Solutions to AHRF15 Test 1

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Write your name on your answer booklets now. Thank you.

As you proceed, label each answer in your answer booklets with the appropriate 'Qn'. In general, strive to write as legibly as possible; thanks.

Q1 Consider this declarative sentence: June is in Cyprus.

By the rules of inference in the propositional calculus, what is an example of another statement that follows deductively from this sentence, if anything does?

Prove that your answer is correct.

Solution

One example, where 'J' represents the English statement in question, is $\neg\neg J$. This answer is provably correct because $\neg\neg J$ follows from J by the inference schema **double negation**.

Q2 Sherlock has three perfectly trustworthy clues to work with in an attempt to place three people. The clues are:

(a) If Jill is in Westchester, then Chris is in Seattle.

(b) It's not the case that: Chris is in Seattle or Kate is not in Ireland.

(c) It's not the case that it's not the case that Chris isn't in Seattle.

Answer the following fill-ins for Sherlock, where what is supplied is either a place-name (where if X is a place-name, we also — rather charitably! — count 'not X' as a place name) or 'UNKNOWN.'

Jill's location:
Kate's location:
Chris's location:

Justify each of these three answers with a separate proof that employs one or more rules of deductive inference from our list of them for the propositional calculus. So that your proof is more perspicuous, make use of abbreviations (e.g., '$\mathrm{J_W}$' for 'Jill is in Westchester'). You are of course free to invent the abbreviations you find most natural, but make sure you explain those you decide to use.

Solution

(c) is $\neg\neg\neg C_S$, which by double negation becomes simply $\neg C_S$; hence Chris's location is: not Seattle.

Next, (a) is this conditional: $J_W \rightarrow C_S$. But since we already know $\neg C_S$, we can reason by *modus tollens* to the negation of the antecedent in this conditional; i.e., we can infer to $\neg J_W$. So we have Jill's location: not Westchester.

Next, (b) can be represented by $\neg(C_S \vee \neg K_I)$. By DeMorgan's Law, we can deduce from this that $\neg C_S \wedge \neg\neg K_I$. Next we can extract the righthand of these two conjuncts (by what we've called 'conjunction elimination') to yield $\neg\neg K_I$. But then by double negation we deduce K_I. Hence our answer here for Kate's location is: Ireland.

Q3 Either the first or second of the following two conditionals is true, but not both:

> If your opponents have the queen, they have the ace; if your opponents don't have the queen, they have the ace.

Additionally: If your opponents have the ace, you should lead with trump. And if they don't have the ace, you should lead with the king.

Given the above information, what is the rational thing for you to lead with?

Prove that you're right.

Solution

Since only one of the two conditionals is true, one of the conditionals must be false. The two conditionals, in the propositional calculus, are:

(1) $Q \to A$

(2) $\neg Q \to A$

Let's consider the two possibilities in turn: Suppose first that (1) is false. Then by the truth table for conditionals, Q is true and A is false — from which, of course, it follows that A is false. Suppose now that (2) is false. Then by the truth table for conditionals, $\neg Q$ is true and A is false. Either way, then, A is false. Yet we are also told that if $\neg A$, you should lead with the king. Hence you should lead with the king.

Q4 Give a proof based on the rules of the propositional calculus that the answer to the problem "NYS 2" given by S Bringsjord in class is correct.

Solution

To represent the declarative information in NYS 2, we use the following: P represents 'You are part of the problem', and S denotes 'You are part of the solution'. So the question then becomes:

"Which one of the following statements is logically equivalent to $\neg S \to P$?" Here are the options in the problem:

1. $S \to \neg P$
2. $\neg P \to S$
3. $P \to \neg S$
4. $\neg P \to \neg S$

The correct answer, as we already know, is option 2. **Proof**: By the rule of contrapositive, from $\neg S \to P$ we can deduce $\neg P \to \neg\neg S$. And then from this formula we can by double negation infer $\neg P \to S$. The sequence of reasoning could be reversed. **QED**

Q5 Assume the exact same context for the Wason Selection Task as S Bringsjord has repeatedly given and explained, but where the four cards presented to you are as follows.

$\boxed{\text{B}}$ $\boxed{\text{A}}$ $\boxed{9}$ $\boxed{6}$

Which card or cards should you flip over in this permutation? Justify your answer.

Solution

The rule is (assuming obvious abbreviation) $V \to E$. If the other side of the A card revealed an odd number, the rule by the truth-table for the conditional would be FALSE (because we'd have a TRUE for V and a FALSE for E); hence A must be turned over. If the other side of the 9 card revealed a vowel, by the same logic we'd have FALSE for the conditional. Hence we should flip A and 9. Neither of the other two cards are relevant, and therefore they shouldn't be flipped. Why are the other two cards irrelevant? That's easy: In the case of the B card, B is not a vowel, hence the antecedent V cannot be true; but the only way $V \to E$ can be false is if V is true. In the case of the 6 card, 6 isn't an odd number; but the only way $V \to E$ can be false is if E is false, i.e. if the number in question isn't even.

Q6 The following four statements are either all true or all false.

(a) If Alvin is happy, so is Betty.

(b) If Betty is happy, Charlie is too.

(c) If Charlie is happy, Darla is happy as well.

