Rational Analysis of Some Shots @ R

Selmer Bringsjord

Are Humans Rational?

9/9/19

Selmer.Bringsjord@gmail.com
Bit of Historical Context …
2019

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2019

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Proof that if there is a body b whose wordline for observer m passes through point p but not through q or through q but not through p, then the events observed by m at p and q are different.

There is a point r reachable at the speed of light from p but not from q for observer m.
Theorem: NTFLIO
(deduced from SpecRel)

Proof that if there is a body b whose wordline for observer m passes through point p but not through q or through r but not through p, then the events observed by m at p and q are different.

There is a point x reachable at the speed of light from p but not from q for observer m.

Assume: In(b, ev(m, x)) ↔ W(m, b, z). Assume: ¬W(m, b, z) ↔ W(m, b, p) ↔ ¬W(m, b, q) ↔ In(b, ev(m, q)).

Theorem: NTFLIO

Rules of Inference

Premises:
1. In(b, ev(m, x)) ↔ W(m, b, z)
2. ¬W(m, b, z) ↔ W(m, b, p) ↔ ¬W(m, b, q)
3. In(b, ev(m, q))

Conclusion:
- NTFLIO

Premises:
1. A body b whose wordline for observer m passes through point p but not through q or through r but not through p.

Conclusion:
- The theorem

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Theorem: NTFLIO
(deduced from SpecRel)

\[ \mathcal{DCEC} \]

2019
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The elements of the branch of logic known as conditional logic: shown in Figure 1. Logics (e.g., FOL and SOL), to logics with intensional operators for always position some particular work he and likeminded collaborators could internally be implemented using computational formalisms (e.g., Turing-level computation; see e.g. (Boolos, Burgess 2019). This abstract view lets us model robots that also has an associated knowledge-base which are not exactly complicated formally speaking, require, when than the material conditional. (Reliance on conditional branching in standard formal logics that include conditionals much more expressive and nuanced induction to the sub-field in formal logic of conditional logic. In the final in formal-logic terms. If the robot had been more empathetic, Officer Smith would have thrived.

\textbf{DCEC*}

\begin{itemize}
\item \textbf{Rules of Inference}
\item \textbf{Substitutions}
\item \textbf{Quantifiers}
\item \textbf{Tautologies}
\item \textbf{Equivalences}
\end{itemize}

\textbf{2019}

\textbf{Are Humans Rational?}

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\begin{center}
\includegraphics[width=0.3\textwidth]{image.png}
\end{center}
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elements of the branch of logic known as logics (e.g., FOL and SOL), to logics with intensional operators for therein, which the reader will note is quite far down the dimension always position some particular work he and likeminded collaborators.

\[
\begin{align*}
\text{R}\text{M} & \text{I}\text{R} \\
\text{DCEC}^* & \text{A}\text{D}\text{R} \\
2019 & \text{A}\text{R} \text{E} \text{H}\text{U}\text{M}\text{A}\text{N}\text{S} \text{R}\text{A}\text{L}\text{I}\text{P}\text{A}\text{R}\text{I}\text{O}\text{N}\text{?} \\
& \text{S}\text{e}\text{l}\text{m}\text{e} \text{r} \text{B}\text{r}\text{i}\text{n}\text{g}\text{i}\text{n}\text{g}\text{d}
\end{align*}
\]

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elements of the branch of logic known as

\[ \text{DCEC}^* \]

\[
\begin{array}{c|c|c}
\text{Agent} & \text{Moment} & \text{Event} \\
\hline
\text{Robot} & \text{Time} & \text{Action} \\
\hline
\end{array}
\]

This language is not exactly complicated formally speaking, requires, when used in our hierarchy is the material conditional; but the material

\[ \text{RAIR} \]

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elements of the branch of logic known as

Syntax = P

Moment (t, a) => Event

Self (t, a) => Contract (t, a)

ActionType (1) => Payoff (t, a)

Boolean

Moral/Ethical Stack

DCEC

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Leibniz 1666

Are Humans Rational? 2019

Leibniz

DCEC

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RAIR

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1.5 centuries < Boole

Leibniz

1666

2019

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1.5 centuries < Boole
Boole

1.5 centuries < Boole

Leibniz

1666

2019

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Solner Dringsiersd

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"Universal Computational Logic"

1666

Leibniz

1.5 centuries < Boole

2019

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Salman Damlaj

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“Universal Computational Logic”

1666  1854  2019

Leibniz

1.5 centuries < Boole
Universal Computational Logic

Logics (e.g., FOL and SOL), to logics with intensional operators for DCEC correspond to the three arrows shown in Figure 2. We have positioned are undertaking within a view of logic that allows a particular...

Fig. 1.

1854

O

x

Fig. 2.

DCEC

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1666

Leibniz

1.5 centuries < Boole

Quantification!
The elements of the branch of logic known as modal logic correspond to the three arrows shown in Figure 2. We have positioned the logical system to be positioned relative to three dimensions, which are undertaking within a view of logic that allows a particular action to happen.

For example, consider the fluent `\( f \)` and the moment `\( t \)`.

The action `\( a \)` happens at the moment `\( t \)`.

The boolean moment `\( B \)` initiates the action `\( a \)` at the moment `\( t \)`.

