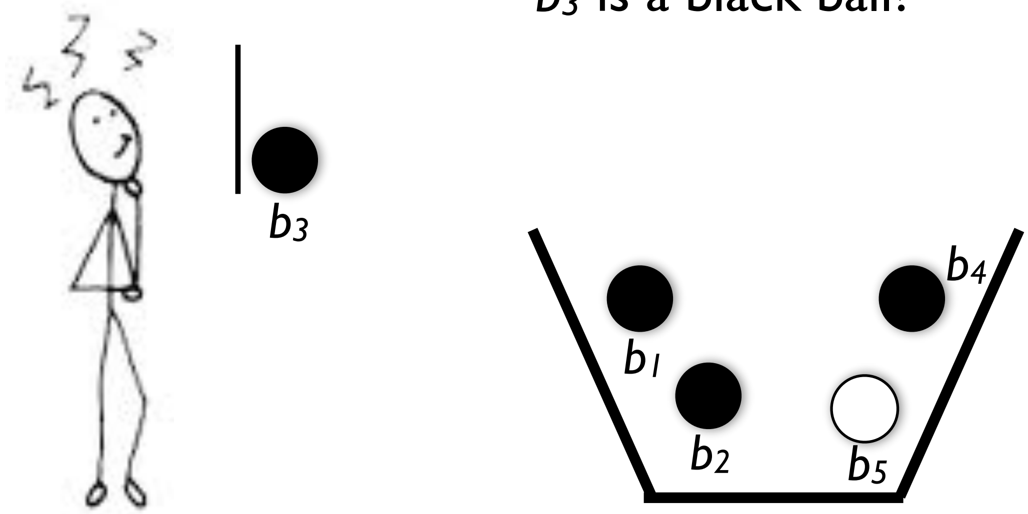

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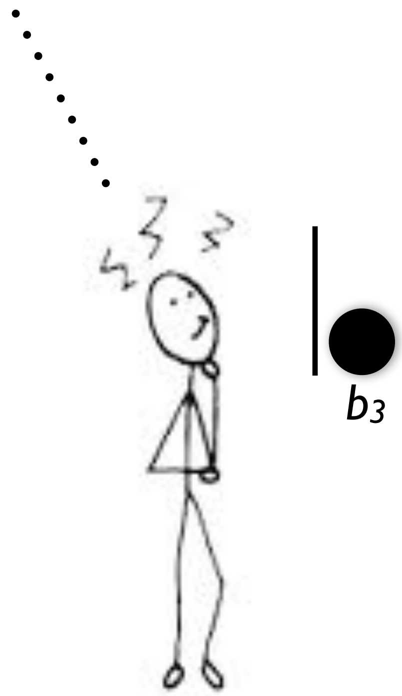

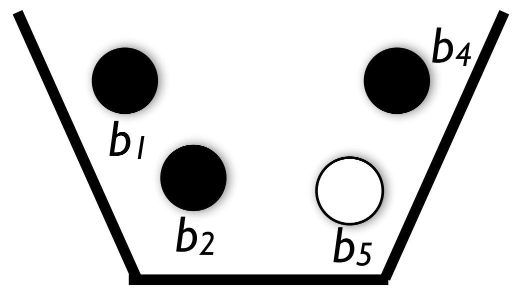

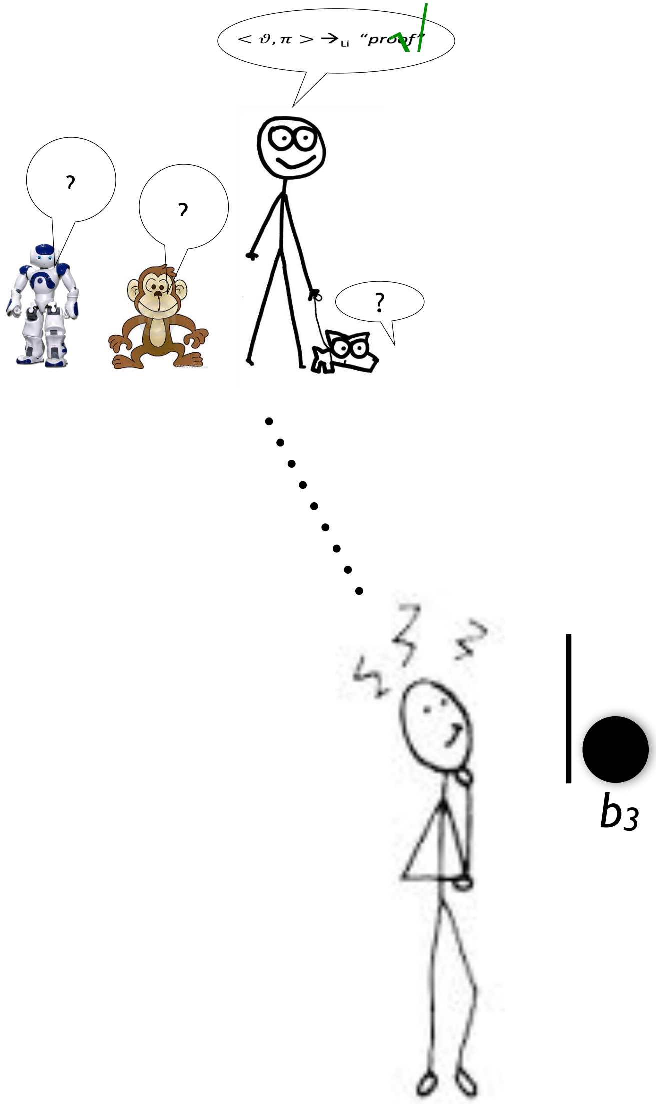

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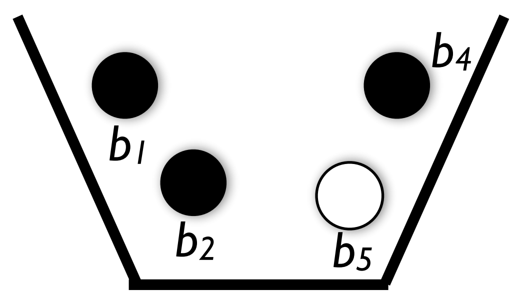