(d) Alvin is happy.

Does it follow deductively from the above information that Darla is happy? Prove that you're right, in significant part by employing the propositional calculus.

Solution

Yes. **Proof**: There are two cases to consider, and in both of them it can be deduced that Darla is happy. Case 1: Here, (a)–(d) are all true. Then, from A and $A \rightarrow B$ we can deduce by *modus ponens* that B, which with $B \rightarrow C$ by *modus ponens* yields C, which with $C \rightarrow D$ by *modus ponens* yields D. Case 2: In this case, all the statements are false. This means that $A \rightarrow B$ is false, which in turn — by the truth-table for the conditional means that A is true and B is false. But since all the statements are assumed in this second case to be false, $B \rightarrow C$ is false too, and from that by the same truth-table we then deduce that B is true (and C is false). This implies that we have a contradiction: $B \wedge \neg B$. By the rule of inference called 'explosion,' anything can be deduced, including then of course D, that Darla is happy. **QED**

Q7 For the following group of statements, rank the members of the group from least probable to more probable, and — using either the Venn-geometric account of probability (recommended), or the urn-based account — prove that your answer is correct.

1. Linda is a medical doctor or Linda is a pilot.
2. Linda is a medical doctor.
3. If Linda is not a medical doctor, then Linda is a pilot.
4. Linda is a medical doctor or Linda is a medical doctor.
5. Linda is a medical doctor and a pilot.
6. If Linda is not a pilot, then Linda is a medical doctor.

Solution

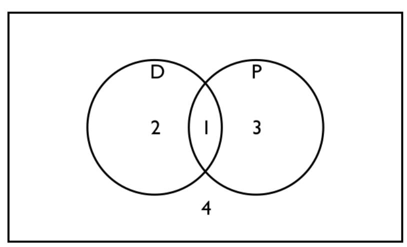
The first key thing to note and apply here is that those statements that are logically equivalent with each other will have exactly the same probabliity. Let's then first note which pairs of these statements are logically equivalent. Statement 1. is equivalent to statement 3., because $\neg D \to P$, a representation of 3. in the propositional calculus, will have exactly the same truth-table as $D \vee P$, a representation of 1. Next, statement 4., $D \vee D$, is logically equivalent to simply D, which means that 4. is logically equivalent to 2. Finally, statement 6. is logically equivalent to $P \vee D$, which is in turn logically equivalent to 1. (since order of disjuncts in a disjunction $\phi \vee \psi$ is irrelevant to the truth-value of that disjunction. Summing up, the upshot is that

$$prob(1.) = prob(3.)$$
$$prob(4.) = prob(2.)$$
$$prob(6.) = prob(1.)$$

In light of these identities, we can locate 1. in our continuum but drop 3. (remembering that 3. would just be where 1. is), locate 2. but drop 4. (remembering that 4. would just be where 2. is), and drop 6. (remembering that 6. would just be where 1. is.

Let's use the following Venn Diagram to complete the solution.

Geometric Interpretation of Probabilities



Assume that the areas of the regions in the diagram represent the probabilities of the formulae they correspond to. I.e.,

$prob(D)$ = size region 2
$prob(P)$ = size region 3
$prob(D \wedge P)$ = size of region 1
$prob(D \vee P)$ = size of region 2 plus region 3

While of course the sizing of the regions could be different, this one gives us the following, which wouldn't be fundamentally different in different size distributions.

$$prob(D \wedge P) < prob(D) = prob(P) < prob(D \vee P)$$

Q8a Think about the wise-man puzzle. Let Roger be a wise man in a puzzle of that variety. Suppose that Roger knows some proposition represented in the propositional calculus as ϕ. Suppose as well that $\{\phi\} \vdash_{\mathrm{PC}} \psi$. Does it follow that Roger knows ψ as well? Does it follow that ψ? Justify your answer.

Solution

Yes. In puzzles of this variety, a background assumption is that the clever people in them know all the propositions that can be deduced from what they know. (Whether or not this is plausible in the case of real humans is another matter.) And a second Yes: Whatever is known is true.

Q8b Susan, a math student, is given the following information to reason from in her geometry class, by her teacher: T is a right triangle. She is also told that T is an equilateral triangle. Susan then tells her teacher: "Well then, on the basis of what you've given me, it follows deductively that there is intelligent life on Mars." Is Susan correct? Why? Make reference to the propositional calculus in your answer.

Solution

Susan is correct. She has paid attention during her class and read the textbook, and hence is familiar with proof by contradiction (or indirect proof). (Recall the citation in class to Pearson's high-school Geometry textbook.) Specifically, Susan knows that everything follows from a contradiction. Put in terms of our collection of rules for the propositional calculus, Susan knows the rule called 'explosion.'

Q9 Susan is told by her math teacher that: If there's a king in the hand, the hand is a winning one. She is also told that: This conditional statement is false. Susan then tells her teacher: "Well then, I know that the hand is a losing one." Is Susan correct? Prove that she is by using the appropriate truth-table.

Solution

Susan is correct. Let's symbolize the relevant statement as $K \to W$. There is only one case when this conditional is false: viz.,

K	W	$K \to W$
T	F	F

It can be deduced from this table that W isn't the case, that is, that the hand, as Susan claims, is a losing one.