The payoff `\( K \)` initiates the action `\( a \)` at the moment `\( t \)`.

The interval `\( I \)` initiates the action `\( a \)` at the moment `\( t \)`.

The agent `\( Agent \)` initiates the action `\( a \)` at the moment `\( t \)`.

The self `\( Self \)` initiates the action `\( a \)` at the moment `\( t \)`.

The type of action `\( ActionType \)` initiates the action `\( a \)` at the moment `\( t \)`.

The rules of inference `\( Rules \)` initiate the action `\( a \)` at the moment `\( t \)`.

The counterfactuals `\( C \)` initiate the action `\( a \)` at the moment `\( t \)`.

The event `\( Event \)` initiates the action `\( a \)` at the moment `\( t \)`.

The rules of inference `\( Rules \)` initiate the action `\( a \)` at the moment `\( t \)`.
"Universal Computational Logic"

1666  1854  2019

1.5 centuries < Boole

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1666

Leibniz

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1854

2019

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1666
Leibniz

1.5 centuries < Boole

1854
Logic Theorist
(birth of modern logicist AI)
(birth of agent-based/behavioral econ)

1956
Simon

2019
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DCEC*
“Universal Computational Logic”

1.5 centuries < Boole

Leibniz

1854

An Investigation of the Laws of Thought
George Boole

1956

Logic Theorist
(birth of modern logicist AI)
(birth of agent-based/behavioral econ)

1666

1.5 centuries < Boole

DCEC

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Simon

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“Universal Computational Logic”

Logic Theorist
(birth of modern logicist AI)
(birth of agent-based/behavioral econ)

1666
1854
1956
2019
2020

Leibniz
1.5 centuries < Boole

Simon

Are Humans Rational?

DCEC*

1

Rensselaer AI and Reasoning Lab
“Universal Computational Logic”

1666
Leibniz

1854
Logic Theorist
(birth of modern logicist AI)
(birth of agent-based/behavioral econ)

1956
Simon

2019
Are Humans Rational?

2020

The Future?
(rationally considered)
1.5 centuries < Boole

Leibniz

1666

The Future? (rationally considered)

1854

The robotic substrate

1956

Are Humans Rational?

Simon

2019

DCEC*

2020

"Universal Computational Logic"
Propositional Calculus!
First Elements of the Propositional Calculus
First Elements of the Propositional Calculus

Variables to represent declarative statements.
First Elements of the Propositional Calculus

Variables to represent declarative statements.

E.g., $k$ to represent ‘There is a king in the hand’.
First Elements of the Propositional Calculus

Variables to represent declarative statements.

E.g., $k$ to represent ‘There is a king in the hand’.

And five simple Boolean connectives:
First Elements of the Propositional Calculus

Variables to represent declarative statements.

E.g., \( k \) to represent ‘There is a king in the hand’.

And five simple Boolean connectives:

\[
\neg \quad \text{and} \quad \land \quad \text{or (inclusive)} \quad \lor \quad \text{if ... then ...} \quad \rightarrow \quad \text{... if and only if ...} \quad \leftrightarrow
\]
Wason Selection Task

Suppose I claim that the following rule is true.

If a card has a vowel on one side, it has an even number on the other side.

Which card or cards should you turn over in order to try to decide whether the rule is true or false?
Suppose I claim that the following rule is true.

If a card has a vowel on one side, it has an even number on the other side.

Which card or cards should you turn over in order to try to decide whether the rule is true or false?
Suppose I claim that the following rule is true.

If a card has a vowel on one side, it has an even number on the other side.

Which card or cards should you turn over in order to try to decide whether the rule is true or false?
Or test. For an overview of Psychometric AI, see:
(the pair represented in mind of $h$)

$\langle \chi, \pi \rangle$

$\chi$ (context)

$\pi$ (problem)
\langle \chi, \pi \rangle
Formalism 1
Formalism 2
Formalism 3

...
Formalism_1
Formalism_2
Formalism_3
...

\langle \chi, \pi \rangle
\langle \chi, \pi \rangle \rightsquigarrow \langle \alpha, \text{argument/proof} \rangle
Today’s machine-learning systems are fundamentally incapable of providing the argument/proof.

\[ \langle \chi, \pi \rangle \rightsquigarrow \langle \alpha, \text{argument/proof} \rangle \]

https://www.darpa.mil/program/explainable-artificial-intelligence
Contrarian view on animal minds in *Nat. Geo.*:
http://ngm.nationalgeographic.com/2008/03/animal-minds/virginia-morell-text
Ok, so where’s the proof (or at least the compelling argument)?

\[ \langle \chi, \pi \rangle \leadsto \langle \alpha, \text{argument}/\text{proof} \rangle \]

Contrarian view on animal minds in Nat. Geo.:
http://ngm.nationalgeographic.com/2008/03/animal-minds/virginia-morell-text
Suppose I claim that the following rule is true.

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Wason Selection Task

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Suppose I claim that the following rule is true.

If a card has a vowel on one side, it has an even number on the other side.

Which card or cards should you turn over in order to try to decide whether the rule is true or false?

