Rational Analysis of Some Shots @ *R*

Selmer Bringsjord

Are Humans Rational?

9/9/19

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Bit of Historical Context …

2019

2019

Are Humans Rational?

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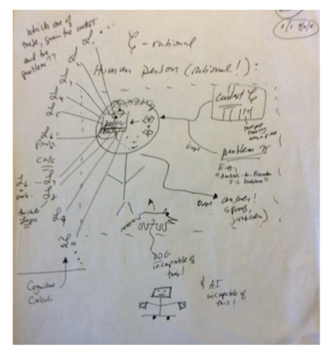

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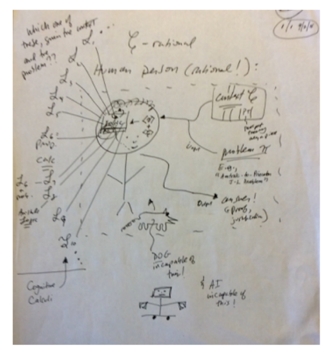

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$\mathcal{DCEC}^*$

Syntax

$$S ::= \text{Object} \mid \text{Agent} \mid \text{Self} \sqsubset \text{Agent} \mid \text{ActionType} \mid \text{Action} \sqsubseteq \text{Event} \mid \text{Moment} \mid \text{Boolean} \mid \text{Fluent} \mid \text{Numeric}$$

$$f ::= \begin{array}{l} \mathit{action} : \text{Agent} \times \text{ActionType} \rightarrow \text{Action} \\ \mathit{initially} : \text{Fluent} \rightarrow \text{Boolean} \\ \mathit{holds} : \text{Fluent} \times \text{Moment} \rightarrow \text{Boolean} \\ \mathit{happens} : \text{Event} \times \text{Moment} \rightarrow \text{Boolean} \\ \mathit{clipped} : \text{Moment} \times \text{Fluent} \times \text{Moment} \rightarrow \mathit{Boolean} \\ \mathit{initiates} : \text{Event} \times \text{Fluent} \times \text{Moment} \rightarrow \text{Boolean} \\ \mathit{terminates} : \text{Event} \times \text{Fluent} \times \text{Moment} \rightarrow \text{Boolean} \\ \mathit{prior} : \text{Moment} \times \text{Moment} \rightarrow \text{Boolean} \\ \mathit{interval} : \text{Moment} \times \text{Boolean} \\ * : \text{Agent} \rightarrow \text{Self} \\ \mathit{payoff} : \text{Agent} \times \text{ActionType} \times \text{Moment} \rightarrow \text{Numeric} \end{array}$$

$$t ::= x : S \mid c : S \mid f(t_1, \ldots, t_n)$$

$$\phi ::= \begin{array}{l} t : \text{Boolean} \mid \neg\phi \mid \phi \wedge \psi \mid \phi \vee \psi \mid \\ \mathbf{P}(a,t,\phi) \mid \mathbf{K}(a,t,\phi) \mid \mathbf{C}(t,\phi) \mid \mathbf{S}(a,b,t,\phi) \mid \mathbf{S}(a,t,\phi) \\ \mathbf{B}(a,t,\phi) \mid \mathbf{D}(a,t,\mathit{holds}(f,t')) \mid \mathbf{I}(a,t,\mathit{happens}(\mathit{action}(a^*,\alpha),t')) \\ \mathbf{O}(a,t,\phi,\mathit{happens}(\mathit{action}(a^*,\alpha),t')) \end{array}$$

Rules of Inference

$$\frac{}{\mathbf{C}(t,\mathbf{P}(a,t,\phi) \rightarrow \mathbf{K}(a,t,\phi))}\ [R_1] \qquad \frac{}{\mathbf{C}(t,\mathbf{K}(a,t,\phi) \rightarrow \mathbf{B}(a,t,\phi))}\ [R_2]$$

$$\frac{\mathbf{C}(t,\phi)\ t \leq t_1 \ldots t \leq t_n}{\mathbf{K}(a_1,t_1,\ldots \mathbf{K}(a_n,t_n,\phi)\ldots)}\ [R_3] \qquad \frac{\mathbf{K}(a,t,\phi)}{\phi}\ [R_4]$$

$$\frac{}{\mathbf{C}(t,\mathbf{K}(a,t_1,\phi_1 \rightarrow \phi_2)) \rightarrow \mathbf{K}(a,t_2,\phi_1) \rightarrow \mathbf{K}(a,t_3,\phi_2)}\ [R_5]$$

$$\frac{}{\mathbf{C}(t,\mathbf{B}(a,t_1,\phi_1 \rightarrow \phi_2)) \rightarrow \mathbf{B}(a,t_2,\phi_1) \rightarrow \mathbf{B}(a,t_3,\phi_2)}\ [R_6]$$

$$\frac{}{\mathbf{C}(t,\mathbf{C}(t_1,\phi_1 \rightarrow \phi_2)) \rightarrow \mathbf{C}(t_2,\phi_1) \rightarrow \mathbf{C}(t_3,\phi_2)}\ [R_7]$$

$$\frac{}{\mathbf{C}(t,\forall x.\ \phi \rightarrow \phi[x \mapsto t])}\ [R_8] \qquad \frac{}{\mathbf{C}(t,\phi_1 \leftrightarrow \phi_2 \rightarrow \neg\phi_2 \rightarrow \neg\phi_1)}\ [R_9]$$

$$\frac{}{\mathbf{C}(t,[\phi_1 \wedge \ldots \wedge \phi_n \rightarrow \phi] \rightarrow [\phi_1 \rightarrow \ldots \rightarrow \phi_n \rightarrow \psi])}\ [R_{10}]$$

$$\frac{\mathbf{B}(a,t,\phi)\quad \phi \rightarrow \psi}{\mathbf{B}(a,t,\psi)}\ [R_{11a}] \qquad \frac{\mathbf{B}(a,t,\phi)\quad \mathbf{B}(a,t,\psi)}{\mathbf{B}(a,t,\psi \wedge \phi)}\ [R_{11b}]$$

$$\frac{\mathbf{S}(s,h,t,\phi)}{\mathbf{B}(h,t,\mathbf{B}(s,t,\phi))}\ [R_{12}]$$

$$\frac{\mathbf{I}(a,t,\mathit{happens}(\mathit{action}(a^*,\alpha),t'))}{\mathbf{P}(a,t,\mathit{happens}(\mathit{action}(a^*,\alpha),t))}\ [R_{13}]$$

$$\frac{\begin{array}{c}\mathbf{B}(a,t,\phi)\quad \mathbf{B}(a,t,\mathbf{O}(a^*,t,\phi,\mathit{happens}(\mathit{action}(a^*,\alpha),t'))) \\ \mathbf{O}(a,t,\phi,\mathit{happens}(\mathit{action}(a^*,\alpha),t'))\end{array}}{\mathbf{K}(a,t,\mathbf{I}(a^*,t,\mathit{happens}(\mathit{action}(a^*,\alpha),t')))}\ [R_{14}]$$

$$\frac{\phi \leftrightarrow \psi}{\mathbf{O}(a,t,\phi,\gamma) \leftrightarrow \mathbf{O}(a,t,\psi,\gamma)}\ [R_{15}]$$

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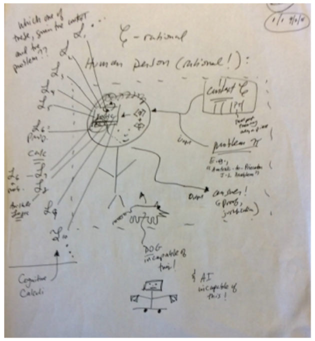

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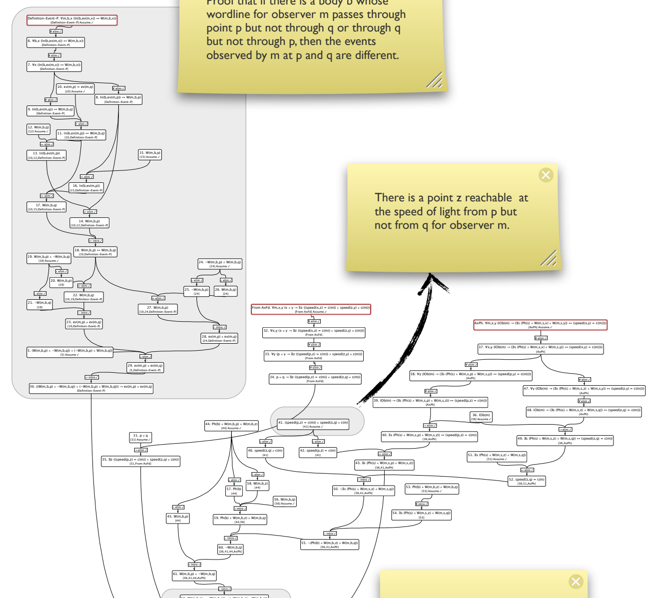
Proof that if there is a body b whose wordline for observer m passes through point p but not through q or through q but not through p, then the events observed by m at p and q are different.



There is a point z reachable at the speed of light from p but not from q for observer m.

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through

The theorem

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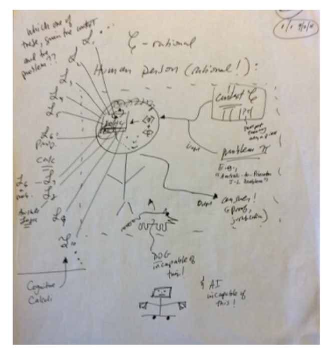

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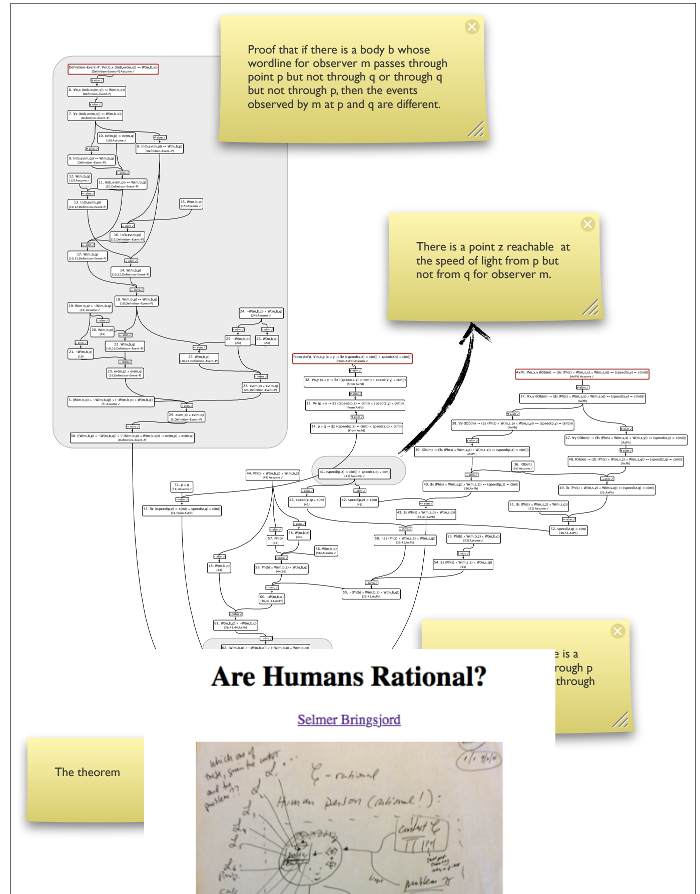
Theorem: NTFLIO

(deduced from **SpecRel**)



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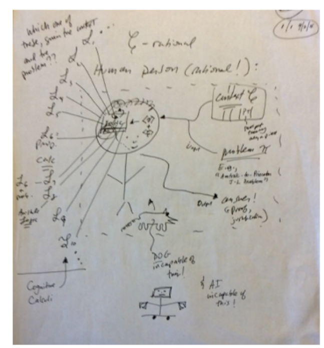

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Theorem: NTFLIO

(deduced from **SpecRel**)