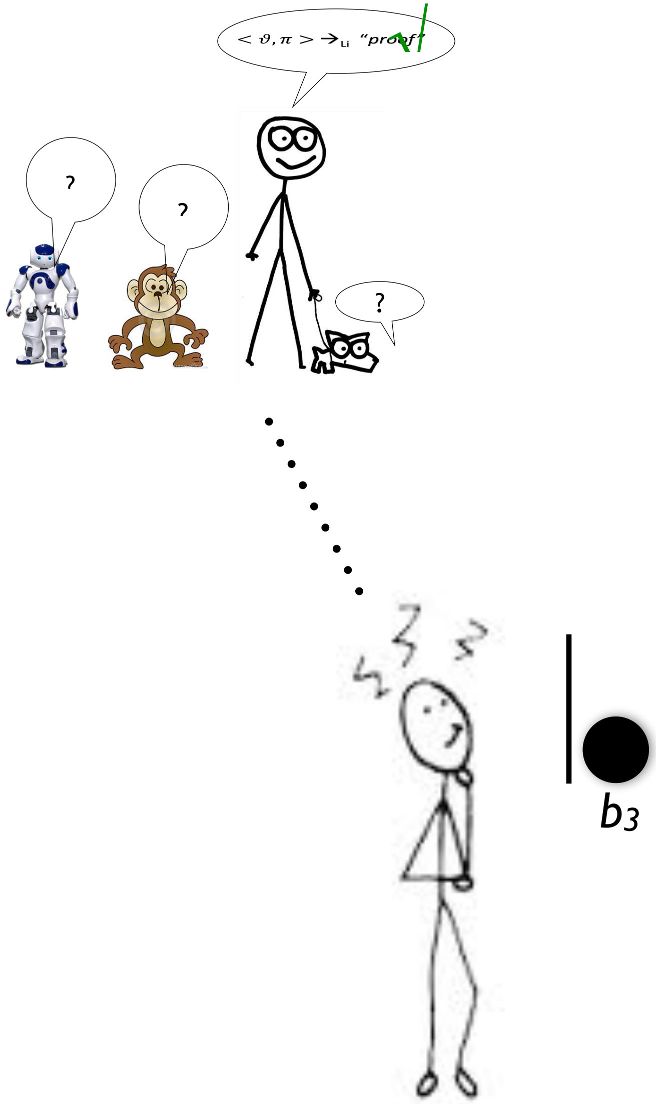

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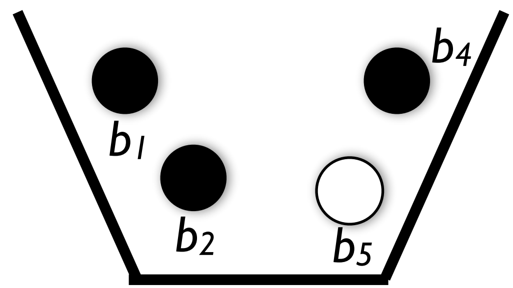

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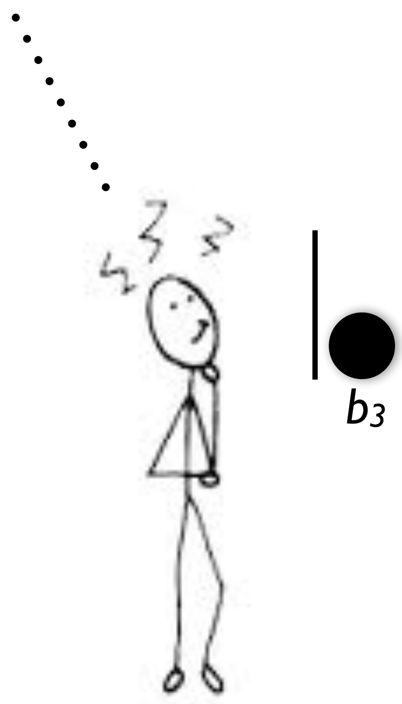

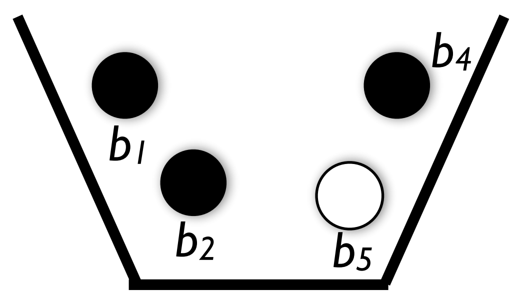

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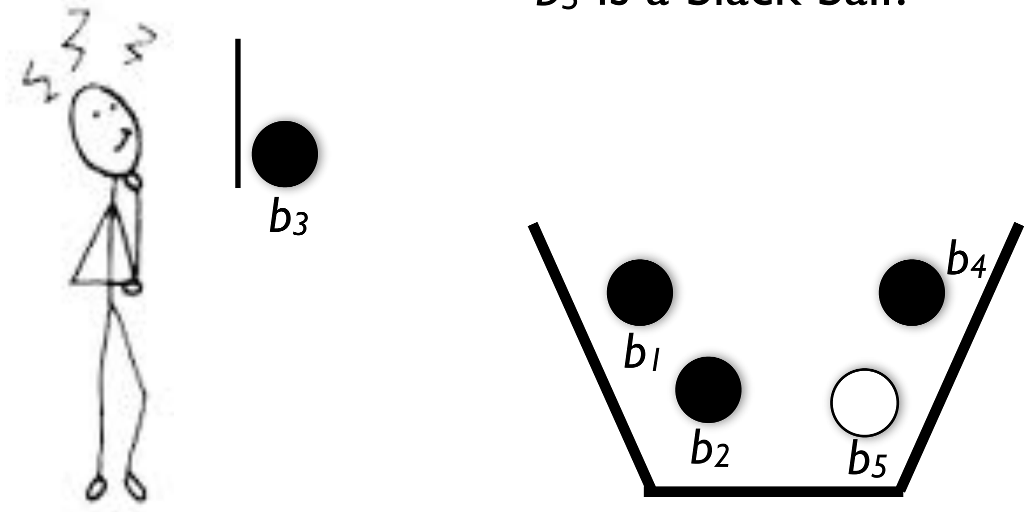