**Proposition 1**: You should flip c1!

**Proof**: Were you to flip c1, there are two and only two general cases that might appear before your eyes: you find an odd number; or else you find an even number. Well, if you find an odd number, you can stop, because the rule in question would then be refuted (since you have a case where the antecedent (vowel on one side) holds, but the consequent (even number on the other side) doesn’t. Since this might well happen for all you know, you should flip over c1. **QED**
Suppose I claim that the following rule is true:

If a card has a vowel on one side, it has an even number on the other side.

Which card or cards should you turn over in order to try to decide whether the rule is true or false?

Proposition 2: You should flip c4!

Proof: Were you to flip c4, there are two and only two general cases that might appear before your eyes: you find a vowel; or else you find a consonant. Well, if you find a vowel, you can stop, because the rule in question would then be refuted (since you have a case where the antecedent (vowel on one side) holds, but the consequent (even number on the other side) doesn’t. Since this might well happen for all you know, you should flip over c4. QED
Suppose I claim that the following rule is true.
If a card has a vowel on one side, it has an even number on
the other side.
Which card or cards should you turn over in order to try to decide
whether the rule is true or false?

**Proposition 2:** You should flip c4!

**Proof:** Were you to flip c4, there are two and only two
general cases that might appear before your eyes: you find a
vowel; or else you find a consonant. Well, if you find a vowel,
you can stop, because the rule in question would then be
refuted (since you have a case where the antecedent (vowel
on one side) holds, but the consequent (even number on the
other side) doesn’t. Since this might well happen for all you
know, you should flip over c4. **QED**

**Proposition 3:** You should not flip c2!

**Proposition 4:** You should *not* flip c3!
Given the statements

\( \neg a \lor \neg b \)

b

c \rightarrow a

which one of the following statements must also be true?

c
\( \neg b \)

\( \neg c \)
h

a

none of the above
Given the statements

\(\neg a \lor \neg b\)

b

c \rightarrow a

which one of the following statements must also be true?

c
\(\neg b\)
\(\neg c\)

h

a

none of the above
Proposition: The correct answer is \( \neg c \).

Proof: We are given that \( b \); that’s the second statement. Well, if \( b \) holds, then \( \neg b \) doesn’t hold. The first statement tells us that either \( \neg a \) or \( \neg b \). So from this and the derived proposition that \( \neg b \) doesn’t hold we can infer \( \neg a \). (If you know \( P \) or \( Q \), and you know not-\( Q \), you immediately know \( P \); this inference rule is called *disjunctive syllogism*.) But from \( \neg a \) and \( c \rightarrow a \) we can deduce that \( c \) can’t be the case; i.e., we can deduce \( \neg c \). (This last inference is sanctioned by the rule of inference called *modus tollens*.) QED
Given the statements
\neg \neg c
\neg c \rightarrow a
\neg a \lor b
b \rightarrow d
\neg (d \lor e)

which one of the following statements must also be true?

\neg c
e
h
\neg a
all of the above
Given the statements
\(-\neg c\)
\(c \to a\)
\(-a \lor b\)
\(b \to d\)
\(- (d \lor e)\)

which one of the following statements must also be true?

\(-c\)
e
h
\(-a\)
all of the above
CSCI 2200: Foundations of Computer Science – Spring 2015

General Information
Instructor: Stacy Patterson  sep@cs.rpi.edu  518-276-2054

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Lectures: MR 10:00am – 11:50 pm, Russell Sage Laboratory 3303

Recitations:
Section 01  W 10:00am – 10:50am  Troy Building 2012
Section 02  W 11:00am – 11:50am  Troy Building 2012
Section 03  W 12:00pm – 12:50pm  Troy Building 2018
Section 04  W 4:00pm – 4:50pm  Walker Laboratory 5113

Course Description
This course introduces important mathematical and theoretical tools for computer science, including topics from logic, number theory, set theory, combinatorics, and probability theory. The course then proceeds to automata theory, the Turing Machine model of computation, and notions of computational complexity. The course will emphasize formal reasoning and proof techniques.

Upon successful completion of this course, each student:
• is able to formulate mathematical proofs using logic
• is able to apply mathematical tools such as induction and recursion
• can recall key definitions from set theory
• is able to formulate combinatorial arguments
• is able to distinguish between various computational models
• is able to think critically on the difficulties of key questions in foundations of computer science
• can recall key facts regarding finite automata and Turing machines.

Pre-requisites: Intro to Calculus (MATH 1010 or MATH 1500); CSCI 1100 (CS I) or CSCI 1200 (Data Structures)

Recitation
Attendance at recitation is not required. Attendance will be taken at recitation, and students who attend regularly will get priority in office hours.

Schedule
An up-to-date schedule will be maintained on the course web site

Homework
There will be 9 homework assignments. The lowest homework grade will be dropped. Homework is due at the beginning of class on the date indicated on the homework assignment. You may turn in an assignment at the beginning of following class for a 50% penalty. No homework will be accepted after that time without a letter from the Student Experience office.