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Syntax

$$S ::= \text{Object} \mid \text{Agent} \mid \text{Self} \sqsubset \text{Agent} \mid \text{ActionType} \mid \text{Action} \sqsubseteq \text{Event} \mid \text{Moment} \mid \text{Boolean} \mid \text{Fluent} \mid \text{Numeric}$$

$$f ::= \begin{cases} \mathit{action} : \text{Agent} \times \text{ActionType} \rightarrow \text{Action} \\ \mathit{initially} : \text{Fluent} \rightarrow \text{Boolean} \\ \mathit{holds} : \text{Fluent} \times \text{Moment} \rightarrow \text{Boolean} \\ \mathit{happens} : \text{Event} \times \text{Moment} \rightarrow \text{Boolean} \\ \mathit{clipped} : \text{Moment} \times \text{Fluent} \times \text{Moment} \rightarrow \mathit{Boolean} \\ \mathit{initiates} : \text{Event} \times \text{Fluent} \times \text{Moment} \rightarrow \text{Boolean} \\ \mathit{terminates} : \text{Event} \times \text{Fluent} \times \text{Moment} \rightarrow \text{Boolean} \\ \mathit{prior} : \text{Moment} \times \text{Moment} \rightarrow \text{Boolean} \\ \mathit{interval} : \text{Moment} \times \text{Boolean} \\ * : \text{Agent} \rightarrow \text{Self} \\ \mathit{payoff} : \text{Agent} \times \text{ActionType} \times \text{Moment} \rightarrow \text{Numeric} \end{cases}$$

$$t ::= x : S \mid c : S \mid f(t_1, \ldots, t_n)$$

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Rules of Inference

$$\frac{}{\mathbf{C}(t,\mathbf{P}(a,t,\phi) \rightarrow \mathbf{K}(a,t,\phi))}\ [R_1] \qquad \frac{}{\mathbf{C}(t,\mathbf{K}(a,t,\phi) \rightarrow \mathbf{B}(a,t,\phi))}\ [R_2]$$

$$\frac{\mathbf{C}(t,\phi)\ t \leq t_1 \ldots t \leq t_n}{\mathbf{K}(a_1,t_1,\ldots \mathbf{K}(a_n,t_n,\phi)\ldots)}\ [R_3] \qquad \frac{\mathbf{K}(a,t,\phi)}{\phi}\ [R_4]$$

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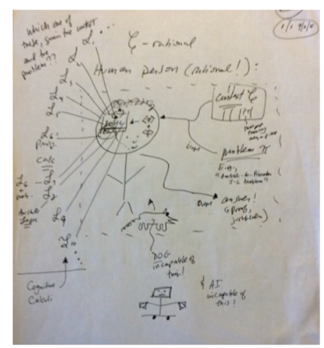

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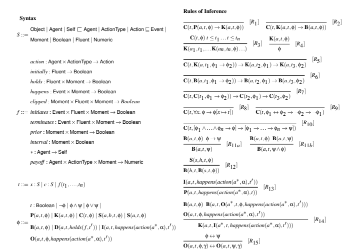

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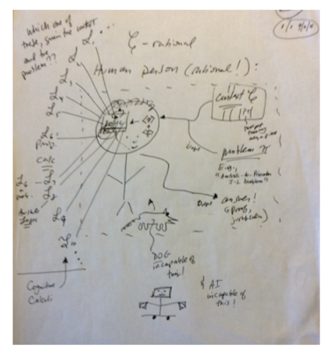

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Rules of Inference

$$\frac{}{\mathbf{C}(t,\mathbf{P}(a,t,\phi) \rightarrow \mathbf{K}(a,t,\phi))} [R_1] \qquad \frac{}{\mathbf{C}(t,\mathbf{K}(a,t,\phi) \rightarrow \mathbf{B}(a,t,\phi))} [R_2]$$

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$$\frac{\mathbf{B}(a,t,\phi)\quad \phi \rightarrow \psi}{\mathbf{B}(a,t,\psi)} [R_{11a}] \qquad \frac{\mathbf{B}(a,t,\phi)\quad \mathbf{B}(a,t,\psi)}{\mathbf{B}(a,t,\psi \wedge \phi)} [R_{11b}]$$

$$\frac{\mathbf{S}(s,h,t,\phi)}{\mathbf{B}(h,t,\mathbf{B}(s,t,\phi))} [R_{12}]$$

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$$\frac{\phi \leftrightarrow \psi}{\mathbf{O}(a,t,\phi,\gamma) \leftrightarrow \mathbf{O}(a,t,\psi,\gamma)} [R_{15}]$$

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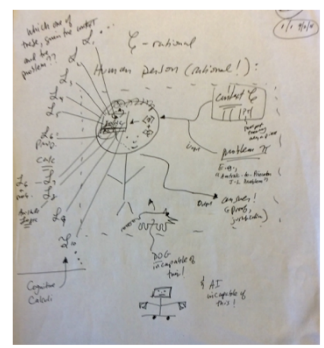

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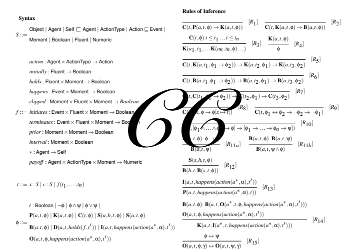

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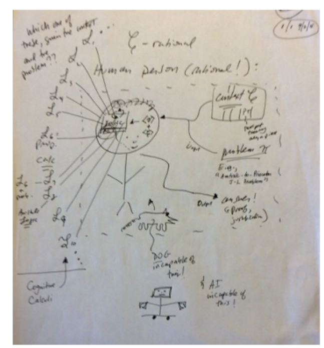

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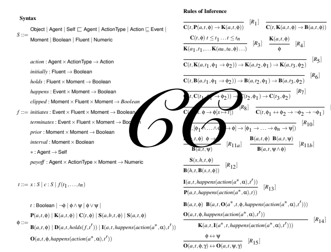

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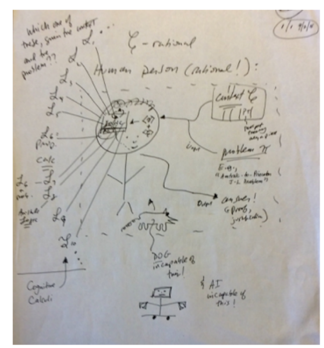

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Syntax

$S ::=$ Object | Agent | Self $\sqsubset$ Agent | ActionType | Action $\sqsubseteq$ Event | Moment | Boolean | Fluent | Numeric

$f ::=$
action : Agent × ActionType → Action
initially : Fluent → Boolean
holds : Fluent × Moment → Boolean
happens : Event × Moment → Boolean
clipped : Moment × Fluent × Moment → *Boolean*
initiates : Event × Fluent × Moment → Boolean
terminates : Event × Fluent × Moment → Boolean
prior : Moment × Moment → Boolean
interval : Moment × Boolean
$*$: Agent → Self
payoff : Agent × ActionType × Moment → Numeric

$t ::= x : S \mid c : S \mid f(t_1, \ldots, t_n)$

$\phi ::=$
t : Boolean $\mid \neg\phi \mid \phi \land \psi \mid \phi \lor \psi \mid$
$\mathbf{P}(a,t,\phi) \mid \mathbf{K}(a,t,\phi) \mid \mathbf{C}(t,\phi) \mid \mathbf{S}(a,b,t,\phi) \mid \mathbf{S}(a,t,\phi)$
$\mathbf{B}(a,t,\phi) \mid \mathbf{D}(a,t,holds(f,t')) \mid \mathbf{I}(a,t,happens(action(a^*,\alpha),t'))$
$\mathbf{O}(a,t,\phi,happens(action(a^*,\alpha),t'))$

Rules of Inference

$$\frac{}{\mathbf{C}(t,\mathbf{P}(a,t,\phi) \to \mathbf{K}(a,t,\phi))}\ [R_1] \qquad \frac{}{\mathbf{C}(t,\mathbf{K}(a,t,\phi) \to \mathbf{B}(a,t,\phi))}\ [R_2]$$

$$\frac{\mathbf{C}(t,\phi)\ t \le t_1 \ldots t \le t_n}{\mathbf{K}(a_1,t_1,\ldots \mathbf{K}(a_n,t_n,\phi)\ldots)}\ [R_3] \qquad \frac{\mathbf{K}(a,t,\phi)}{\phi}\ [R_4]$$

$$\frac{}{\mathbf{C}(t,\mathbf{K}(a,t_1,\phi_1 \to \phi_2)) \to \mathbf{K}(a,t_2,\phi_1) \to \mathbf{K}(a,t_3,\phi_2)}\ [R_5]$$

$$\frac{}{\mathbf{C}(t,\mathbf{B}(a,t_1,\phi_1 \to \phi_2)) \to \mathbf{B}(a,t_2,\phi_1) \to \mathbf{B}(a,t_3,\phi_2)}\ [R_6]$$

$$[R_7] \qquad [R_8] \qquad \frac{}{\mathbf{C}(t,\phi_1 \leftrightarrow \phi_2 \to \neg\phi_2 \to \neg\phi_1)}\ [R_9]$$

$$[R_{10}]$$

$$[R_{11a}] \qquad \frac{\mathbf{B}(a,t,\phi)\ \ \mathbf{B}(a,t,\psi)}{\mathbf{B}(a,t,\psi \land \phi)}\ [R_{11b}]$$

$$\frac{\mathbf{S}(s,h,t,\phi)}{\mathbf{B}(h,t,\mathbf{B}(s,t,\phi))}\ [R_{12}]$$

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$$\frac{\mathbf{B}(a,t,\phi)\ \ \mathbf{B}(a,t,\mathbf{O}(a^*,t,\phi,happens(action(a^*,\alpha),t')))\quad \mathbf{O}(a,t,\phi,happens(action(a^*,\alpha),t'))}{\mathbf{K}(a,t,\mathbf{I}(a^*,t,happens(action(a^*,\alpha),t')))}\ [R_{14}]$$

$$\frac{\phi \leftrightarrow \psi}{\mathbf{O}(a,t,\phi,\gamma) \leftrightarrow \mathbf{O}(a,t,\psi,\gamma)}\ [R_{15}]$$

$\mathcal{CC}$

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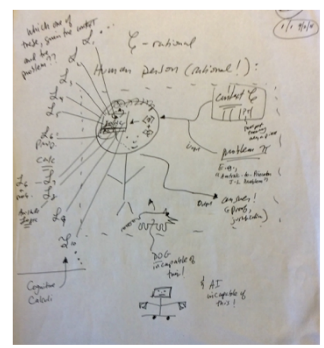

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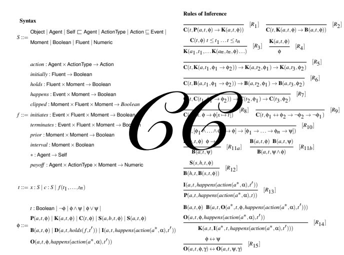

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Leibniz

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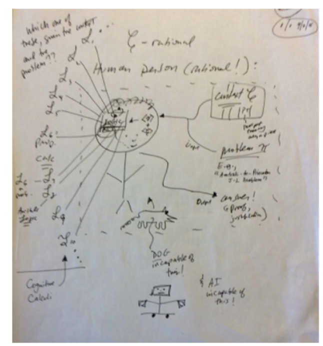

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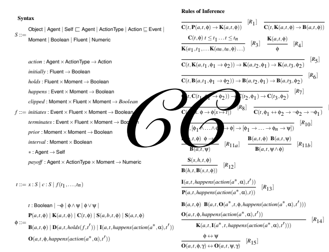
$\mathcal{DCEC}^*$



1666

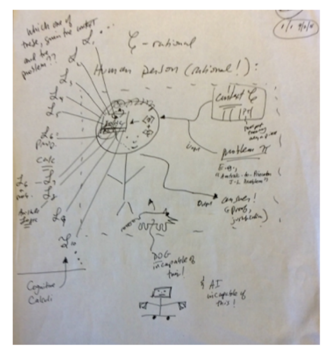
Leibniz

1.5 centuries < Boole

2019

Are Humans Rational?

Selmer Bringsjord



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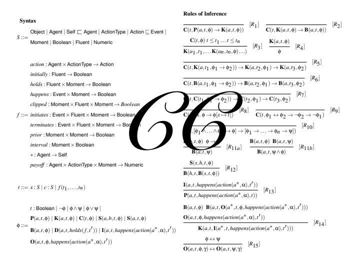

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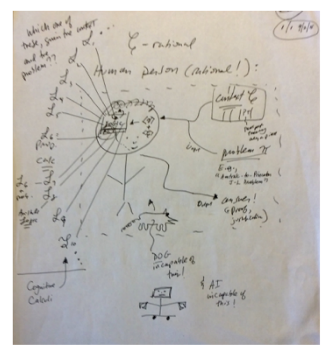

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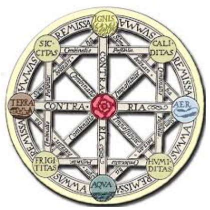
"Universal Computational Logic"



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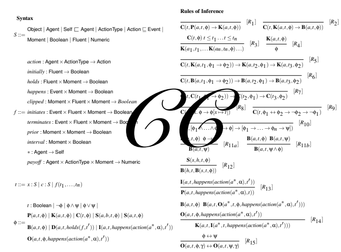

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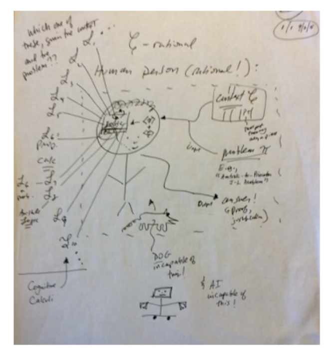

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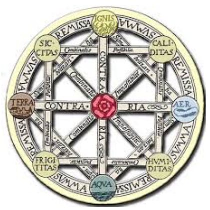

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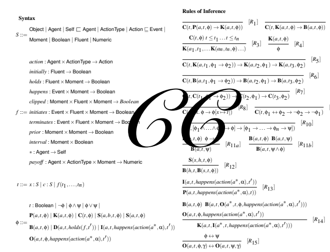

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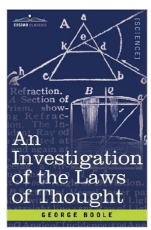

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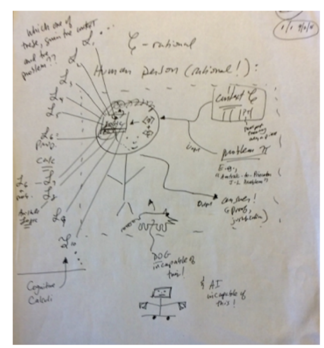

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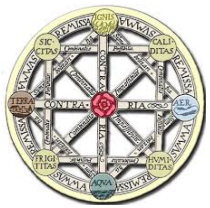
Quantification!