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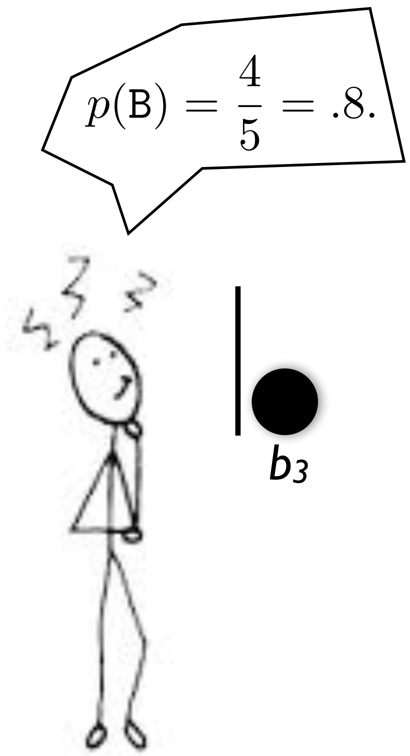

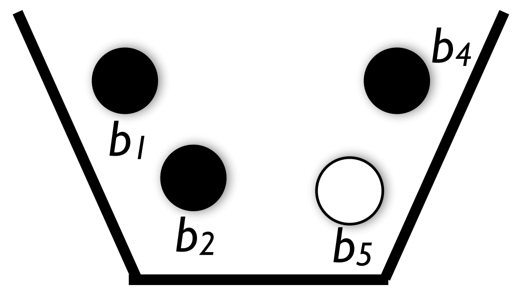

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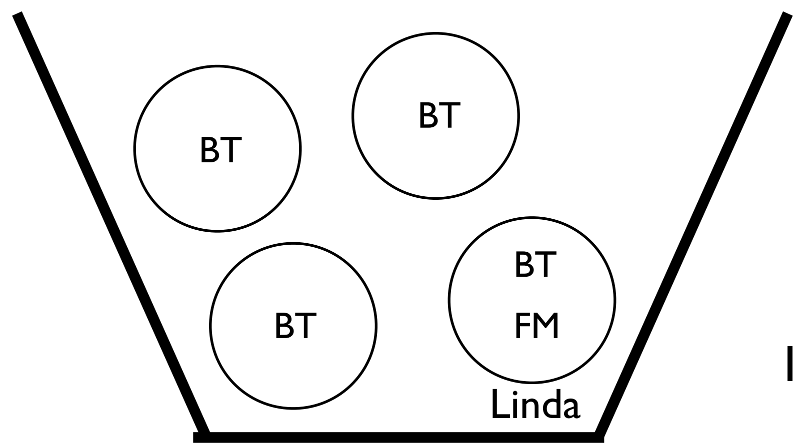

k people

4 BTs

1 BT-and-FM

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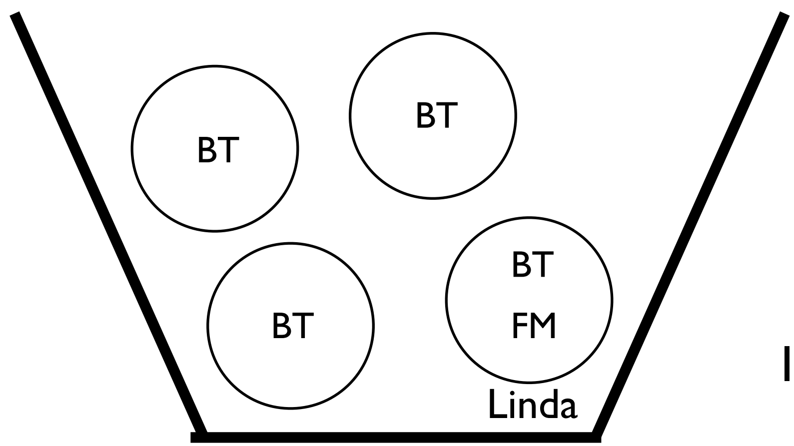

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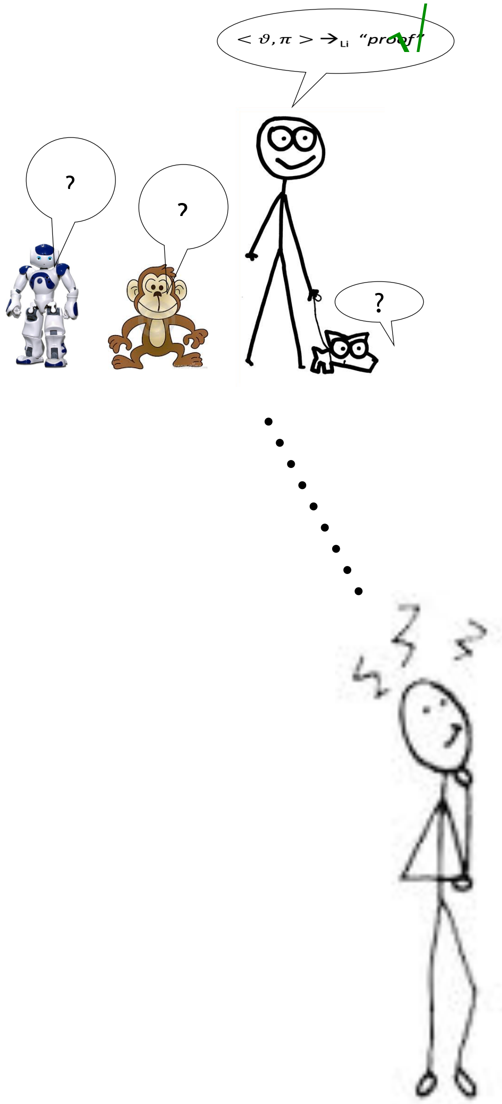

