AHRF16 Test 1 (with solutions)

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Please use a pen, not a pencil. Write your name on your answer booklets now. Thank you.

As you proceed, label each answer in your answer booklets with the appropriate 'Qn'. Please strive to write as legibly as possible. Thank you.

Q1 Here are two statements that are givens: (1) Jill is in Cyprus or Bill is in Cyprus, and (2) Jill isn't in Cyprus or Phil is in Cyprus.

By the rules of inference in the propositional calculus that we have specified and affirmed, does it follow deductively that (3) Bill is in Cyprus or Phil is in Cyprus? (I.e., is this equation true: $\{(1),(2)\} \vdash_{\texttt{PC}}$ (3)}.) Please answer 'Yes' or 'No.' In addition, prove that you are correct.

> **Solution**: Yes. This can be proved in one step using the inference rule `resolution`, as follows.
>
> $$\frac{\begin{array}{c} c_j \vee c_b \\ \neg c_j \vee c_p \end{array}}{c_b \vee c_p}$$
>
> There are many other ways of proving the affirmative response is correct. Here e.g. is a route based on an informal proof:
>
> **Proof**: Our second premise gives us two cases: either Jill isn't in Cyprus, or Phil is there. We are done if we can show that in either case the conclusion (that either Bill is in Cyprus or Phil is). Suppose that the first case holds, i.e. that Jill isn't in Cyprus. This supposition, combined with (1) leads deductively to the proposition that Bill is in Cyprus (by `disjunctive syllogism`. But if Bill is in Cyprus, it follows that Bill is in Cyprus or Phil is. Now suppose on the other hand that Phil is in Cyprus. This leads immediately to (3), because whenever ϕ holds, $\phi \vee \psi$ holds, for any ψ. **QED**

Q2 In order to make it to RPI (congratulations, by the way!), you had to study math for many years, starting in Kindergarten (or at least First Grade). Hence you will no doubt agree that throughout the math education you received, where ϕ is any mathematical statement in your textbooks along the way, that statement either holds, or does not hold. Let's call this principle 'The Principle of Bivalence' (TPB). More pedantically put, TPB is: $\phi \vee \neg\phi$, for every mathematical statement ϕ in your K–12 math textbooks. Johnny tells his math teacher in high school that TPB is false. Teacher Tony replies that Johnny's position is irrational. Who's right? Why, exactly?

> **Solution**: Teacher Tony is right. Here's why: He's affirming TPB, which is a very circumspect principle, since it specifically

refers (and only refers) to mathematical statements in Johnny's K–12 textbooks. (It doesn't say that bivalence holds for any kind of statement whatever — though certainly many people have affirmed the more ambitious version of the principle.) And it's not hard to prove TPB using what's in those textbooks. Here's one way: **Proof**: Suppose that TBP is false. Suppose in addition that $\neg\phi$ holds. It follows immediately from our second supposition that $\phi \vee \neg\phi$ holds, but then given this combined with our original supposition yields a contradiction: viz. both $\phi \vee \neg\phi$ and $\neg(\phi \vee \neg\phi)$. Therefore, we reject $\neg\phi$ by indirect proof; i.e. we have ϕ. But from ϕ we can immediately deduce $\phi \vee \neg\phi$. But this combined with our original supposition is a contradiction. Hence our original supposition must be rejected; i.e. we have established the negation of our original supposition, viz. TPB itself. **QED**

(Many other such proofs, including formal ones, are possible.)

Q3 Sherlock has three perfectly trustworthy clues to work with in an attempt to place four people. The clues are:

(a) It's not the case that: Jill is in Westchester if and only if Chris is in Seattle.

(b) It's not the case that: Henry is in Seattle or Kate is not in Ireland.

(c) Chris is in Seattle or Chris is in Seattle.

Answer the following fill-ins for Sherlock, where what is supplied is either a place-name (where if X is a place-name, we also — rather charitably! — count 'not X' as a place name) or 'UNKNOWN.'