"Universal Computational Logic"



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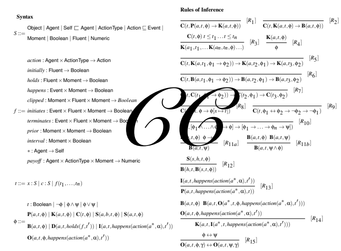

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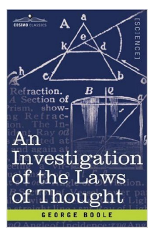

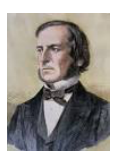

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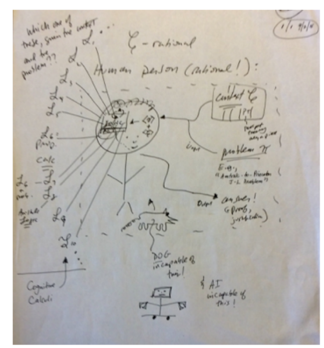

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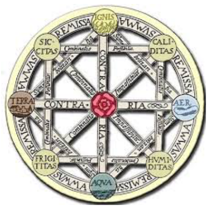

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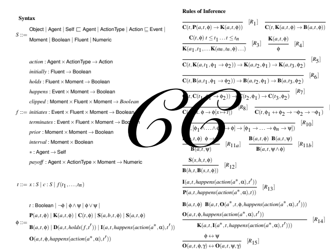

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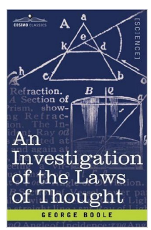

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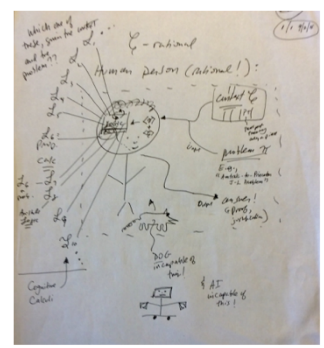

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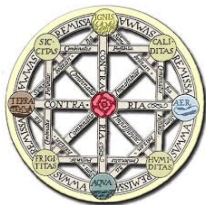

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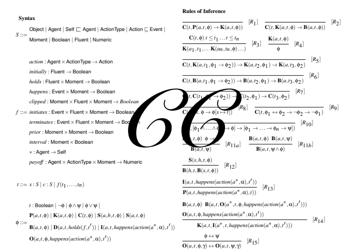

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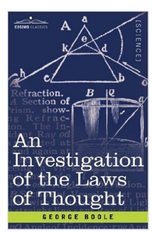

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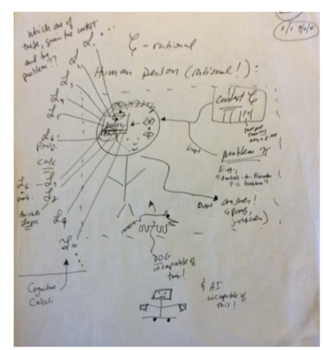

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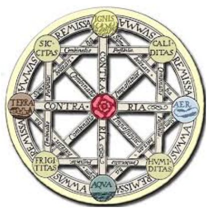

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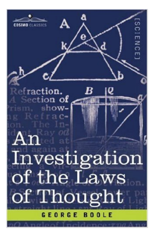

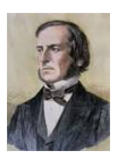

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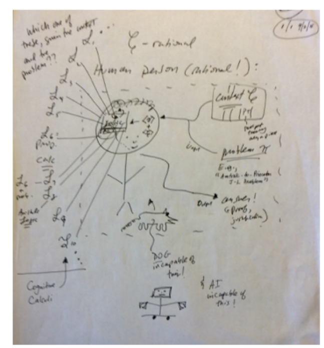

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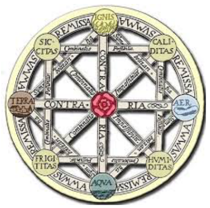

Logic Theorist

(birth of modern logicist AI)

(birth of agent-based/ behavioral econ)

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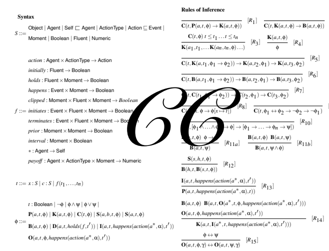

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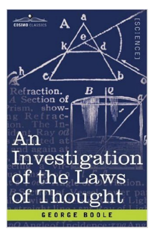

1956

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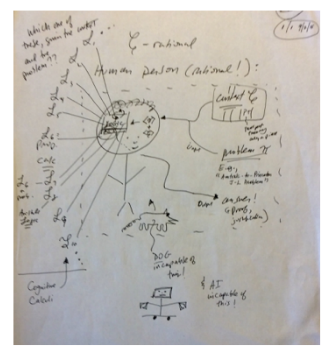

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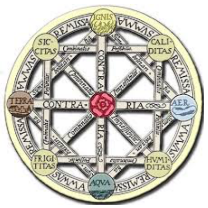

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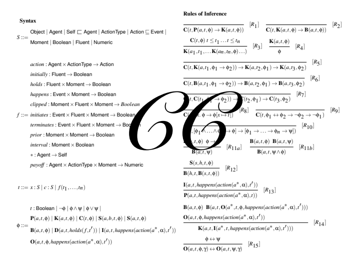

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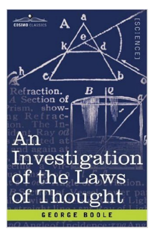

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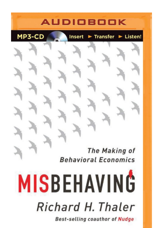

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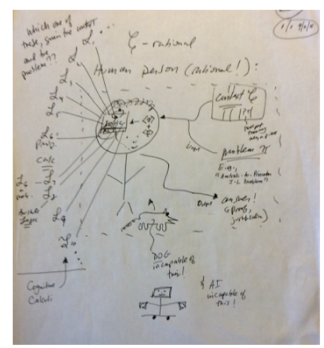

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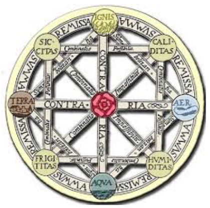

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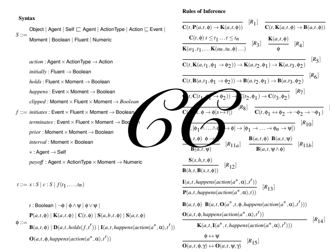

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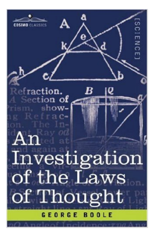

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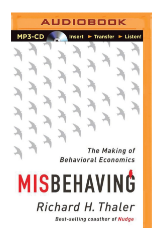

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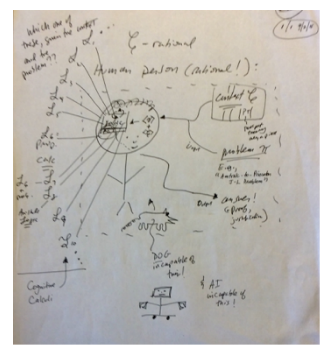

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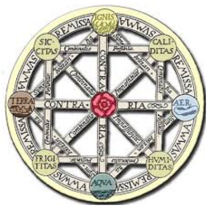

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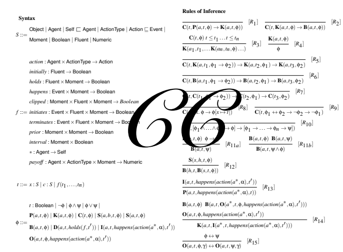

The Future?

(rationally considered)

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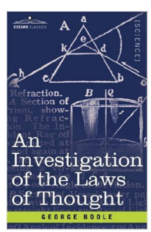

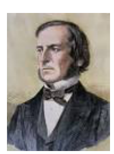

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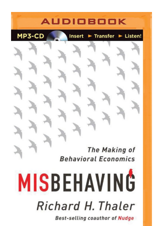

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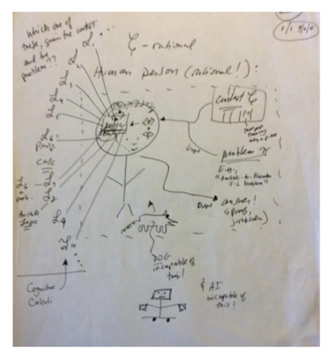

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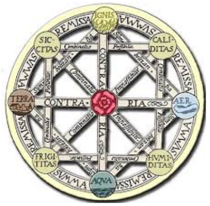

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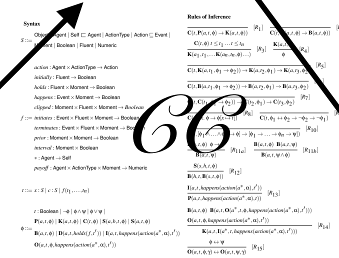

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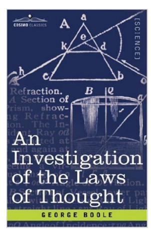

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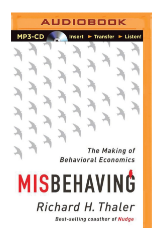

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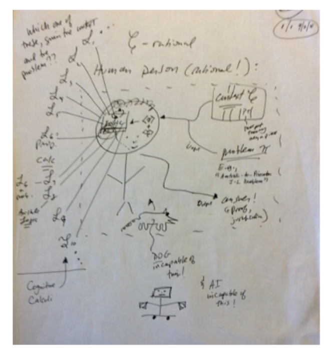

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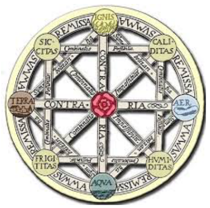

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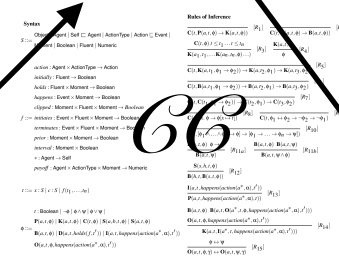

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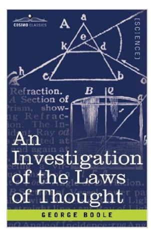

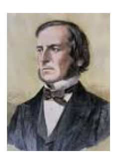

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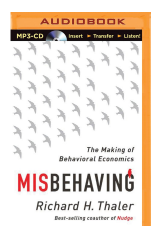

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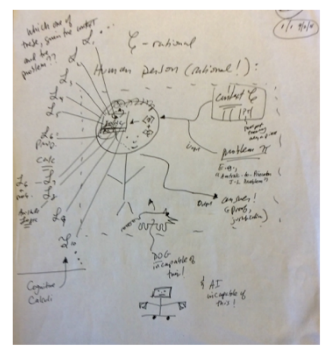

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2020

Propositional Calculus!

First Elements of the Propositional Calculus

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Variables to represent declarative statements.

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E.g., *k* to represent ‘There is a king in the hand’.

First Elements of the Propositional Calculus

Variables to represent declarative statements.

E.g., k to represent 'There is a king in the hand'.

And five simple Boolean connectives:

First Elements of the Propositional Calculus

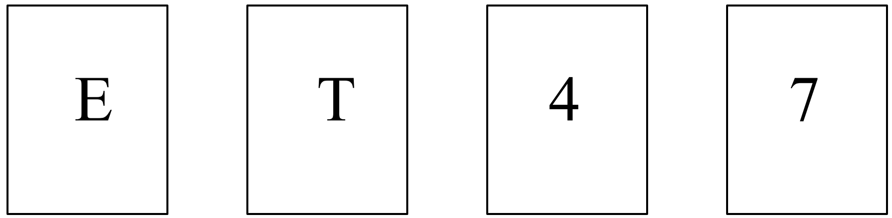
Variables to represent declarative statements.