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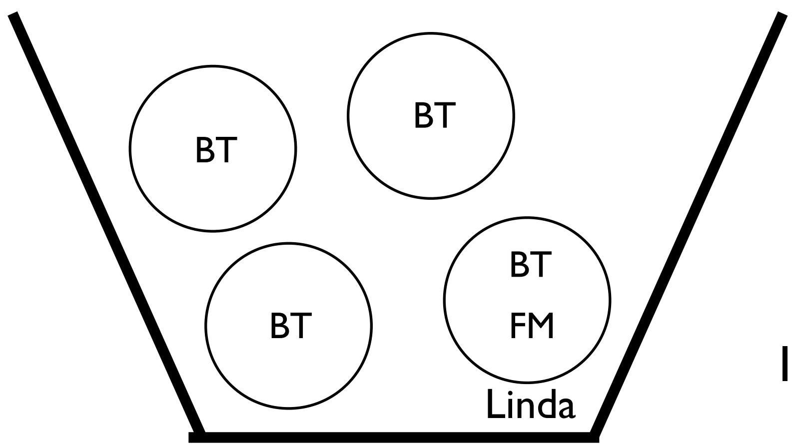

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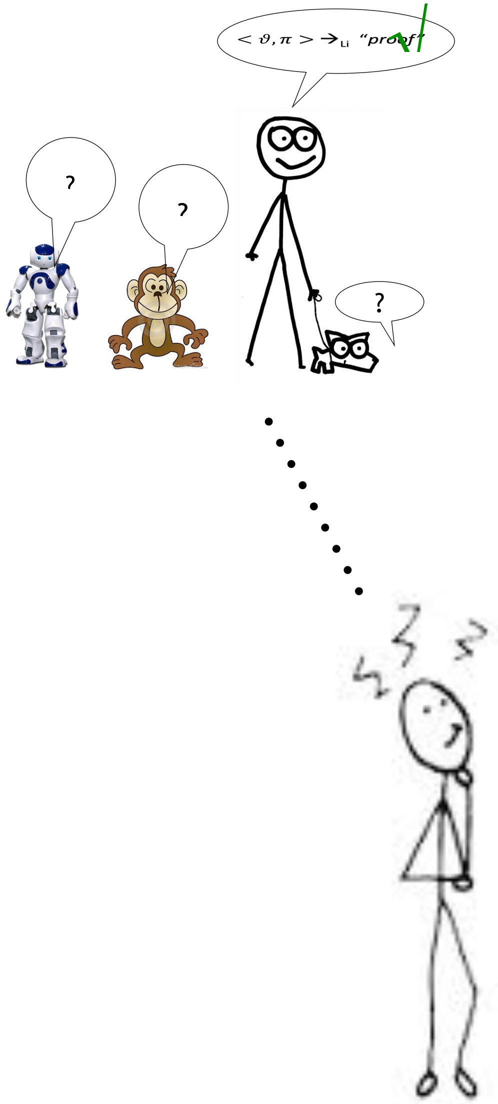

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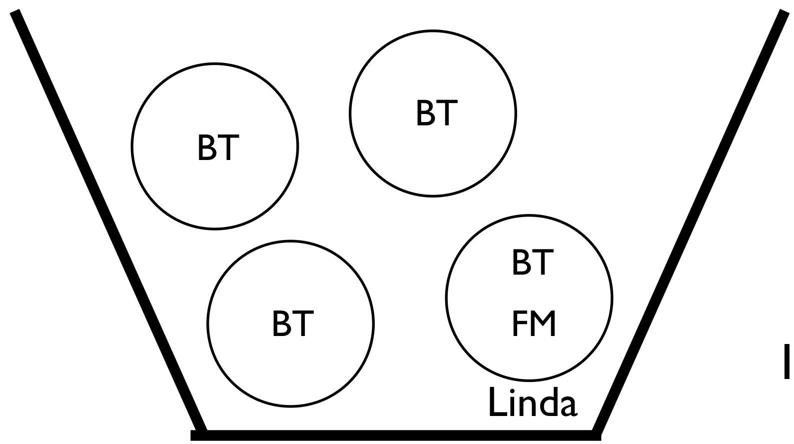

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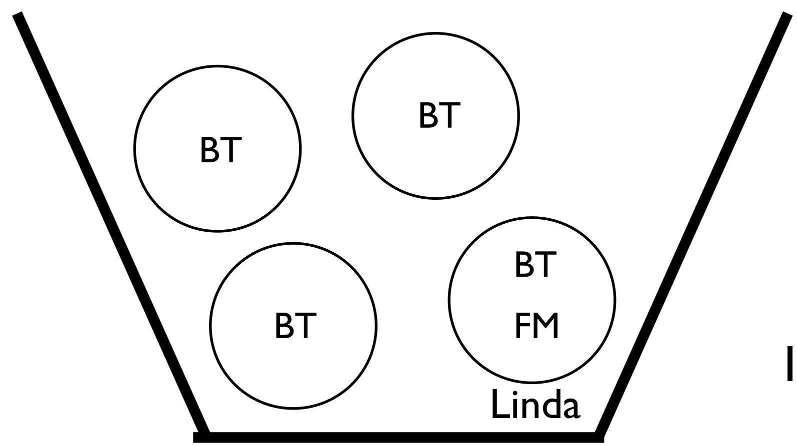

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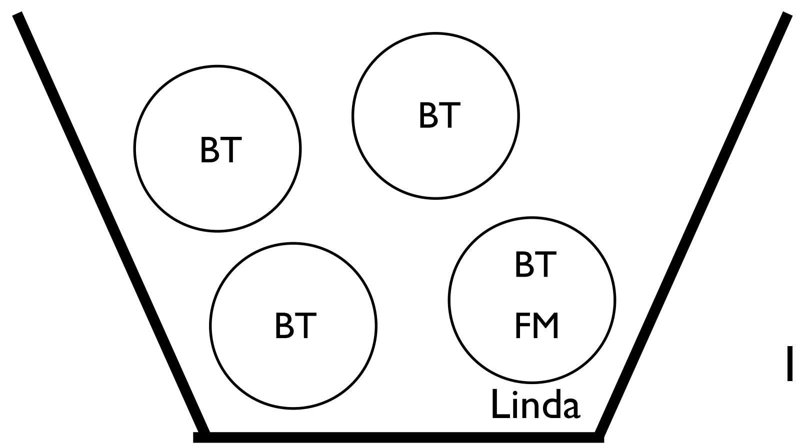