Jill's location:
Kate's location:
Henry's location:
Chris's location:

Justify each of these four answers with a separate proof that employs one or more rules of deductive inference from our list of them for the propositional calculus, or that employs a new rule of deductive inference that you introduce. If you make use of a new rule of deductive inference, show that that rule is in fact valid before using it. So that your proof is more perspicuous, make use of abbreviations (e.g., '$\mathrm{J_W}$' for 'Jill is in Westchester'). You are of course free to invent the abbreviations you find most natural, but make sure you explain those you decide to use.

Solution: Using what should be obvious representation, we have the following in the propositional calculus:

(a) $\neg(w_j \leftrightarrow s_c)$

(b) $\neg(s_h \vee \neg i_k)$

(c) $s_c \vee s_c$

- Jill is not in Westchester ($\neg w_j$). This one is trickier than the others. From (a) we know (and we've done this before on a much tougher one; do you remember?) that there are two cases, corresponding to the only two ways that a biconditional can be false: Either $w_j \wedge \neg s_c$ or $\neg w_j \wedge s_c$. There are thus two cases given to us to consider. The first of these cases contradicts that Chris is in Seattle (s_c), which we have directly from (c) (see below); hence we have $\neg w_j \wedge s_c$, from which it follows immediately that $\neg w_j$.
- Kate is in Ireland (i_k). DeMorgan's Law applied to (b) yiedls $\neg s_h \wedge i_k$, from which it directly follows that i_k.
- Henry is not in Seattle ($\neg s_h$). DeMorgan's Law applied to (b) yiedls $\neg s_h \wedge i_k$, from which it directly follows that $\neg s_h$.
- Chris is in Seattle (s_c). This is the easiest one, perhaps. Chris must be in Seattle, for if this is not the case (i.e. if $\neg s_c$), it follows from this and (c) by disjunctive syllogism that s_c, which is a contradiction. (Also, $(\phi \vee \phi) \rightarrow \phi$ is a theorem in the propositional calculus, and from it and (c) s_c follows.)

Q4 If one of the following three assertions is true, then so are the other two.

1. There is a smart giraffe in the zoo if and only if there is a male horse in the zoo.
2. There is a smart giraffe in the zoo.
3. There is a lazy llama in the zoo.

Given the above information, which animal among the three is most likely to be in the zoo (if in fact one is most likely)?

Prove that you're right.

Solution: Essentially a direct carry-over from class/slides. Answer: the male horse! Do impromptu on board?

Q5 Give a proof based on our set of inference rules for the propositional calculus (a set that includes the explosion rule) that the answer to the problem “NYS 3” given by S Bringsjord in class is correct.

Solution: Was assigned as a homework. Any student want to do it on the board?

Q6 Assume the exact same context for the Wason Selection Task as S Bringsjord has repeatedly given and explained, but where the four cards presented to you are as follows.

$\boxed{\text{A}}$ $\boxed{\text{D}}$ $\boxed{6}$ $\boxed{9}$

Which card or cards should you flip over in this permutation? Justify your answer.

Solution: This is nothing more than an isomorph of the original version we considered and solved. Questions?

Q7 Consider the following argument:

All the Swedes in the room are skiers.
Some of the skiers in the room are athletic.
Therefore:
Some of the Swedes in the room are athletic.

Is this argument logically valid? Why, exactly?

Solution: Recall Ju’s solution, shown in class. Would someone like a diagrammatic solution (e.g. Venn-diagrammatic one)?

Q8 The following four statements are all false.

(a) If Alvin is happy, so is Betty.
(b) If Betty is happy, Charlie is too.
(c) If Charlie is happy, Darla is happy as well.
(d) Alvin is happy.

Does it follow deductively from the above information that Darla is happy? Prove that you're right, by employing the propositional calculus.