See also e.g. http://www.cs.rpi.edu/~magdon/courses/focs.html
CSCI 2200: Foundations of Computer Science – Spring 2015

General Information
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Website: http://www.cs.rpi.edu/~magdon/courses/focs.html


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See also e.g. http://www.cs.rpi.edu/~magdon/courses/focs.html
Or ...
### List of Symbols

<table>
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<th>Subject</th>
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<td>Logic</td>
<td>(\neg p)</td>
<td>not (p)</td>
<td>25</td>
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<td></td>
<td>(p \land q)</td>
<td>(p) and (q)</td>
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<td>(p \lor q)</td>
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<td></td>
<td>(p \oplus q) or (p \mathrm{NOR} q)</td>
<td>(p) or (q) but not both (p) and (q)</td>
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<td>(p \implies q)</td>
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<td>(\therefore)</td>
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<td>(P(x))</td>
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<td>(P(x) \implies Q(x))</td>
<td>every element in the truth set for (P(x)) is in the truth set for (Q(x))</td>
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<td></td>
<td>(Q(x))</td>
<td>(P(x)) and (Q(x)) have identical truth sets</td>
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<td>(\forall)</td>
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<td>(\exists)</td>
<td>there exists</td>
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<table>
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<th>Applications</th>
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<td>of Logic</td>
<td>(\text{NOT})</td>
<td>NOT-gate</td>
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<td>Theory and</td>
<td>(n)</td>
<td>number written in binary notation</td>
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<tr>
<td>Applications</td>
<td>(n_{10})</td>
<td>number written in decimal notation</td>
<td>78</td>
</tr>
<tr>
<td></td>
<td>(n_{16})</td>
<td>number written in hexadecimal notation</td>
<td>91</td>
</tr>
</tbody>
</table>

<p>|                     | (d) | (d) divides (n)                         | 170  |
|                     | (d) | (d) does not divide (n)                | 172  |
|                     | (n) | the integer quotient of (n) divided by (d) | 181  |
| Theory and         | (n) | the integer remainder of (n) divided by (d) | 181  |
| Applications       | (x) | the floor of (x)                          | 191  |
|                     | (x) | the ceiling of (x)                        | 191  |
|                     | (x) | the absolute value of (x)                | 187  |
|                     | (\gcd(a, b)) | the greatest common divisor of (a) and (b) | 220  |
|                     | (x := e) | (x) is assigned the value (e)          | 214  |</p>
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<tr>
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<td>( P \equiv Q )</td>
<td>( P ) is logically equivalent to ( Q )</td>
<td>30</td>
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<tr>
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<td>if ( p ) then ( q )</td>
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<td>( p ) if and only if ( q )</td>
<td>45</td>
</tr>
<tr>
<td></td>
<td>\therefore</td>
<td>therefore</td>
<td>51</td>
</tr>
<tr>
<td></td>
<td>( P(x) )</td>
<td>predicate in ( x )</td>
<td>97</td>
</tr>
<tr>
<td></td>
<td>( P(x) \implies Q(x) )</td>
<td>every element in the truth set for ( P(x) ) is in the truth set for ( Q(x) )</td>
<td>104</td>
</tr>
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<td>( P(x) \iff Q(x) )</td>
<td>( P(x) ) and ( Q(x) ) have identical truth sets</td>
<td>104</td>
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<td></td>
<td>( \forall )</td>
<td>for all</td>
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</tr>
<tr>
<td></td>
<td>( \exists )</td>
<td>there exists</td>
<td>103</td>
</tr>
</tbody>
</table>

**Applications of Logic**

- [NOT-gate](#)
- [AND-gate](#)
- [OR-gate](#)
- [NAND-gate](#)
- [NOR-gate](#)
<table>
<thead>
<tr>
<th>Subject</th>
<th>Symbol</th>
<th>Meaning</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Logic</td>
<td>(~p)</td>
<td>not (p)</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>(p \land q)</td>
<td>(p) and (q)</td>
<td>25</td>
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<td>(p \lor q)</td>
<td>(p) or (q)</td>
<td>25</td>
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<tr>
<td></td>
<td>(p \oplus q) or (p \text{ XOR } q)</td>
<td>(p) or (q) but not both (p) and (q)</td>
<td>28</td>
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<td></td>
<td>(P \equiv Q)</td>
<td>(P) is logically equivalent to (Q)</td>
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<tr>
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<td>(p \implies q)</td>
<td>if (p) then (q)</td>
<td>40</td>
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<td></td>
<td>(p \iff q)</td>
<td>(p) if and only if (q)</td>
<td>45</td>
</tr>
<tr>
<td></td>
<td>\therefore</td>
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<td>51</td>
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<td>for all</td>
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</tr>
<tr>
<td></td>
<td>(\exists)</td>
<td>there exists</td>
<td>103</td>
</tr>
</tbody>
</table>

**Applications of Logic**

- **NOT-gate**: 
- **AND-gate**: 
- **OR-gate**: 
- **NAND-gate**: 
- **NOR-gate**: 
### List of Symbols

<table>
<thead>
<tr>
<th>Subject</th>
<th>Symbol</th>
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<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Logic</td>
<td>( \neg p )</td>
<td>not ( p )</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>( p \land q )</td>
<td>( p ) and ( q )</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>( p \lor q )</td>
<td>( p ) or ( q )</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>( p \oplus q ) or ( p \mathrm{NOR} q )</td>
<td>( p ) or ( q ) but not both ( p ) and ( q )</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td>( p \Rightarrow q )</td>
<td>if ( p ) then ( q )</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>( p \iff q )</td>
<td>( p ) if and only if ( q )</td>
<td>45</td>
</tr>
<tr>
<td></td>
<td>( \therefore )</td>
<td>therefore</td>
<td>54</td>
</tr>
<tr>
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<td>( P(x) )</td>
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<td>every element in the truth set for ( P(x) ) in the truth set for ( Q(x) )</td>
<td>104</td>
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<td></td>
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<td>there exists</td>
<td>103</td>
</tr>
</tbody>
</table>

