Rationality & Paradox, Part II: The Lottery Paradox

Selmer Bringsjord
Are Humans Rational?
10/29/15 & 11/2/15
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The Lottery Paradox

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Some Logistics:
Grades on Test 1 are rounded up.
Retake will be up on Monday, by class, & will then cover submission details.
Re-assessed Test 1s will be back on Monday.
Graded Test 2 will be returned and grades explained next Thursday.
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10/29/15 & 11/2/15
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- A perfectly rational person can never believe $P$ and believe $\neg P$ at the same time.
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- The Lottery Paradox shows, courtesy of its two Sequences (of Reasoning), that a perfectly rational person can indeed have such a belief (upon considering a fair, large lottery).
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- Sanguine defenders and would-be cultivators of potential and present rationality in *homo sapiens sapiens*, including specifically anyone who affirms $R$, would be shown to be incoherent babblers if a perfectly present-rational human can believe (at the same time) some $P$ and its negation $\neg P$. 
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• The Lottery Paradox shows, courtesy of its two Sequences, that a perfectly rational person can indeed have such a belief (upon considering a fair, large lottery).

• Ergo: Bringsjord et al. are incoherent babblers!
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- Because it can light the path forward in logic, mathematics, AI, etc.
Types of Paradoxes

- Deductive Paradoxes — corresponding to Area 1, going all the way back to our syllabus
- Inductive Paradoxes — corresponding to Area 2
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Today; Part II
The Lottery Paradox …
E: “Please go down to Stewart’s & get the T U.”
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S: “I’m sorry, that would be irrational.”
E: “Please go down to Stewart’s & get the T U.”

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E: “Please go down to Stewart’s & get the T U.”

S: “I’m sorry, that would be irrational.”

...
Sequence I
Sequence 1
Sequence 1
Sequence 1  Sequence 2
Sequence 1

Sequence 2
Contradiction; Selmer is an Incoherent Babbler
Sequence 1
Sequence I

Let D be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent a is fully apprised.
Sequence 1

Let $D$ be a meticulous and perfectly accurate description of a $1,000,000,000,000$-ticket lottery, of which rational agent $a$ is fully apprised.

From $D$ it obviously can be proved that either ticket 1 will win or ticket 2 will win or … or ticket $1,000,000,000,000$ will win. Let’s write this (exclusive) disjunction as follows:
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$$Wt_1 \oplus Wt_2 \oplus \ldots \oplus Wt_{1T} \quad (1)$$
Sequence I

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W_{t_1} \oplus W_{t_2} \oplus \ldots \oplus W_{t_{1T}} \quad (1)
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We then deduce from this that there is at least one ticket that will win, a proposition represented — using standard notation — as:
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$$\exists t_i W_{t_i} \quad (2)$$
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Very well; perfectly clear so far. And now we can add another deductive step: Since our rational agent $a$ can follow this deduction sequence to this point, and since $D$ is assumed to be indubitable, it follows that our rational agent in turn believes (2); i.e., we conclude Sequence 1 by obtaining the following:
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$$B_a \exists t_i Wt_i \quad (3)$$
Sequence 1
Sequence 1
Sequence 2
Sequence 2

As in Sequence 1, once again let $D$ be a meticulous and perfectly accurate description of a $1,000,000,000,000$-ticket lottery, of which rational agent $a$ is fully apprised.
Sequence 2

As in Sequence 1, once again let $D$ be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent $a$ is fully apprised.

From $D$ it obviously can be proved that the probability of a particular ticket $t_i$ winning is 1 in 1,000,000,000,000. Using ‘1T’ to denote 1 trillion, we can write the probability for each ticket to win as a conjunction:
Sequence 2

As in Sequence 1, once again let \( D \) be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent \( a \) is fully apprised.