E.g., *k* to represent 'There is a king in the hand'.

And five simple Boolean connectives:

not $\neg$ and $\wedge$ or (inclusive) $\vee$ if ... then ... $\rightarrow$ ... if and only if ... $\leftrightarrow$

Wason Selection Task

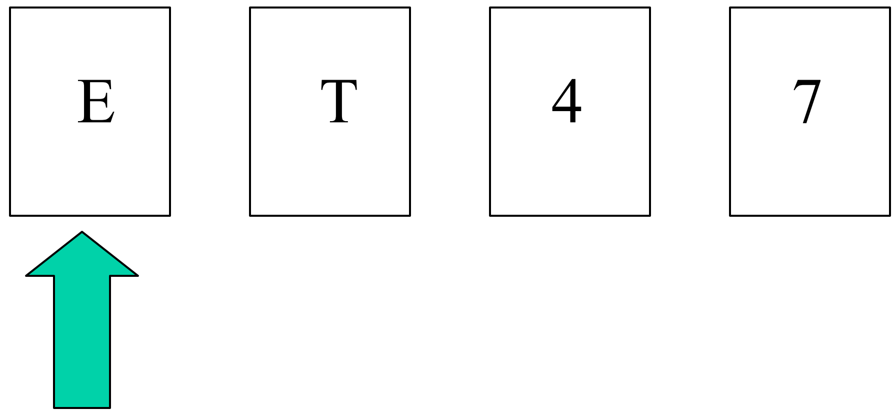


Suppose I claim that the following rule is true.

> If a card has a vowel on one side, it has an even number on the other side.

Which card or cards should you turn over in order to try to decide whether the rule is true or false?

Wason Selection Task



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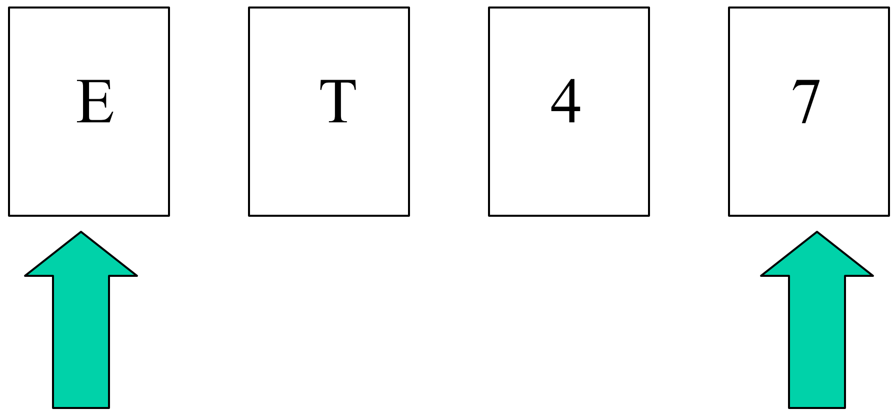
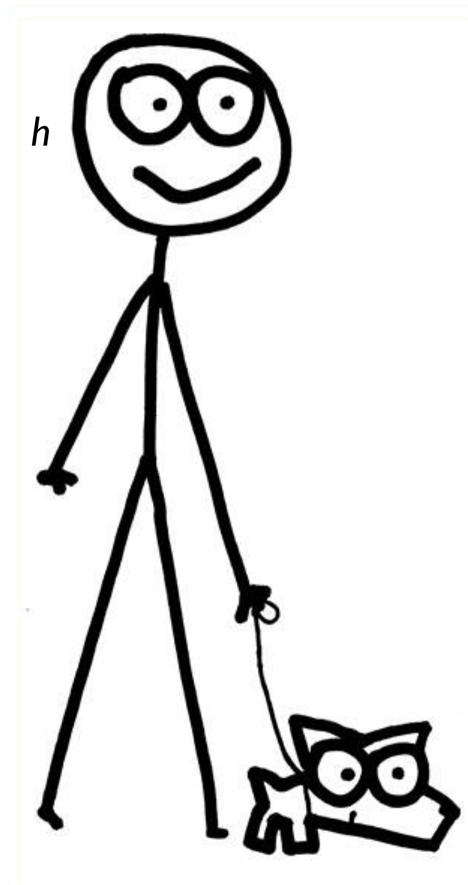
Wason Selection Task

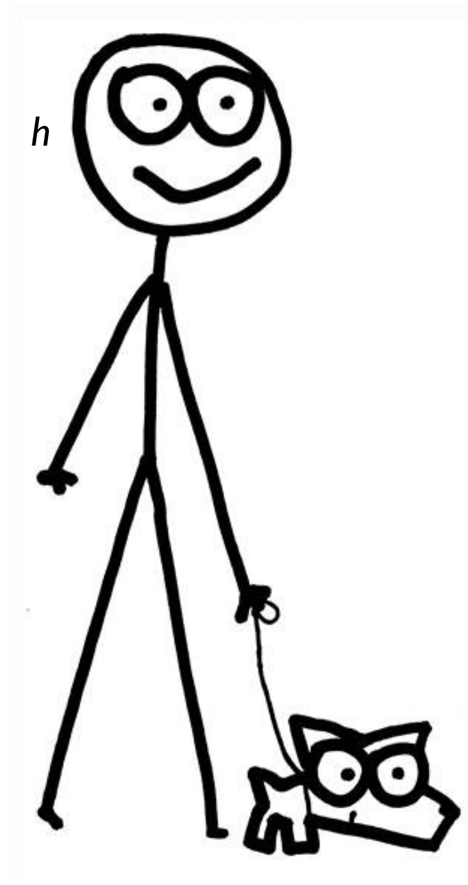


Suppose I claim that the following rule is true.

> If a card has a vowel on one side, it has an even number on the other side.

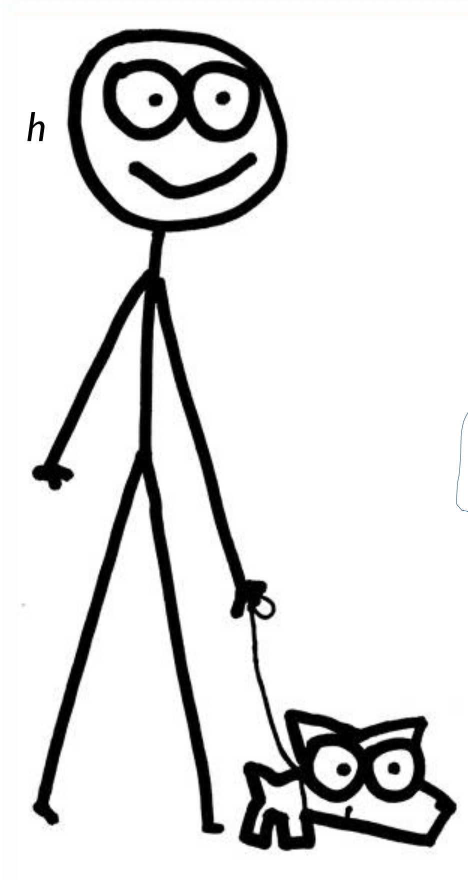
Which card or cards should you turn over in order to try to decide whether the rule is true or false?


h



Problem!