### Applications of Logic

- **NOT**
  - Symbol: \( \neg \)
  - Meaning: NOT-gate
  - Page: 62
- **AND**
  - Symbol: \( \land \)
  - Meaning: AND-gate
  - Page: 67
- **OR**
  - Symbol: \( \lor \)
  - Meaning: OR-gate
  - Page: 67
- **NAND**
  - Symbol: \( \neg \land \)
  - Meaning: NAND-gate
  - Page: 75
- **NOR**
  - Symbol: \( \neg \lor \)
  - Meaning: NOR-gate
  - Page: 75

- **|**
  - Meaning: Sheffer stroke
  - Page: 74
- **↓**
  - Meaning: Peirce arrow
  - Page: 74
- \( n_2 \)
  - Meaning: number written in binary notation
  - Page: 78
- \( n_{10} \)
  - Meaning: number written in decimal notation
  - Page: 78
- \( n_{16} \)
  - Meaning: number written in hexadecimal notation
  - Page: 91

### Number Theory and Applications

- \( d | n \)
  - Meaning: \( d \) divides \( n \)
  - Page: 170
- \( d \nmid n \)
  - Meaning: \( d \) does not divide \( n \)
  - Page: 172
- \( n \div d \)
  - Meaning: the integer quotient of \( n \) divided by \( d \)
  - Page: 181
- \( n \mod d \)
  - Meaning: the integer remainder of \( n \) divided by \( d \)
  - Page: 181
- \( \lfloor x \rfloor \)
  - Meaning: the floor of \( x \)
  - Page: 191
- \( \lceil x \rceil \)
  - Meaning: the ceiling of \( x \)
  - Page: 191
- \( |x| \)
  - Meaning: the absolute value of \( x \)
  - Page: 187
- \( \gcd(a, b) \)
  - Meaning: the greatest common divisor of \( a \) and \( b \)
  - Page: 220
- \( x := e \)
  - Meaning: \( x \) is assigned the value \( e \)
  - Page: 214
But we’ll instead go with …
But we’ll instead go with ...
<table>
<thead>
<tr>
<th>Rule of Inference</th>
<th>Tautology</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p )</td>
<td>((p \land (p \rightarrow q)) \rightarrow q)</td>
<td>Modus ponens</td>
</tr>
<tr>
<td>( p \rightarrow q )</td>
<td>(\therefore q)</td>
<td></td>
</tr>
<tr>
<td>( \neg q )</td>
<td>((\neg q \land (p \rightarrow q)) \rightarrow \neg p)</td>
<td>Modus tollens</td>
</tr>
<tr>
<td>( p \rightarrow q )</td>
<td>(\therefore \neg p)</td>
<td></td>
</tr>
<tr>
<td>( q \rightarrow r )</td>
<td>((p \rightarrow q) \land (q \rightarrow r) \rightarrow (p \rightarrow r))</td>
<td>Hypothetical syllogism</td>
</tr>
<tr>
<td>( p \lor q )</td>
<td>((p \lor q) \land \neg p \rightarrow q)</td>
<td>Disjunctive syllogism</td>
</tr>
<tr>
<td>( \neg p )</td>
<td>(\therefore q)</td>
<td></td>
</tr>
<tr>
<td>( p )</td>
<td>(p \rightarrow (p \lor q))</td>
<td>Addition</td>
</tr>
<tr>
<td>( \therefore p \lor q)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( p \land q )</td>
<td>((p \land q) \rightarrow p)</td>
<td>Simplification</td>
</tr>
<tr>
<td>( \therefore p)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( q )</td>
<td>((p \land (q)) \rightarrow (p \land q))</td>
<td>Conjunction</td>
</tr>
<tr>
<td>( \therefore p \land q)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( p \lor q )</td>
<td>((p \lor q) \land (\neg p \lor r) \rightarrow (q \lor r))</td>
<td>Resolution</td>
</tr>
<tr>
<td>( \neg p \lor r)</td>
<td>(\therefore q \lor r)</td>
<td></td>
</tr>
</tbody>
</table>

**EXAMPLE 3** State which rule of inference is the basis of the following argument: “It is below freezing. Therefore, it is either below freezing or raining now.”

**Solution:** Let \( p \) be the proposition “It is below freezing now” and \( q \) the proposition “It is raining now.” Then this argument is of the form

\[
\begin{align*}
\quad & \quad p \\
\therefore & \quad p \lor q \\
\end{align*}
\]

This is an argument that uses the addition rule.

**EXAMPLE 4** State which rule of inference is the basis of the following argument: “It is below freezing now. Therefore, it is below freezing now.”