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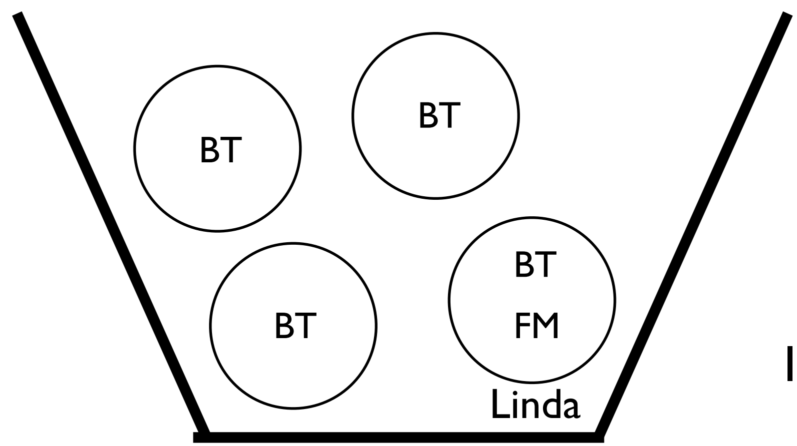

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Kolmogorov's Axioms …

Kolmogorov's Axioms of Probability
(= his *probability calculus*, viewed propositionally)

K1 $\forall\phi(0 \leq p(\phi) \leq 1)$.
Each formula in the propositional calculus has a probability between 0 and 1, inclusive.
K2 If $\vdash \phi$, then $p(\phi) = 1$.
All formulas that are deductively provable without remaining suppositions are certain.
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A formula that can be used to prove another has a probability less than or equal to the proved one.
K4 If $\{\phi, \psi\} \vdash \delta \wedge \neg\delta$, then $p(\phi \vee \psi) = p(\phi) + p(\psi)$.
Two inconsistent formulas, disjoined, have a probability equal to the sum of the probability of each.

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Hence it can never be the case that 'Linda is a bank teller and Linda is in the feminist movement' is more probable than 'Linda is a bank teller.'

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$$p(\text{Will be a 5.}|\text{Will be odd.}) = \frac{p(5 \wedge \text{odd})}{p(\text{odd})} = \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{3}$$

Re. K4

E.g., consider a fair die again. The set composed of the two propositions *The die will be odd* and *The die will be even* leads deductively to a contradiction. So K4 "predicts" that the probability of the disjunction of these two propositions is the sum of the probability of each independently. Does the prediction pan out?

Probability Logic …

Geometric Interpretation of Probability Logic

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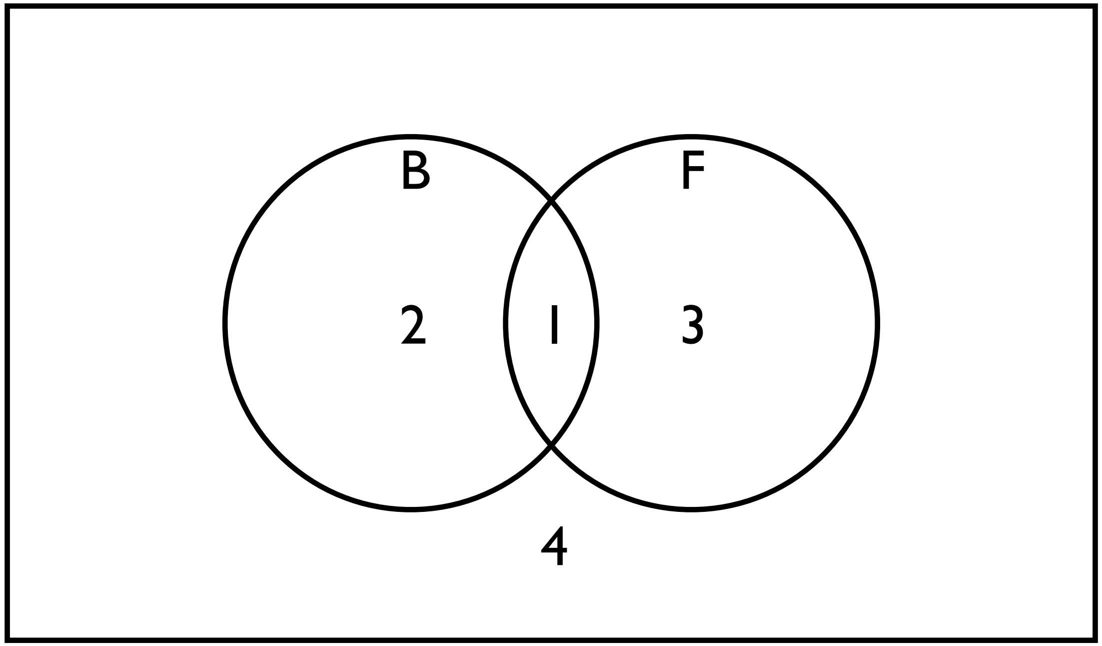
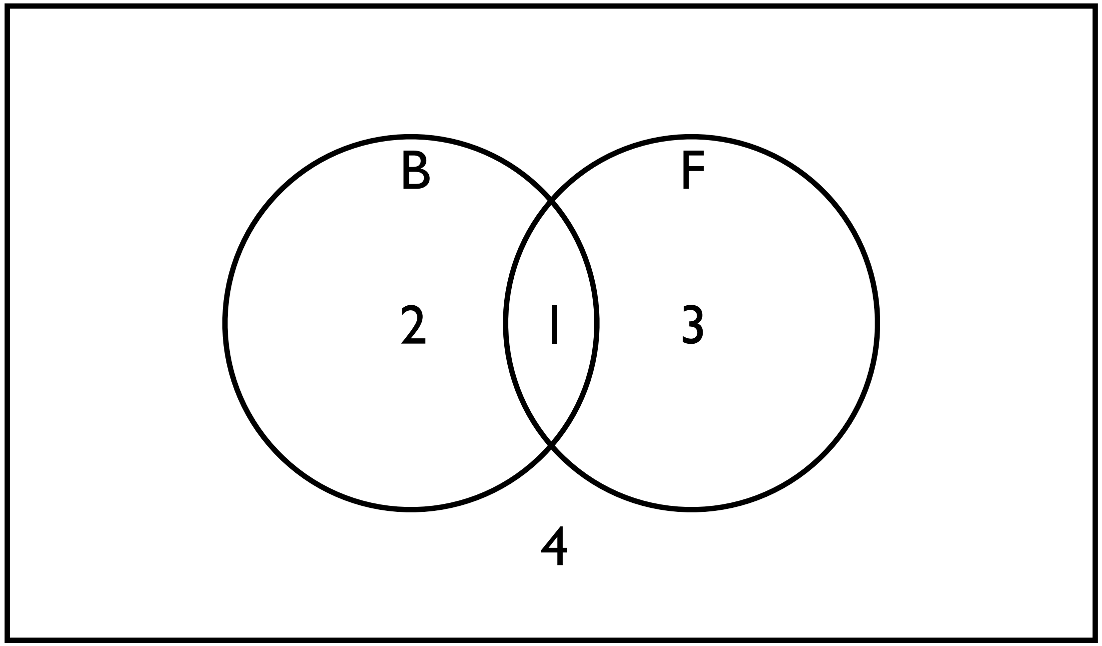