Or test. For an overview of Psychometric AI, see:
http://www.tandfonline.com/doi/pdf/10.1080/0952813X.2010.502314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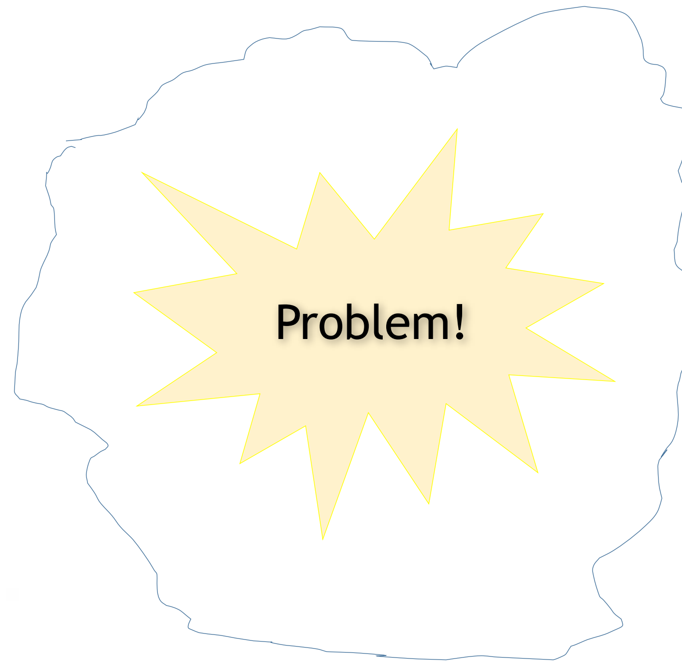
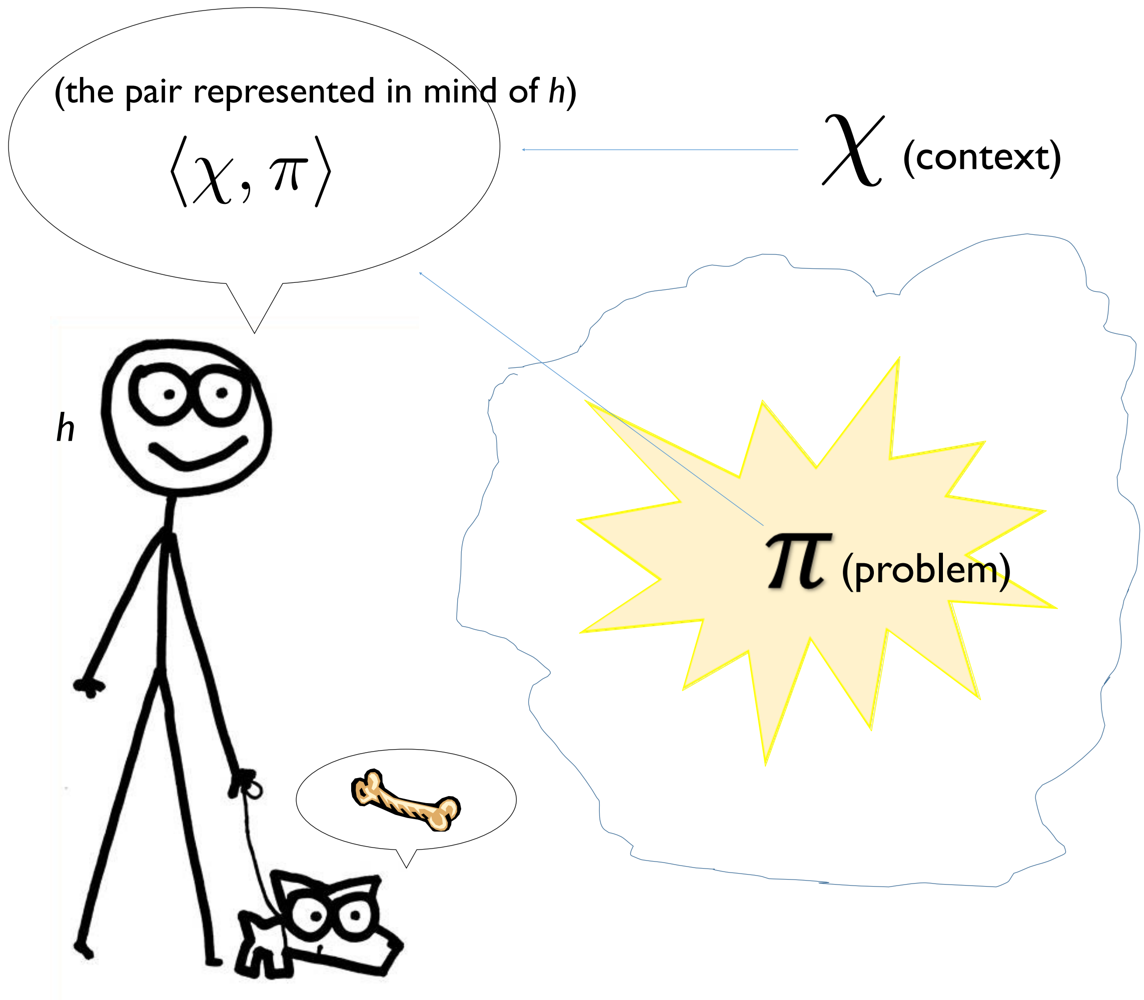
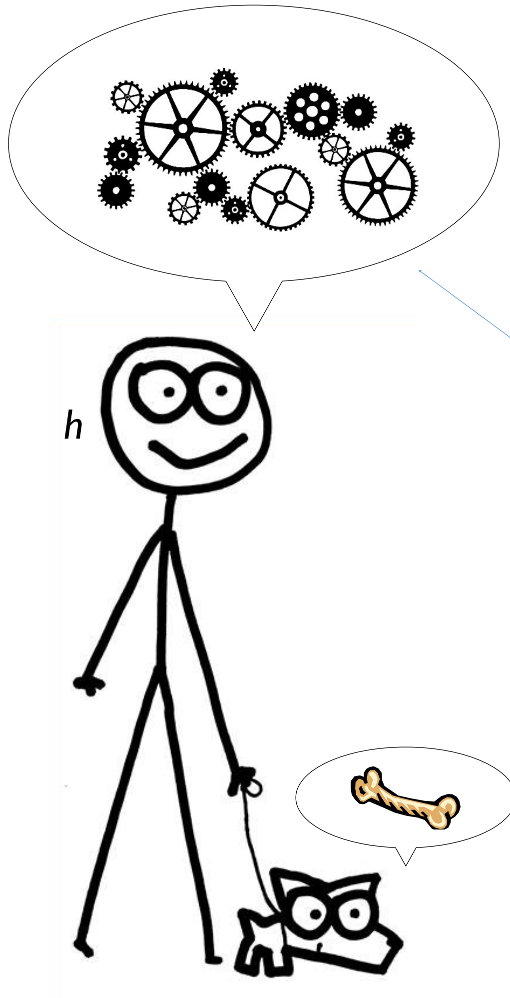
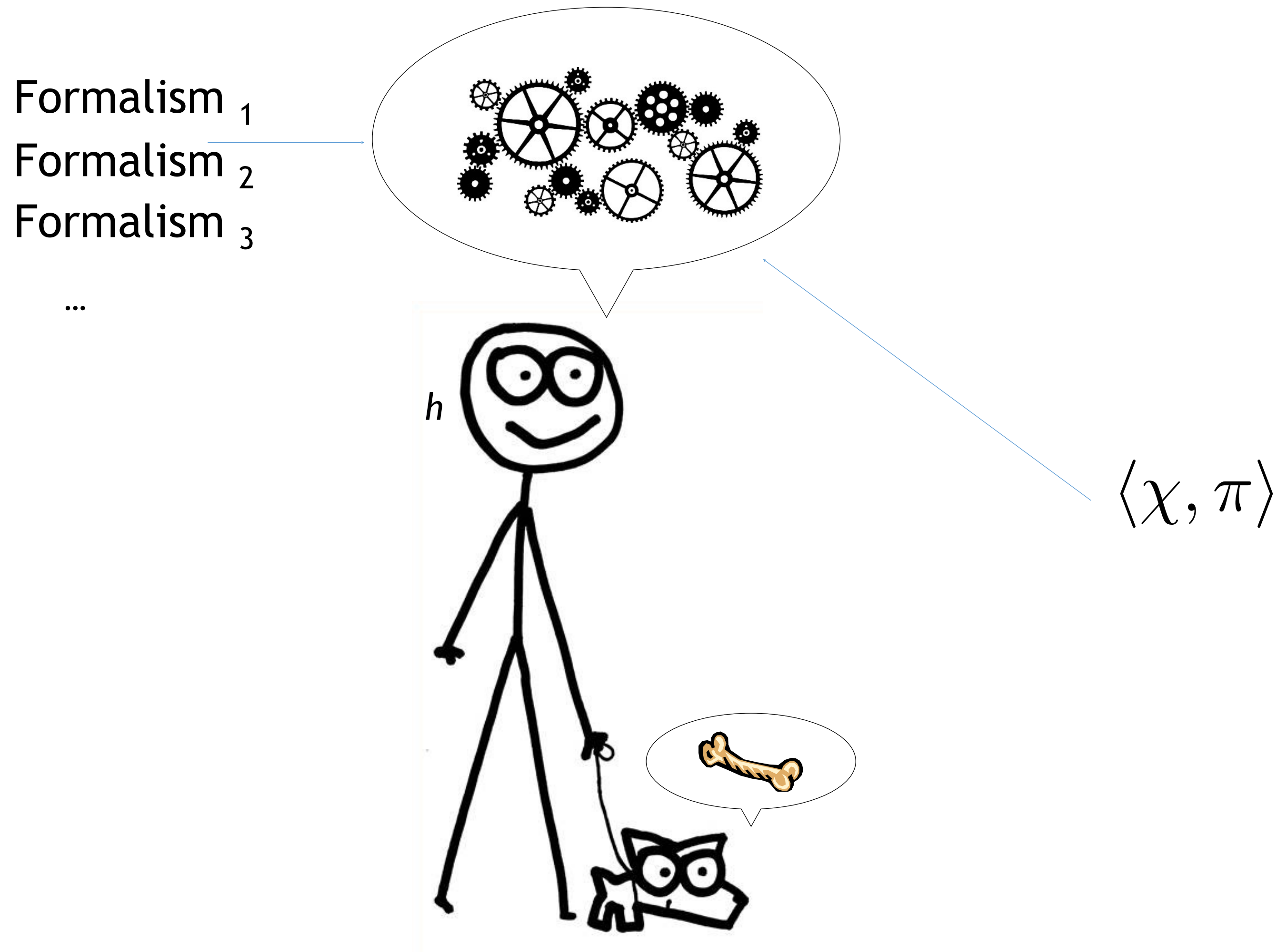
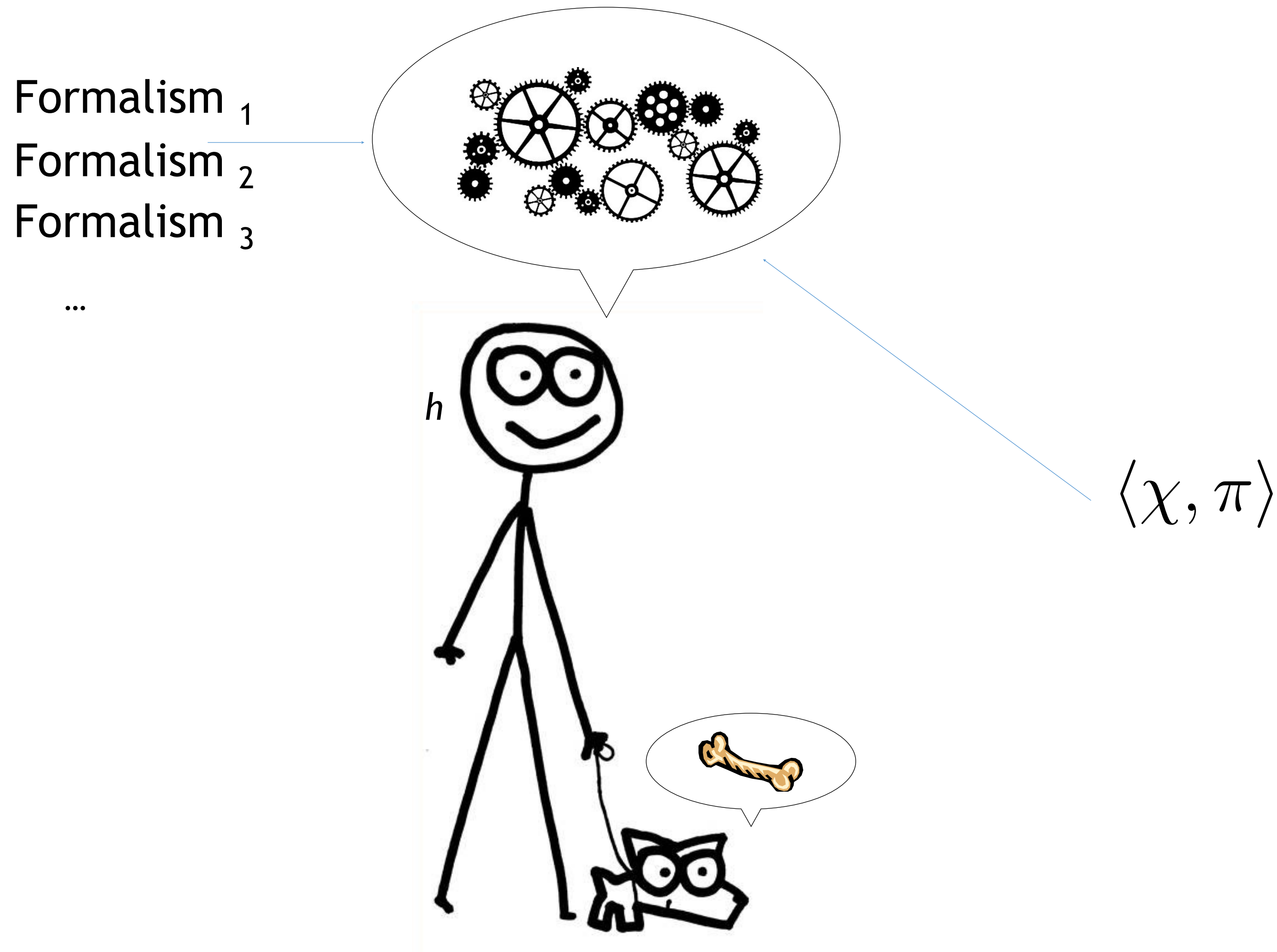

Problem!


(the pair represented in mind of *h*)
$\langle \chi, \pi \rangle$
χ (context)
h
π (problem)



$\langle \chi, \pi \rangle$


Formalism $_1$
Formalism $_2$
Formalism $_3$
…
h
$\langle \chi, \pi \rangle$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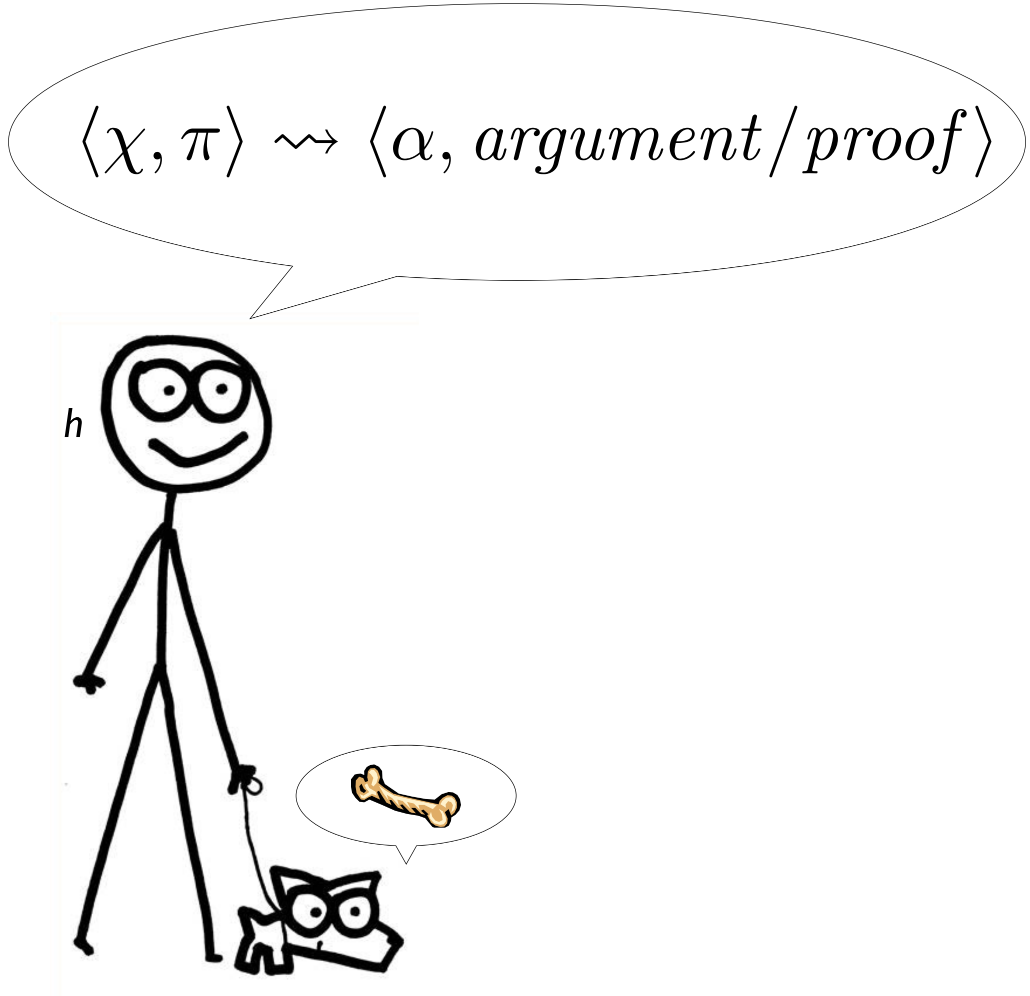
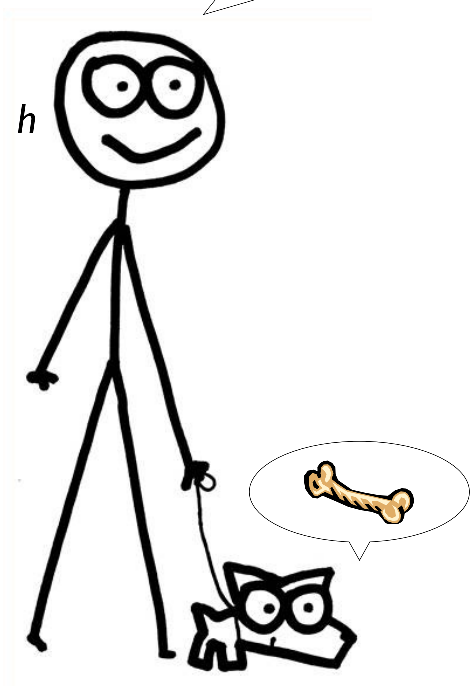