**Solution:** Let \( p \) be the proposition “It is below freezing now,” and let \( q \) be the proposition “It is raining now.” This argument is of the form

\[
\begin{align*}
\quad & \quad p \land q \\
\therefore & \quad p \\
\end{align*}
\]

This argument uses the simplification rule.
<table>
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</tr>
</thead>
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</tr>
<tr>
<td>( p \land q )</td>
<td>(p \lor q)</td>
<td></td>
</tr>
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<td>((p \land q) \rightarrow p)</td>
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<td>Conjunction</td>
</tr>
<tr>
<td>( \neg p \lor q )</td>
<td>(\therefore \neg p \lor (p \land q))</td>
<td>Resolution</td>
</tr>
<tr>
<td>( p \lor q )</td>
<td>((\neg p \lor (p \lor q)) \land (q \lor r))</td>
<td></td>
</tr>
<tr>
<td>( \neg q \lor r )</td>
<td>(\therefore q \lor r)</td>
<td></td>
</tr>
</tbody>
</table>

**EXAMPLE 3** State which rule of inference is the basis of the following argument: “It is below freezing now. Therefore, it is either below freezing or raining now.”

**Solution:** Let \( p \) be the proposition “It is below freezing now” and \( q \) the proposition “It is raining now.” The rule of inference used is the Law of Addition.
Some

### TABLE 1  Rules of Inference.

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<td>( \neg q )</td>
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<tr>
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<td></td>
</tr>
<tr>
<td>\therefore p \rightarrow r</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( p \lor q )</td>
<td>((p \lor q) \land \neg p \rightarrow q)</td>
<td>Disjunctive syllogism</td>
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<td>( \neg p )</td>
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<td></td>
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<td>\therefore q</td>
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<td></td>
</tr>
<tr>
<td>( p )</td>
<td>(p \rightarrow (p \lor q))</td>
<td>Addition</td>
</tr>
<tr>
<td>\therefore p \lor q</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( p \land q )</td>
<td>((p \land q) \rightarrow p)</td>
<td>Simplification</td>
</tr>
<tr>
<td>\therefore p</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( p \lor q )</td>
<td>((p \lor (q \land q)) \rightarrow (p \land q))</td>
<td>Conjunction</td>
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<td></td>
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<td></td>
<td></td>
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<tr>
<td>( p \lor q )</td>
<td>((p \lor q) \land (\neg p \lor r) \rightarrow (q \lor r))</td>
<td>Resolution</td>
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<tr>
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<td></td>
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</tr>
</tbody>
</table>

**EXAMPLE 3** State which rule of inference is the basis of the following argument: “It is below freezing. Therefore, it is either below freezing or raining now.”

**Solution:** Let \( p \) be the proposition “It is below freezing now” and \( q \) the proposition “It is raining.” The conclusions are both of the form \( p \lor q \).
Explosion Rule!
Explosion Rule!

\[ p \land \neg p \]

\[ \frac{q}{q} \]
Explosion Rule!

\[ p \land \neg p \]

\[ q \]

Easy peasy to prove in Rosen:
Explosion Rule!

\[ p \land \neg p \]

\[ \frac{}{q} \]

Easy peasy to prove in Rosen:

(1) \( p \land \neg p \)  Premise
(2) \( p \)  Simplification using (1)
(3) \( p \lor q \)  Addition using (2)
(4) \( \neg p \)  Simplification using (1)
(5) \( q \)  Disjunctive Syllogism using (3) and (4)
EXAMPLE 6

Show that the premises “It is not sunny this afternoon and it is colder than yesterday,” “We will go swimming only if it is sunny,” “If we do not go swimming, then we will take a canoe trip,” and “If we take a canoe trip, then we will be home by sunset” lead to the conclusion “We will be home by sunset.”

Solution: Let $p$ be the proposition “It is sunny this afternoon,” $q$ the proposition “It is colder than yesterday,” $r$ the proposition “We will go swimming,” $s$ the proposition “We will take a canoe trip,” and $t$ the proposition “We will be home by sunset.” Then the premises become $\neg p \land q$, $r \rightarrow p$, $\neg r \rightarrow s$, and $s \rightarrow t$. The conclusion is simply $t$. We need to give a valid argument with premises $\neg p \land q$, $r \rightarrow p$, $\neg r \rightarrow s$, and $s \rightarrow t$ and conclusion $t$.

We construct an argument to show that our premises lead to the desired conclusion as follows.

<table>
<thead>
<tr>
<th>Step</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\neg p \land q$</td>
<td>Premise</td>
</tr>
<tr>
<td>2. $\neg p$</td>
<td>Simplification using (1)</td>
</tr>
<tr>
<td>3. $r \rightarrow p$</td>
<td>Premise</td>
</tr>
<tr>
<td>4. $\neg r$</td>
<td>Modus tollens using (2) and (3)</td>
</tr>
<tr>
<td>5. $\neg r \rightarrow s$</td>
<td>Premise</td>
</tr>
<tr>
<td>6. $s$</td>
<td>Modus ponens using (4) and (5)</td>
</tr>
<tr>
<td>7. $s \rightarrow t$</td>
<td>Premise</td>
</tr>
<tr>
<td>8. $t$</td>
<td>Modus ponens using (6) and (7)</td>
</tr>
</tbody>
</table>

Note that we could have used a truth table to show that whenever each of the four hypotheses is true, the conclusion is also true. However, because we are working with five propositional variables, $p$, $q$, $r$, $s$, and $t$, such a truth table would have 32 rows.
Given the statements
\( \neg \neg c \)
\( c \rightarrow a \)
\( \neg a \lor b \)
\( b \rightarrow d \)
\( \neg (d \lor e) \)

which one of the following statements must also be true?