Geometric Interpretation of Probability Logic



Assume that the areas of the regions in the diagram represent the probabilities of the formulae they correspond to.

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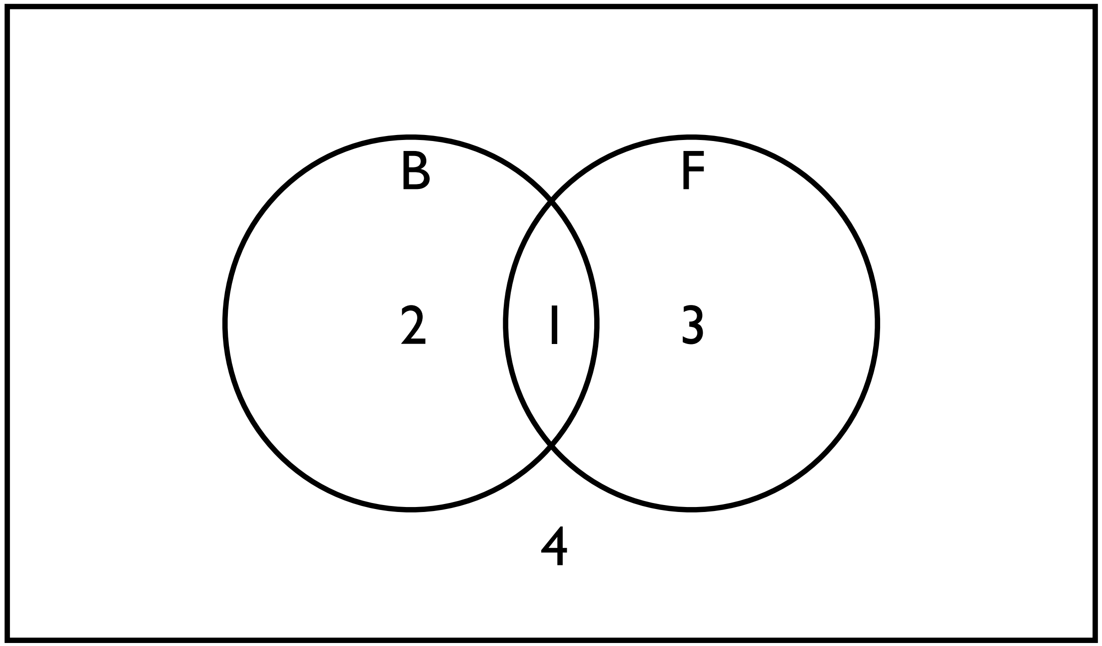

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Very well. Then why is it true that:

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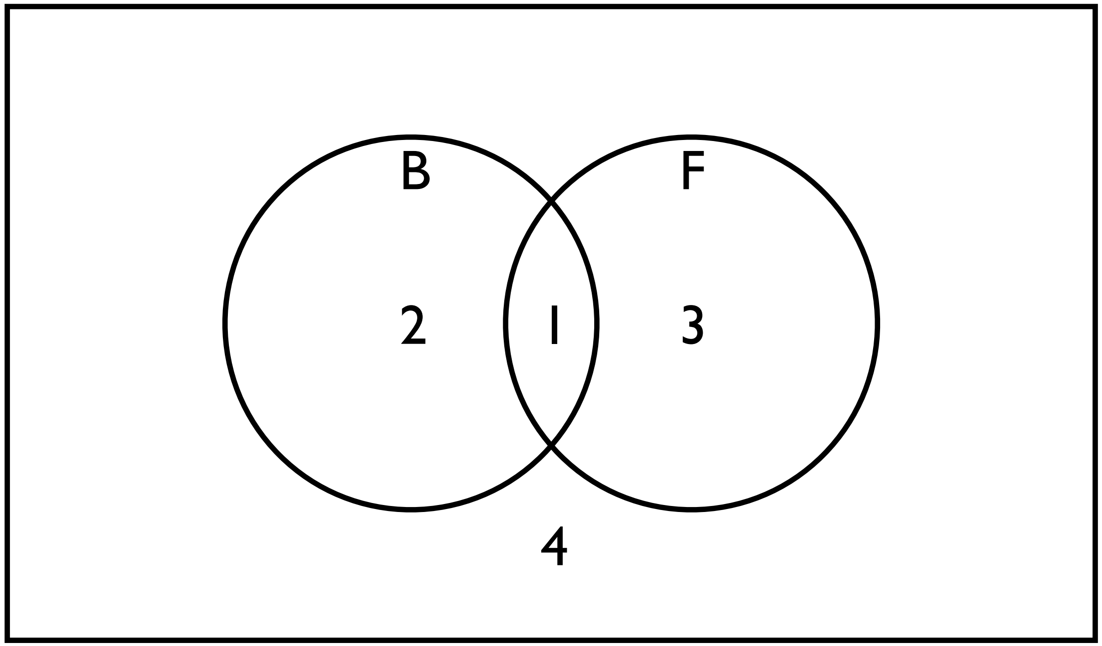