$\langle \chi, \pi \rangle \rightsquigarrow \langle \alpha, argument/proof \rangle$
h

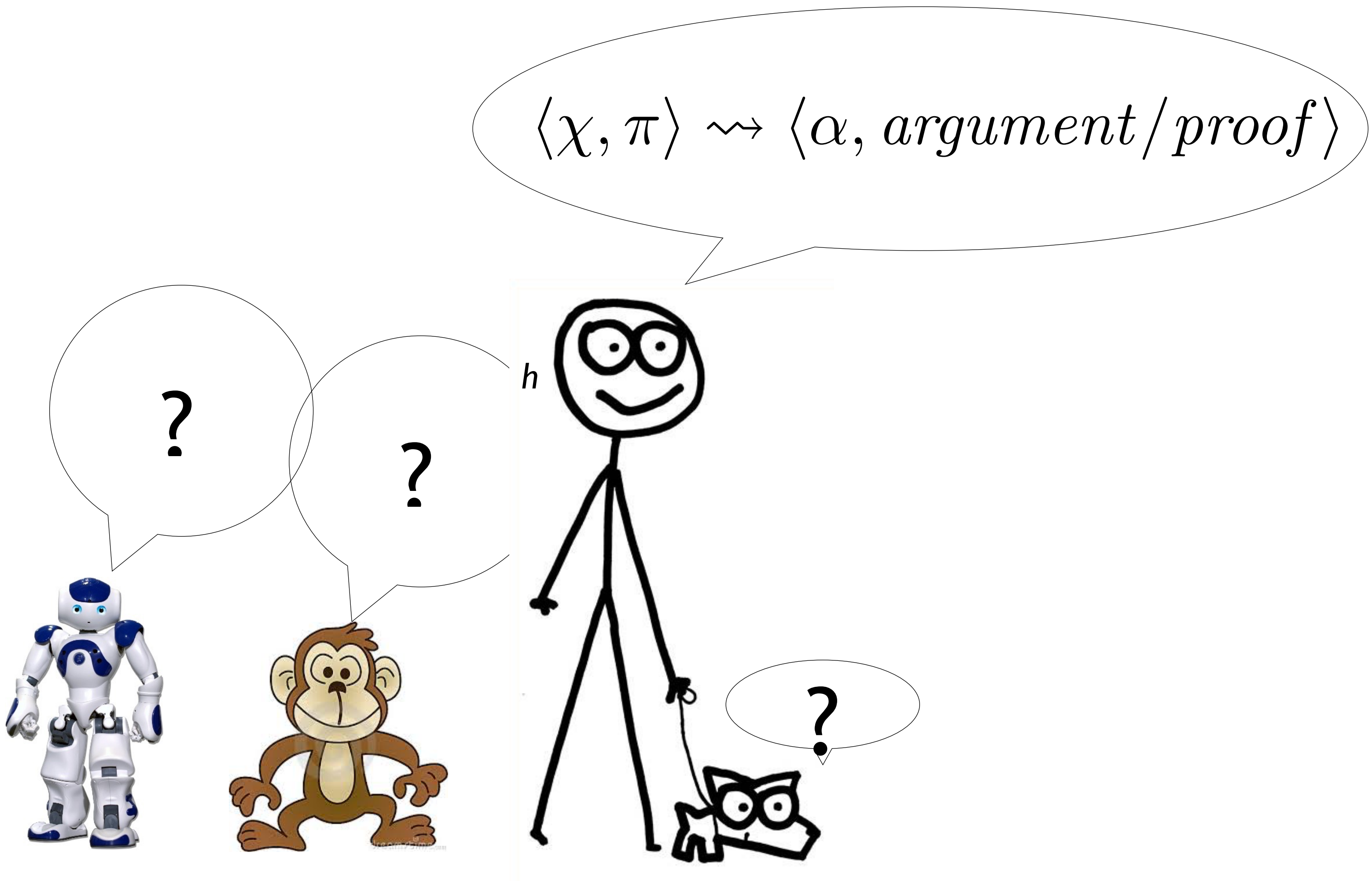
$$\langle \chi, \pi \rangle \rightsquigarrow \langle \alpha, argument/proof \rangle$$

Today's machine-learning systems are fundamentally incapable of providing the argument/proof.

?

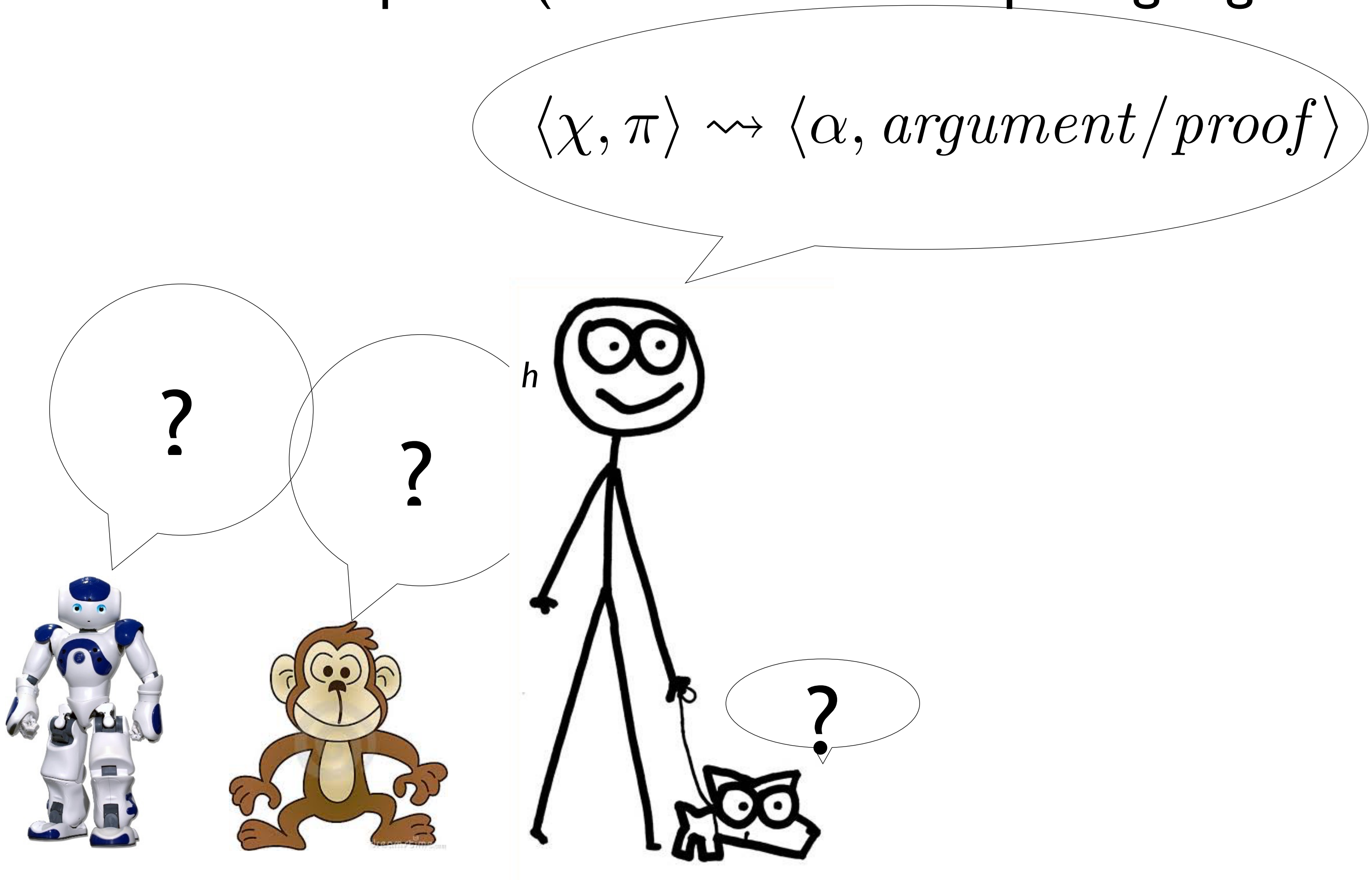


https://www.darpa.mil/program/explainable-artificial-intelligence



Contrarian view on animal minds in *Nat. Geo.*:
http://ngm.nationalgeographic.com/2008/03/animal-minds/virginia-morell-text

Ok, so where's the proof (or at leas the compelling argument)?



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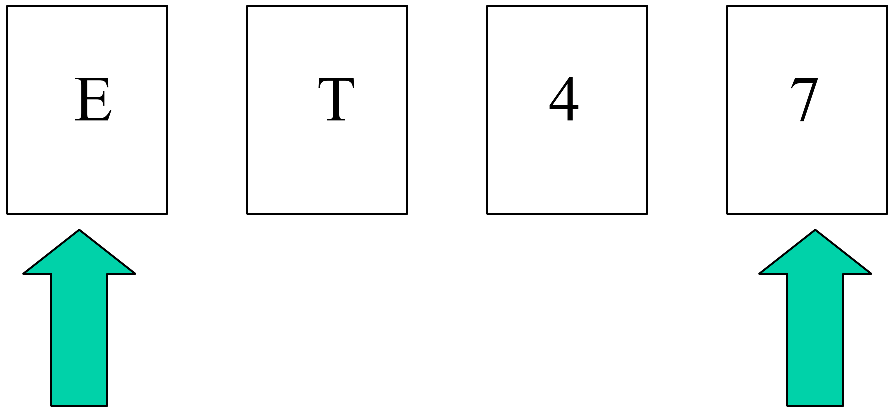
Wason Selection Task



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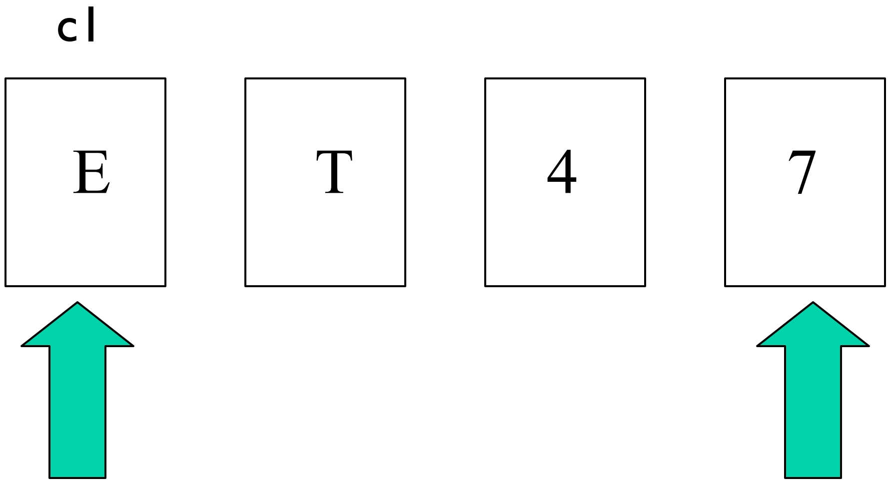

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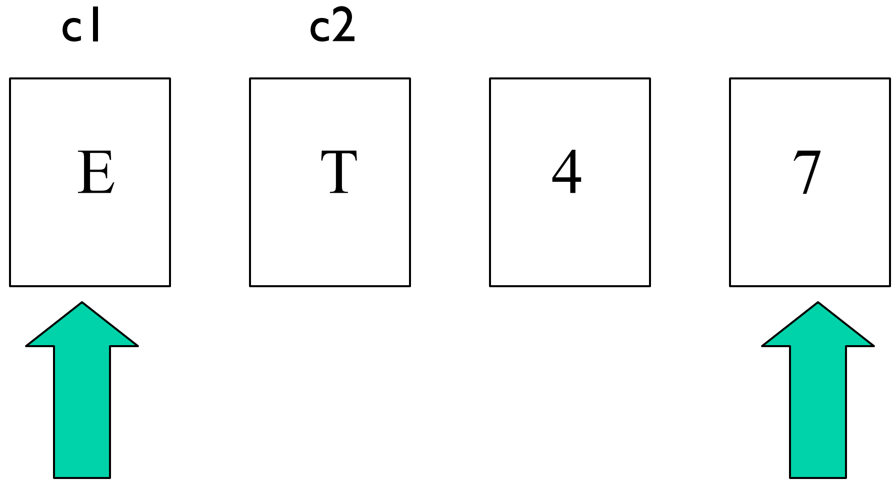