\( \neg c \)
\( e \)
\( h \)
\( \neg a \)
all of the above
Given the statements:
\[ \neg \neg c \]
\[ c \rightarrow a \]
\[ \neg a \lor b \]
\[ b \rightarrow d \]
\[ \neg (d \lor e) \]

which one of the following statements must also be true?

\[ \neg c \]
\[ e \]
\[ h \]
\[ \neg a \]

all of the above
Given the statements
¬¬c
¬¬c → a
¬a ∨ b
b → d
¬(d ∨ e)

which one of the following statements must also be true?

¬c
e
h
¬a
all of the above
Given the statements
\( \neg \neg c \rightarrow c \)
\( c \rightarrow a \)
\( \neg a \lor b \)
\( b \rightarrow d \)
\( \neg (d \lor e) \)

which one of the following statements must also be true?

\( \neg c \)
\( e \)
\( h \)
\( \neg a \)
all of the above
Given the statements

\(-\neg c \rightarrow c\)
\(c \rightarrow a\)
\(\neg a \lor b\)
\(b \rightarrow d\)
\(\neg (d \lor e)\)

which one of the following statements must also be true?

\(-c\)
\(e\)
\(h\)
\(\neg a\)
all of the above
Given the statements
¬c → c

a → b
¬b ∨ d
b → d
¬(d ∨ e) → ¬d ∧ ¬e

which one of the following statements must also be true?
¬c
e
h
¬a
all of the above
Given the statements

\[ \neg \neg c \rightarrow c \]
\[ c \rightarrow a \]
\[ \neg a \lor b \]
\[ b \rightarrow d \]
\[ \neg (d \lor e) \rightarrow \neg d \land \neg e \]

which one of the following statements must also be true?

\[ \neg c \]
\[ e \]
\[ h \]
\[ \neg a \]
\[ all \ of \ the \ above \]

Homework 1: Prove that the answer to this problem is indeed “all of the above,” using tools provided to you in the present slide deck.
Homework 1 Solution

**Proposition:** The answer is “all of the above.”

**Proof:** We know from the rule of inference explosion that everything follows from a contradiction, so we simply need to find a contradiction in the given statements. We do so as follows. We already have \( \sim d \) by DeMorgan’s Law, as indicated on the previous slide. On that slide, we also have c from the first statement. This, combined with the second given, yields by modus ponens a in one step. Next, by disjunctive syllogism we have b from a and \( \sim a \lor b \). Another use of modus ponens with b and \( b \Rightarrow d \) gives d, and we have our contradiction. \( \text{QED} \)
Which one of the following statements is logically equivalent to the following statement: “If you are not part of the solution, then you are part of the problem.”

If you are part of the solution, then you are not part of the problem.

If you are not part of the problem, then you are part of the solution.

If you are part of the problem, then you are not part of the solution.

If you are not part of the problem, then you are not part of the solution.
Which one of the following statements is logically equivalent to the following statement: “If you are not part of the solution, then you are part of the problem.”

If you are part of the solution, then you are not part of the problem.

If you are not part of the problem, then you are part of the solution.

If you are part of the problem, then you are not part of the solution.

If you are not part of the problem, then you are not part of the solution.
Which one of the following statements is logically equivalent to the following statement: “If you are not part of the solution, then you are part of the problem.”

If you are part of the solution, then you are not part of the problem.

If you are not part of the problem, then you are part of the solution.

If you are part of the problem, then you are not part of the solution.

If you are not part of the problem, then you are not part of the solution.

Homework 2: Prove that the answer to this problem is indeed the second option, using tools provided to you in the present slide deck.
Homework 2 Solution

**Proposition**: The answer is the second option.

**Proof**: From a conditional $P \Rightarrow Q$ it can be immediately deduced that $\sim Q \Rightarrow \sim P$ (and *vice versa*) by the rule of inference *contrapositive*, and contrapositive applied to the given statement yield the second option in one step. Now we obtain contrapositive itself. Suppose that a given conditional $P \Rightarrow Q$ holds, and suppose as well that $\sim Q$ holds. We are done when we can deduce $\sim P$ from what we now have to work with, and what’s available to us in the present slide deck. The rule of inference *modus tollens* allows us to infer $\sim P$ in one step from $P \Rightarrow Q$ and $\sim Q$. **QED**
More-Recent Shots …
The Original King-Ace

Suppose that the following premise is true:

If there is a king in the hand, then there is an ace in the hand, or else if there isn’t a king in the hand, then there is an ace.

What can you infer from this premise?
The Original King-Ace

Suppose that the following premise is true:

\[
\text{If there is a king in the hand, then there is an ace in the hand, or else if there isn’t a king in the hand, then there is an ace.}
\]

What can you infer from this premise?

There is an ace in the hand.
The Original King-Ace

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The Original King-Ace

Suppose that the following premise is true:

If there is a king in the hand, then there is an ace in the hand, or else if there isn’t a king in the hand, then there is an ace.