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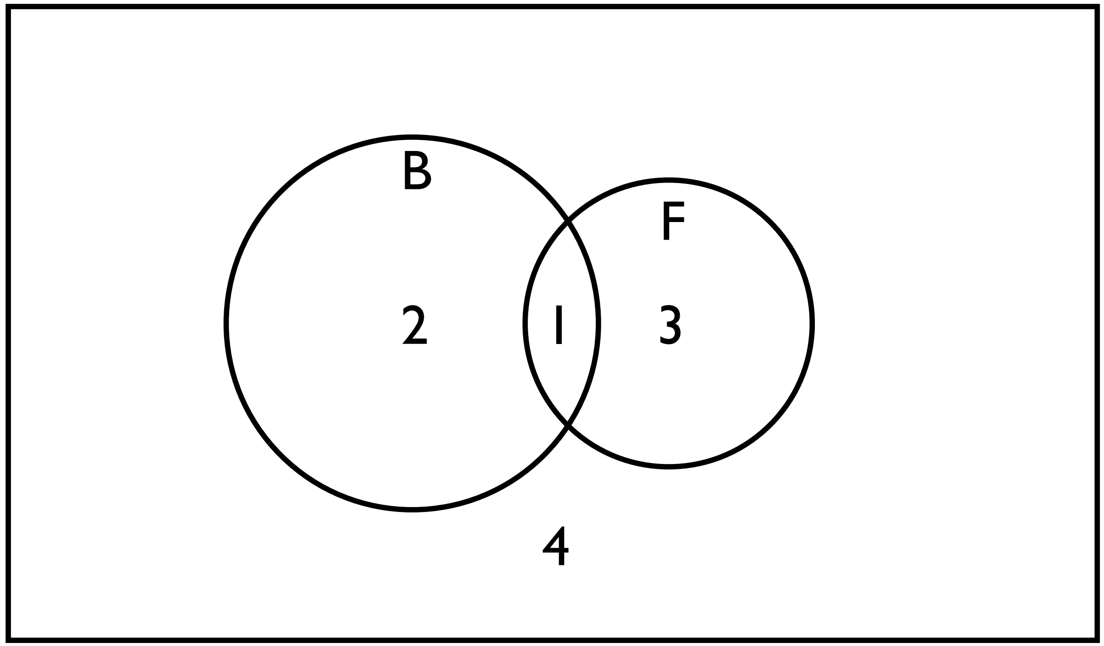
Geometric Interpretation of Probability Logic



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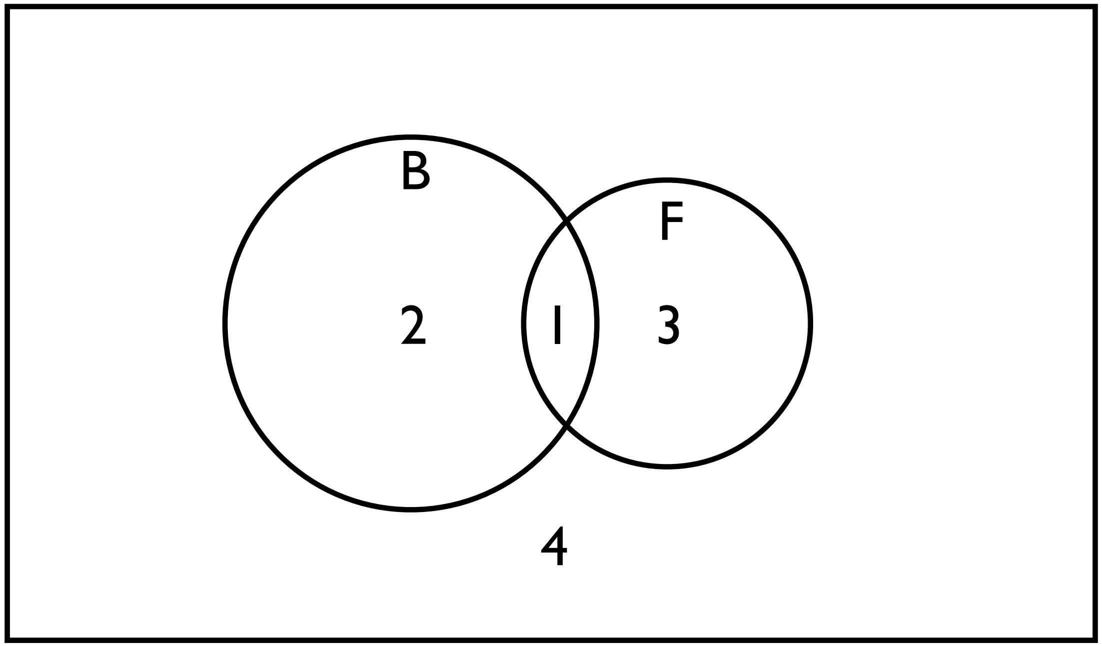

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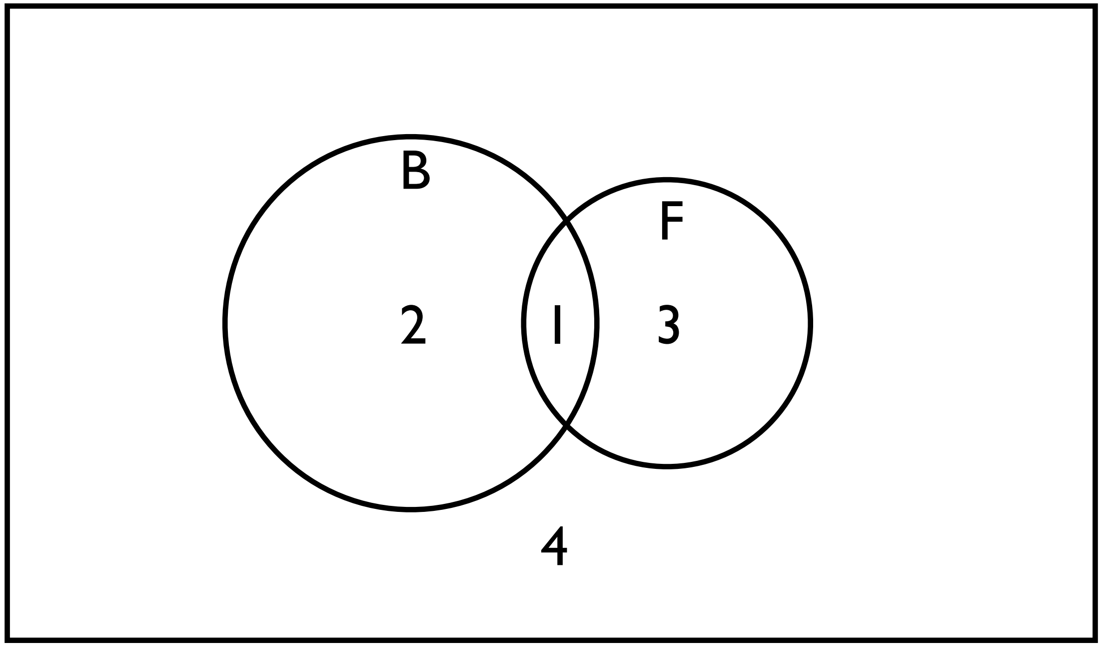