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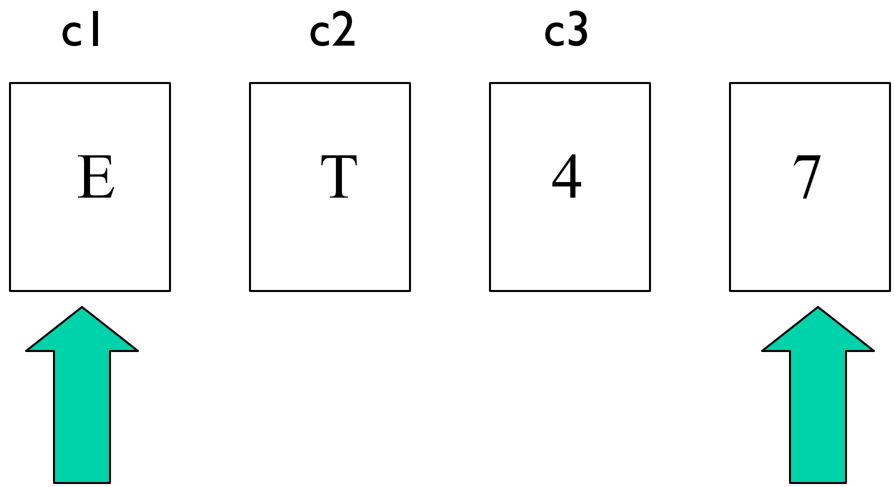

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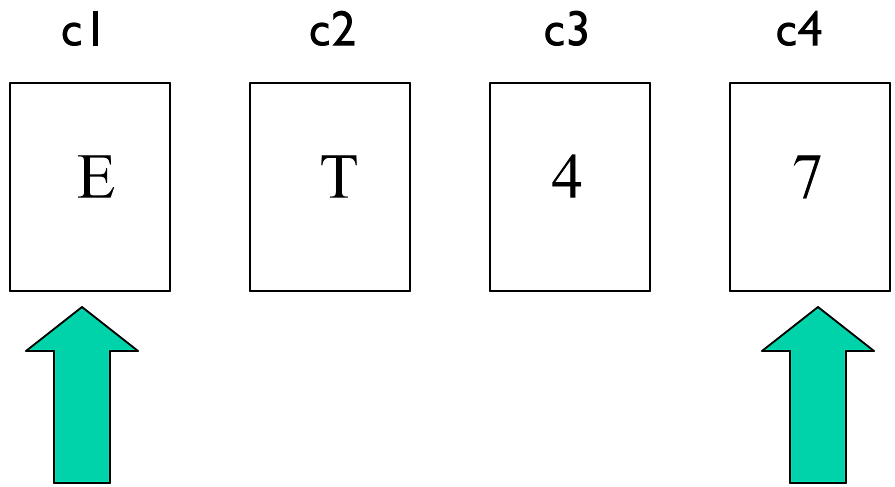

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Which card or cards should you turn over in order to try to decide whether the rule is true or false?

Wason Selection Task

Proposition 1: You should flip c1!

Proof: Were you to flip c1, there are two and only two general cases that might appear before your eyes: you find an odd number; or else you find an even number. Well, if you find an odd number, you can stop, because the rule in question would then be refuted (since you have a case where the antecedent (vowel on one side) holds, but the consequent (even number on the other side) doesn't. Since this might well happen for all you know, you should flip over c1. **QED**

Which card or cards should you turn over in order to try to decide whether the rule is true or false?

Wason Selection Task

Proposition 2: You should flip c4!

Proof: Were you to flip c4, there are two and only two general cases that might appear before your eyes: you find a vowel; or else you find a consonant. Well, if you find a vowel, you can stop, because the rule in question would then be refuted (since you have a case where the antecedent (vowel on one side) holds, but the consequent (even number on the other side) doesn't. Since this might well happen for all you know, you should flip over c4. **QED**

Which card or cards should you turn over in order to try to decide whether the rule is true or false?

Proposition 2: You should flip c4!

Proof: Were you to flip c4, there are two and only two general cases that might appear before your eyes: you find a vowel; or else you find a consonant. Well, if you find a vowel, you can stop, because the rule in question would then be refuted (since you have a case where the antecedent (vowel on one side) holds, but the consequent (even number on the other side) doesn't. Since this might well happen for all you know, you should flip over c4. **QED**

?

Proposition 3: You should *not* flip c2!

Proposition 4: You should *not* flip c3!

“NYS 1”

Given the statements

$\neg a \vee \neg b$

b

$c \rightarrow a$

which one of the following statements must also be true?

c

$\neg b$

$\neg c$

h

a

none of the above

“NYS 1”

Given the statements

$\neg a \lor \neg b$

b

$c \rightarrow a$

which one of the following statements must also be true?

c

$\neg b$

$\neg c$

h

a

none of the above

Proposition: The correct answer is ¬c.

Proof: We are given that b; that's the second statement. Well, if b holds, then ¬b doesn't hold. The first statement tells us that either ¬a or ¬b. So from this and the derived proposition that ¬b doesn't hold we can infer ¬a. (If you know P or Q, and you know not-Q, you immediately know P; this inference rule is called *disjunctive syllogism*.) But from ¬a and c → a we can deduce that c can't be the case; i.e., we can deduce ¬c. (This last inference is sanctioned by the rule of inference called *modus tollens*.) **QED**

“NYS 3”

Given the statements

$\neg\neg c$

$c \rightarrow a$

$\neg a \vee b$

$b \rightarrow d$

$\neg(d \vee e)$

which one of the following statements must also be true?

$\neg c$

e

h

$\neg a$

all of the above

"NYS 3"

Given the statements

$\neg\neg c$

$c \rightarrow a$

$\neg a \vee b$

$b \rightarrow d$

$\neg(d \vee e)$

which one of the following statements must also be true?

$\neg c$

e

h

$\neg a$

all of the above

CSCI 2200: Foundations of Computer Science – Spring 2015

General Information

Instructor: Stacy Patterson sep@cs.rpi.edu 518-276-2054

Teaching Assistants

Ashwin Bahulkar	bahula@rpi.edu
Lingxun Hu	hul5@rpi.edu
Md. Ridwan Al Iqbal	iqbalm@rpi.edu
Jai Wadhwani	wadhwj@rpi.edu

Web site: http://www.cs.rpi.edu/~sep/csci2200

Textbook: Kenneth H. Rosen, *Discrete Mathematics and Its Applications*, 7th ed., McGraw Hill, 2012

Lectures: MR 10:00am – 11:50 pm, Russell Sage Laboratory 3303

Recitations:

Section 01	W 10:00am – 10:50am	Troy Building 2012
Section 02	W 11:00am – 11:50am	Troy Building 2012
Section 03	W 12:00pm – 12:50pm	Troy Building 2018
Section 04	W 4:00pm – 4:50pm	Walker Laboratory 5113

Course Description

This course introduces important mathematical and theoretical tools for computer science, including topics from logic, number theory, set theory, combinatorics, and probability theory. The course then proceeds to automata theory, the Turing Machine model of computation, and notions of computational complexity. The course will emphasize formal reasoning and proof techniques.

Upon successful completion of this course, each student:

- is able to formulate mathematical proofs using logic
- is able to apply mathematical tools such as induction and recursion
- can recall key definitions from set theory
- is able to formulate combinatorial arguments
- is able to distinguish between various computational models
- is able to think critically on the difficulties of key questions in foundations of computer science
- can recall key facts regarding finite automata and Turing machines.

Pre-requisites: Intro to Calculus (MATH-1010 or MATH-1500); CSCI-1100 (CS I) or CSCI-1200 (Data Structures)

Recitation

Attendance at recitation is not required. Attendance will be taken at recitation, and students who attend regularly will get priority in office hours.

Schedule

An up-to-date schedule will be maintained on the course web site

Homework

There will be 9 homework assignments. The lowest homework grade will be dropped. Homework is due at the beginning of class on the date indicated on the homework assignment. You may turn in an assignment at the beginning of following class for a 50% penalty. No homework will be accepted after that time without a letter from the Student Experience office.

See also e.g. http://www.cs.rpi.edu/~magdon/courses/focs.html

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Teaching Assistants

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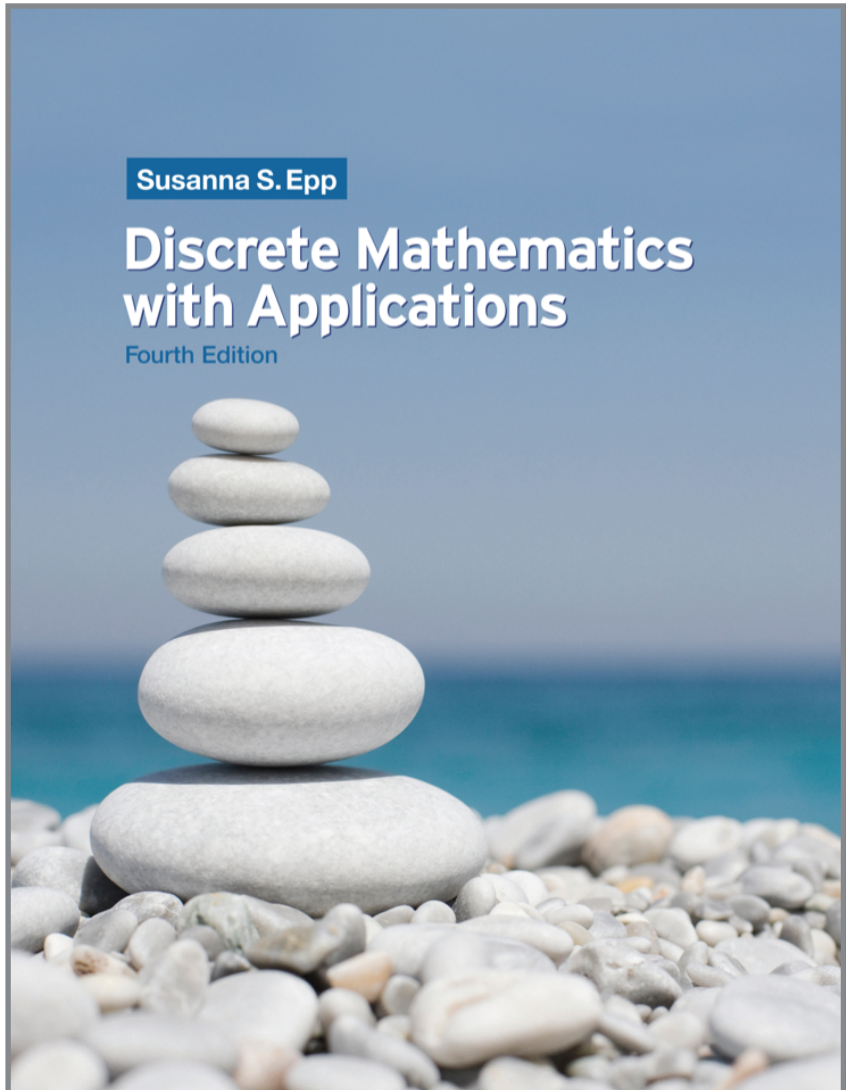
Or …


Susanna S. Epp
Discrete Mathematics with Applications
Fourth Edition

List of Symbols

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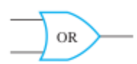
Applications of Logic		NOT-gate	67
		AND-gate	67
		OR-gate	67
		NAND-gate	75
		NOR-gate	75
	$\mid$	Sheffer stroke	74
	$\downarrow$	Peirce arrow	74
	n_2	number written in binary notation	78
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	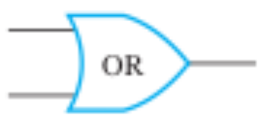	OR-gate	67
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	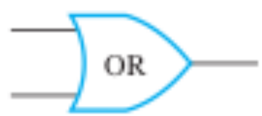	OR-gate	67
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	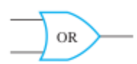	OR-gate	67
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But we’ll instead go with …

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	n *mod* d	the integer remainder of n divided by d	181
	$\lfloor x \rfloor$	the floor of x	191
	$\lceil x \rceil$	the ceiling of x	191
	$\lvert x \rvert$	the absolute value of x	187
	$\gcd(a, b)$	the greatest common divisor of a and b	220
	$x := e$	x is assigned the value e	214

But we'll instead go with …

TABLE 1 Rules of Inference.