What can you infer from this premise?

NO! There is an ace in the hand.
The Original King-Ace

Suppose that the following premise is true:

If there is a king in the hand, then there is an ace in the hand, or else if there isn’t a king in the hand, then there is an ace.

What can you infer from this premise?

NO! There is an ace in the hand. NO!
The Original King-Ace

Suppose that the following premise is true:

If there is a king in the hand, then there is an ace in the hand, or else if there isn’t a king in the hand, then there is an ace.

What can you infer from this premise?

NO! There is an ace in the hand. NO!

In fact, what you *can* infer is that there *isn’t* an ace in the hand!
Suppose that the following premise is true:

If there is a king in the hand, then there is an ace in the hand; or if there isn’t a king in the hand, then there is an ace; but not both of these if-then statements are true.

What can you infer from this premise?
Suppose that the following premise is true:

*If there is a king in the hand, then there is an ace in the hand; or if there isn’t a king in the hand, then there is an ace; but not both of these if-then statements are true.*

What can you infer from this premise?

There is an ace in the hand.
Suppose that the following premise is true:

*If there is a king in the hand, then there is an ace in the hand; or if there isn’t a king in the hand, then there is an ace; but not both of these if-then statements are true.*

What can you infer from this premise?

*There is an ace in the hand.*
King-Ace 2

Suppose that the following premise is true:

If there is a king in the hand, then there is an ace in the hand; or if there isn’t a king in the hand, then there is an ace; but not both of these if-then statements are true.

What can you infer from this premise?

NO! There is an ace in the hand.
Suppose that the following premise is true:

*If there is a king in the hand, then there is an ace in the hand; or if there isn’t a king in the hand, then there is an ace; but not both of these if-then statements are true.*

What can you infer from this premise?

**NO! There is an ace in the hand. NO!**
Suppose that the following premise is true:

If there is a king in the hand, then there is an ace in the hand; or if there isn’t a king in the hand, then there is an ace; but not both of these if-then statements are true.

What can you infer from this premise?

NO! There is an ace in the hand. NO!

In fact, what you can infer is that there isn’t an ace in the hand!
King-Ace Solved

**Proposition:** There is *not* an ace in the hand.

**Proof:** We know that at least one of the if-thens (i.e., at least one of the **conditionals**) is false. We know this because we are told that either the first if-then holds, or the second if-then holds, but not both.

So we have two cases to consider, viz., that $K \rightarrow A$ is false, and (the other case) that $\neg K \rightarrow A$ is false. ($\rightarrow$ is the same as the arrow we have used.)

Take first the first case; accordingly, suppose that $K \rightarrow A$ is false. Then it follows that $K$ is true (since, when a conditional is false, its antecedent holds but its consequent doesn’t), and $A$ is false; i.e., $\neg A$.

Now consider the second case, which consists in $\neg K \rightarrow A$ being false. Here, in a direct parallel, we know $\neg K$ and, once again, since the consequent of the conditional must be false, $\neg A$.

In both of our two cases, which are exhaustive, there is no ace in the hand. The proposition is established. **QED**
Proposition: There is not an ace in the hand.

Proof: We know that at least one of the if-thens (i.e., at least one of the conditionals) is false. We know this because we are told that either the first if-then holds, or the second if-then holds, but not both.

So we have two cases to consider, viz., that \( K \rightarrow A \) is false, and (the other case) that \( \neg K \rightarrow A \) is false. (\( \rightarrow \) is the same as the arrow we have used.)

Take first the first case; accordingly, suppose that \( K \rightarrow A \) is false. Then it follows that \( K \) is true (since, when a conditional is false, its antecedent holds but its consequent doesn’t), and \( A \) is false; i.e., \( \neg A \).

Now consider the second case, which consists in \( \neg K \rightarrow A \) being false. Here, in a direct parallel, we know \( \neg K \) and, once again, since the consequent of the conditional must be false, \( \neg A \).

In both of our two cases, which are exhaustive, there is no ace in the hand. The proposition is established. QED
Yours soon?
“Show-me-the-$” Problem (AI Version)

If one of the following assertions is true then so is the other:

(1) If there is an apple in the cup then there is a battery in the cup; and, if there is a battery in the cup then there is an apple in the cup.

(2) There is an apple in the cup.

Which is more likely to be in the cup, if either: the apple or the battery?
“Show-me-the-$” Problem (AI Version)

If one of the following assertions is true then so is the other:

(1) If there is an apple in the cup then there is a battery in the cup; and, if there is a battery in the cup then there is an apple in the cup.

(2) There is an apple in the cup.

Which is more likely to be in the cup, if either: the apple or the battery?

Now class, here’s a robot. Notice the cup next to it. The robot has been programmed in a simple way: the code consists of three conditional statements: (1) If the answer to the problem above is “apple,” place only an apple in the empty cup. (2) If the answer to the above problem is “battery,” place only a battery in the empty cup. (3) If the answer is that neither is more likely to be in the cup, leave the cup empty. Earlier, this code was executed and the robot performed accordingly (having before this assimilated and solved the above problem). So: Tell me, assuming that the code all worked perfectly, what’s in the cup, if anything! If you’re right, and can prove that you are, here’s a $20 for you, on the spot.