> **Solution**: **Proof**: Since all four statements are false, (a) and (b) are false. Recall that a conditional is false if and only if its antecedent is true and it's consequent is false. Therefore from (a) we deduce a and $\neg b$, and from (b) we deduce b and $\neg c$. But then we have a contradiction: $b \wedge \neg b$. Hence by the explosion rule we can prove anything, including d (Darla is happy). **QED**

D

Q9 For the following group of statements, rank the members of the group from least probable to more probable, and — using either the Venn-geometric account of probability (recommended), the urn-based account, or Kolmogorov's axioms — prove that your answer is correct.

(a) It's not the case that: Linda is a medical doctor or Linda is a pilot.

(b) Linda is a medical doctor.

(c) If Linda is not a medical doctor, then Linda is a pilot or Linda is a pilot.

(d) Linda is a medical doctor if and only if Linda is a medical doctor.

(e) Linda is neither a medical doctor nor a pilot.

Solution

The first thing to note is that there is an error in this problem: We didn't cover how to handle formulae like the one corresponding to (e): viz. $\neg D \wedge \neg P$. So the student is not required to place (e) in the asked-for continuum.

The second key thing to note and apply here is that those statements that are logically equivalent with each other will have exactly the same probabliity. Let's then first note which pairs of these statements are logically equivalent. Statement (a) is equivalent to statement (e) because $\neg(D \vee P)$ is by DeMorgan's Law

$\neg D \wedge \neg P$, a direct representation of (e) in the propositional calculus.

We can also simplify to make things easier on ourselves: (c) is represented in the propositional caclculus as $\neg D \rightarrow (P \vee P)$, which has the same truth table as $\neg D \rightarrow P$ (since $P \vee P$ has the same truth-table as just P), which in turn is equivalent to simply $D \vee P$.

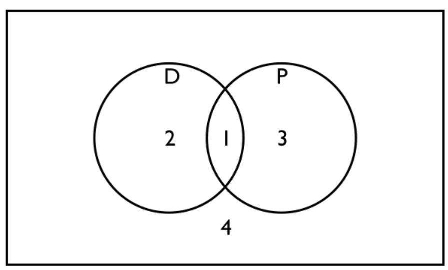
Another thing we can do is note that (d) is a theorem, so it must have a probability of 1.

Summing up, the upshot is that:

$$prob((a)) = prob((e))$$
$$prob((d)) = 1$$

In light of the first of these two, we can just go with (e) but drop (a) (remembering that (a) would just be where (e) is), or vice versa. But of course, as noted above, (a) and (e) have been thrown out.

Geometric Interpretation of Probabilities



Assume that the areas of the regions in the diagram represent the probabilities of the formulae they correspond to. I.e.,

$$prob(D) = \text{size region 2}$$
$$prob(P) = \text{size region 3}$$
$$prob(D \wedge P) = \text{size of region 1}$$
$$prob(D \vee P) = \text{size of region 2 plus region 3}$$

While of course the sizing of the regions could be different, this one gives us the following, which wouldn't be fundamentally different in different size distributions.

$$prob(D) \leq prob(D \vee P) < prob(D \leftrightarrow D)$$

Q10 Recall our variant of the Monty Hall Problem (MHP) considered in class: the Monty *Fall* Problem (MFP). (Make sure that you understand MFP exactly as we specified it, rather than versions online.) We have established that a switching policy (instead of a staying policy) is rational for an agent to affirm before that agent is confronted with an instance of MHP, and rational to apply when that agent is in an instance of the problem on a particular occasion. Now ...

(a) Suppose that we consider a variant of MHP that has not three doors, but seven. Full Monty in this variant reveals what's behind one of the six doors not initially picked, everything else remaining constant from the original MHP. Is switching still a rational policy in this variant? Why, exactly?

(b) Is a switching policy still rational to affirm before an agent is confronted with an instance of MFP (notice, MFP, not MHP), and to apply when that agent is in an instance of MFP on a particular occasion? Why, exactly?

Solution: ??