Rule of Inference	***Tautology***	***Name***
p $p \rightarrow q$ $\therefore q$	$(p \wedge (p \rightarrow q)) \rightarrow q$	Modus ponens
$\neg q$ $p \rightarrow q$ $\therefore \neg p$	$(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$	Modus tollens
$p \rightarrow q$ $q \rightarrow r$ $\therefore p \rightarrow r$	$((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$	Hypothetical syllogism
$p \vee q$ $\neg p$ $\therefore q$	$((p \vee q) \wedge \neg p) \rightarrow q$	Disjunctive syllogism
p $\therefore p \vee q$	$p \rightarrow (p \vee q)$	Addition
$p \wedge q$ $\therefore p$	$(p \wedge q) \rightarrow p$	Simplification
p q $\therefore p \wedge q$	$((p) \wedge (q)) \rightarrow (p \wedge q)$	Conjunction
$p \vee q$ $\neg p \vee r$ $\therefore q \vee r$	$((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$	Resolution

EXAMPLE 3 State which rule of inference is the basis of the following argument: “It is below freezing Therefore, it is either below freezing or raining now.”

Solution: Let p be the proposition “It is below freezing now” and q the proposition “It is ra now.” Then this argument is of the form

$$\begin{array}{l} \quad p \\ \hline \therefore p \vee q \end{array}$$

This is an argument that uses the addition rule.

EXAMPLE 4 State which rule of inference is the basis of the following argument: “It is below freezing raining now. Therefore, it is below freezing now.”

Solution: Let p be the proposition “It is below freezing now,” and let q be the proposition raining now.” This argument is of the form

$$\begin{array}{l} \quad p \wedge q \\ \hline \therefore p \end{array}$$

This argument uses the simplification rule.

TABLE 1 **Rules of Inference.**

Rule of Inference	*Tautology*	*Name*
p $p \to q$ $\therefore\ q$	$(p \wedge (p \to q)) \to q$	Modus ponens
$\neg q$ $p \to q$ $\therefore\ \neg p$	$(\neg q \wedge (p \to q)) \to \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore\ p \to r$	$((p \to q) \wedge (q \to r)) \to (p \to r)$	Hypothetical syllogism
$p \vee q$ $\neg p$ $\therefore\ q$	$((p \vee q) \wedge \neg p) \to q$	Disjunctive syllogism
p $\therefore\ p \vee q$	$p \to (p \vee q)$	Addition
$p \wedge q$ $\therefore\ p$	$(p \wedge q) \to p$	Simplification
p q $\therefore\ p \wedge q$	$((p) \wedge (q)) \to (p \wedge q)$	Conjunction
$p \vee q$ $\neg p \vee r$ $\therefore\ q \vee r$	$((p \vee q) \wedge (\neg p \vee r)) \to (q \vee r)$	Resolution

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Explosion Rule!

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$$\frac{p \wedge \neg p}{q}$$

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Easy peasy to prove in Rosen:

Explosion Rule!

$$\frac{p \wedge \neg p}{q}$$

Easy peasy to prove in Rosen:

(1)	$p \wedge \neg p$	Premise
(2)	p	Simplification using (1)
(3)	$p \vee q$	Addition using (2)
(4)	$\neg p$	Simplification using (1)
(5)	q	Disjunctive Syllogism using (3) and (4)

EXAMPLE 6 Show that the premises "It is not sunny this afternoon and it is colder than yesterday," "We will go swimming only if it is sunny," "If we do not go swimming, then we will take a canoe trip," and "If we take a canoe trip, then we will be home by sunset" lead to the conclusion "We will be home by sunset."

Extra Examples

Solution: Let p be the proposition "It is sunny this afternoon," q the proposition "It is colder than yesterday," r the proposition "We will go swimming," s the proposition "We will take a canoe trip," and t the proposition "We will be home by sunset." Then the premises become $\neg p \wedge q, r \rightarrow p, \neg r \rightarrow s$, and $s \rightarrow t$. The conclusion is simply t. We need to give a valid argument with premises $\neg p \wedge q, r \rightarrow p, \neg r \rightarrow s$, and $s \rightarrow t$ and conclusion t.

We construct an argument to show that our premises lead to the desired conclusion as follows.

Step	Reason
1. $\neg p \wedge q$	Premise
2. $\neg p$	Simplification using (1)
3. $r \rightarrow p$	Premise
4. $\neg r$	Modus tollens using (2) and (3)
5. $\neg r \rightarrow s$	Premise
6. s	Modus ponens using (4) and (5)
7. $s \rightarrow t$	Premise
8. t	Modus ponens using (6) and (7)

Note that we could have used a truth table to show that whenever each of the four hypotheses is true, the conclusion is also true. However, because we are working with five propositional variables, p, q, r, s, and t, such a truth table would have 32 rows. ◂

"NYS 3"

Given the statements

$\neg\neg c$

$c \rightarrow a$

$\neg a \vee b$

$b \rightarrow d$

$\neg(d \vee e)$

which one of the following statements must also be true?

$\neg c$

e

h

$\neg a$

all of the above

“NYS 3”

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$c \rightarrow a$

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$b \rightarrow d$

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"NYS 3"

Given the statements

$\neg\neg c \longrightarrow c$

$c \rightarrow a$

$\neg a \vee b$

$b \rightarrow d$

$\neg(d \vee e)$

which one of the following statements must also be true?

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e

h

$\neg a$

all of the above

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Given the statements

$\neg\neg c \longrightarrow c$

$c \rightarrow a$

$\neg a \vee b$

$b \rightarrow d$

$\neg(d \vee e) \longrightarrow$

which one of the following statements must also be true?

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e

h

$\neg a$

all of the above

"NYS 3"

Given the statements

$\neg\neg c \longrightarrow c$

$c \rightarrow a$

$\neg a \vee b$

$b \rightarrow d$

$\neg(d \vee e) \longrightarrow \neg d \wedge \neg e$

which one of the following statements must also be true?

$\neg c$

e

h

$\neg a$

all of the above

"NYS 3"

Given the statements

$\neg\neg c \longrightarrow c$

$c \rightarrow a$

$\neg a \vee b$

$b \rightarrow d$

$\neg(d \vee e) \longrightarrow \neg d \wedge \neg e$

> Homework 1: Prove that the answer to this problem is indeed "all of the above," using tools provided to you in the present slide deck.

which one of the following statements must also be true?

$\neg c$

e

h

$\neg a$

all of the above

Homework 1 Solution

Proposition: The answer is "all of the above."

Proof: We know from the rule of inference *explosion* that everything follows from a contradiction, so we simply need to find a contradiction in the given statements. We do so as follows. We already have ~d by DeMorgan's Law, as indicated on the previous slide. On that slide, we also have c from the first statement. This, combined with the second given, yields by *modus ponens* a in one step. Next, by disjunctive syllogism we have b from a and ~a v b. Another use of modus ponens with b and b => d gives d, and we have our contradiction. **QED**

“NYS 2”

Which one of the following statements is logically equivalent to the following statement: “If you are not part of the solution, then you are part of the problem.”

If you are part of the solution, then you are not part of the problem.

If you are not part of the problem, then you are part of the solution.

If you are part of the problem, then you are not part of the solution.

If you are not part of the problem, then you are not part of the solution.

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If you are not part of the problem, then you are part of the solution.

If you are part of the problem, then you are not part of the solution.

If you are not part of the problem, then you are not part of the solution.

“NYS 2”

Homework 2: Prove that the answer to this problem is indeed the second option, using tools provided to you in the present slide deck.

Which one of the following statements is logically equivalent to the following statement: “If you are not part of the solution, then you are part of the problem.”

If you are part of the solution, then you are not part of the problem.

If you are not part of the problem, then you are part of the solution.

If you are part of the problem, then you are not part of the solution.

If you are not part of the problem, then you are not part of the solution.

Homework 2 Solution

Proposition: The answer is the second option.

Proof: From a conditional P => Q it can be immediately deduced that ~Q => ~P (and *vice versa*) by the rule of inference *contrapositive*, and contrapositive applied to the given statement yield the second option in one step. Now we obtain contrapositive itself. Suppose that a given conditional P => Q holds, and suppose as well that ~Q holds. We are done when we can deduce ~P from what we now have to work with, and what's available to us in the present slide deck. The rule of inference modus tollens allows us to infer ~P in one step from P => Q and ~Q. **QED**

More-Recent Shots …

The Original King-Ace

Suppose that the following premise is true:

> If there is a king in the hand, then there is an ace in the hand, or else if there isn't a king in the hand, then there is an ace.

What can you infer from this premise?

The Original King-Ace

Suppose that the following premise is true:

> If there is a king in the hand, then there is an ace in the hand, or else if there isn't a king in the hand, then there is an ace.

What can you infer from this premise?

There is an ace in the hand.

The Original King-Ace

Suppose that the following premise is true:

> If there is a king in the hand, then there is an ace in the hand, or else if there isn't a king in the hand, then there is an ace.

What can you infer from this premise?

~~There is an ace in the hand.~~

The Original King-Ace

Suppose that the following premise is true:

> If there is a king in the hand, then there is an ace in the hand, or else if there isn't a king in the hand, then there is an ace.

What can you infer from this premise?

NO! ~~There is an ace in the hand.~~

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What can you infer from this premise?

NO! ~~There is an ace in the hand.~~ NO!

In fact, what you *can* infer is that there *isn't* an ace in the hand!

King-Ace 2

Suppose that the following premise is true:

If there is a king in the hand, then there is an ace in the hand; or if there isn't a king in the hand, then there is an ace; but not both of these if-then statements are true.

What can you infer from this premise?

King-Ace 2

Suppose that the following premise is true:

If there is a king in the hand, then there is an ace in the hand; or if there isn't a king in the hand, then there is an ace; but not both of these if-then statements are true.

What can you infer from this premise?

There is an ace in the hand.

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Suppose that the following premise is true:

If there is a king in the hand, then there is an ace in the hand; or if there isn't a king in the hand, then there is an ace; but not both of these if-then statements are true.

What can you infer from this premise?

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NO! ~~There is an ace in the hand.~~ NO!

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In fact, what you *can* infer is that there *isn't* an ace in the hand!

King-Ace Solved

Proposition: There is *not* an ace in the hand.

Proof: We know that at least one of the if-thens (i.e., at least one of the **conditionals**) is false. We know this because we are told that either the first if-then holds, or the second if-then holds, but not both.

So we have two cases to consider, viz., that $K \rightarrow A$ is false, and (the other case) that $\neg K \rightarrow A$ is false. ($\rightarrow$ is the same as the arrow we have used.)

Take first the first case; accordingly, suppose that $K \rightarrow A$ is false. Then it follows that K is true (since, when a conditional is false, its antecedent holds but its consequent doesn't), and A is false; i.e., $\neg A$.

Now consider the second case, which consists in $\neg K \rightarrow A$ being false. Here, in a direct parallel, we know $\neg K$ and, once again, since the consequent of the conditional must be false, $\neg A$.

In both of our two cases, which are exhaustive, there is no ace in the hand. The proposition is established. **QED**

King-Ace Solved

Homework 3: Study to understand.

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20
FEDERAL RESERVE NOTE
20
THE UNITED STATES OF AMERICA
EE 00000000 D
E5
SPECIMEN
SPECIMEN
EE 00000000 D
20
20
TWENTY DOLLARS

Yours soon?


20
FEDERAL
RESERVE NOTE
THE
UNITED STATES
OF AMERICA
EE 00000000 D
E5
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“Show-me-the-$” Problem (AI Version)

If one of the following assertions is true then so is the other:

(1) If there is an apple in the cup then there is a battery in the cup; and, if there is a battery in the cup then there is an apple in the cup.

(2) There is an apple in the cup.

Which is more likely to be in the cup, if either: the apple or the battery?

“Show-me-the-$” Problem (AI Version)

If one of the following assertions is true then so is the other:

(1) If there is an apple in the cup then there is a battery in the cup; and, if there is a battery in the cup then there is an apple in the cup.

(2) There is an apple in the cup.

Which is more likely to be in the cup, if either: the apple or the battery?

Now class, here’s a robot. Notice the cup next to it. The robot has been programmed in a simple way: the code consists of three conditional statements: (1) If the answer to the problem above is “apple,” place only an apple in the empty cup. (2) If the answer to the above problem is “battery,” place only a battery in the empty cup. (3) If the answer is that neither is more likely to be in the cup, leave the cup empty. Earlier, this code was executed and the robot performed accordingly (having before this assimilated and solved the above problem). So: Tell me, assuming that the code all worked perfectly, what’s in the cup, if anything! If you’re right, and can prove that you are, here’s a $20 for you, on the spot.