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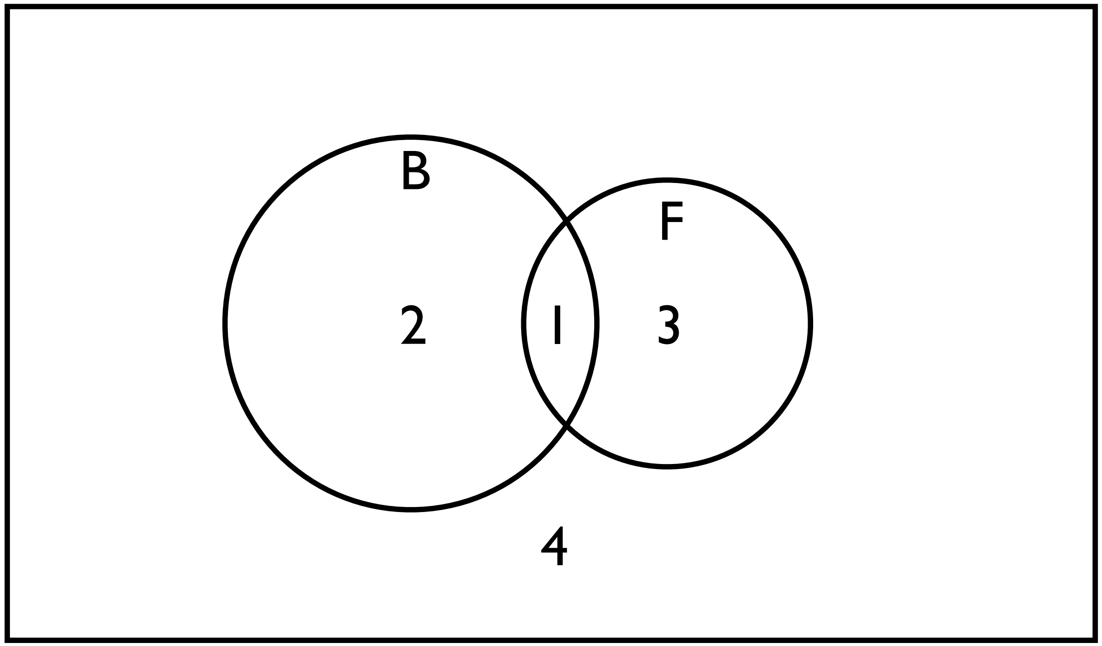
Geometric Interpretation of Probability Logic



Assume, again, that the areas of the regions in the diagram represent the probabilities of the formulae they correspond to. Here, the population of Fs is assumed to be smaller than that of Bs.

Very well. Then why is it true that: $p(\mathtt{B}) > p(\mathtt{B} \wedge \mathtt{F})$

What’s really going on with Linda …

“Narratological” Probability

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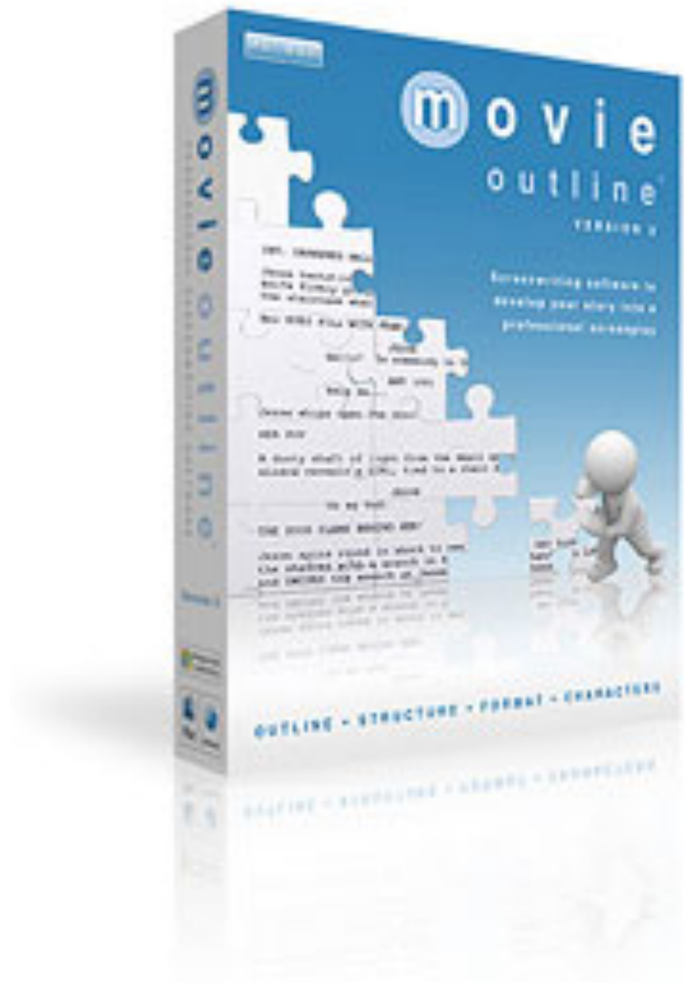

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Which is more probable [

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Which is more probable [as a heading-toward-3D-picture of a person]? **A** alone or **A** and **B** together?

LABE Formalization

$$\frac{\mathcal{S} \Vdash_{c.} \phi, \mathcal{S} \Vdash_{p} \mathbf{L}, \mathbf{L} \cup \{\beta\} \Vdash_{\sigma \geq c.} \phi}{\mathcal{S} \Vdash_{\sigma \geq c.} \phi \wedge \beta}$$