From \( D \) it obviously can be proved that the probability of a particular ticket \( t_i \) winning is 1 in 1,000,000,000,000. Using ‘\( 1T \)’ to denote 1 trillion, we can write the probability for each ticket to win as a conjunction:

\[
\text{prob}(Wt_1) = \frac{1}{1,000,000,000,000} = \frac{1}{1T} \quad \text{and} \quad \text{prob}(Wt_2) = \frac{1}{1T} \quad \text{and} \quad \ldots \quad \text{and} \quad \text{prob}(Wt_{1T}) = \frac{1}{1T} \quad (1)
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$$\text{prob}(Wt_1) = \frac{1}{1,000,000,000,000} = \frac{1}{1T}$$

For the next step, note that the probability of ticket $t_i$ winning is lower than, say, the probability that if you walk outside a minute from now you will be flattened on the spot by a door from a 747 that falls from a jet of that type cruising at 35,000 feet. Since you have the rational belief that death won't ensue if you go outside (and have this belief precisely because you believe that the odds of your sudden demise in this manner are vanishingly small), the inference to the rational belief on the part of $a$ that $t_i$ won’t win sails through—and this of course works for each ticket. Hence we have:
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$$prob(Wt_1) = \frac{1}{1,000,000,000,000} = \frac{1}{1T} \land prob(Wt_2) = \frac{1}{1T} \land \ldots \land prob(Wt_{1T}) = \frac{1}{1T} \quad (1)$$

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$$B_a \neg Wt_1 \land B_a \neg Wt_2 \land \ldots \land B_a \neg Wt_{1T} \quad (2)$$
Sequence 2

As in Sequence 1, once again let \( D \) be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent \( a \) is fully apprised.

From \( D \) it obviously can be proved that the probability of a particular ticket \( t_i \) winning is 1 in 1,000,000,000,000. Using ‘1T’ to denote 1 trillion, we can write the probability for each ticket to win as a conjunction:

\[
\begin{align*}
\text{prob}(Wt_1) &= \frac{1}{1,000,000,000,000} = \frac{1}{1T} \\
\text{prob}(Wt_2) &= \frac{1}{1T} \\
&\quad \quad \quad \vdots \\
\text{prob}(Wt_{1T}) &= \frac{1}{1T} \quad (1)
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B_a \neg Wt_1 \land B_a \neg Wt_2 \land \ldots \land B_a \neg Wt_{1T} \quad (2)
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Of course, if a rational agent believes \( P \), and believes \( Q \) as well, it follows that that agent will believe the conjunction \( P \& Q \). Applying this principle to (2) yields:
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For the next step, note that the probability of ticket $t_i$ winning is lower than, say, the probability that if you walk outside a minute from now you will be flattened on the spot by a door from a 747 that falls from a jet of that type cruising at 35,000 feet. Since you have the rational belief that death won't ensue if you go outside (and have this belief precisely because you believe that the odds of your sudden demise in this manner are vanishingly small), the inference to the rational belief on the part of $a$ that $t_i$ won't win sails through—and this of course works for each ticket. Hence we have:

$$B_a(-Wt_1 \land -Wt_2 \land \ldots \land -Wt_{1T})$$  \hspace{1cm} (2)

Of course, if a rational agent believes $P$, and believes $Q$ as well, it follows that that agent will believe the conjunction $P \land Q$. Applying this principle to (2) yields:

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B_a \neg Wt_1 \land B_a \neg Wt_2 \land \ldots \land B_a \neg Wt_{1T} \tag{2}
\]

Of course, if a rational agent believes \( P \), and believes \( Q \) as well, it follows that that agent will believe the conjunction \( P \& Q \). Applying this principle to (2) yields:

\[
B_a(\neg Wt_1 \land \neg Wt_2 \land \ldots \land \neg Wt_{1T}) \tag{3}
\]

But (3) is logically equivalent to the statement that there doesn’t exist a winning ticket; i.e., (3) is logically equivalent to this result from Sequence 2:
Sequence 2
As in Sequence 1, once again let $D$ be a meticulous and perfectly accurate description of a $1,000,000,000,000,000$-ticket lottery, of which rational agent $a$ is fully apprised.

From $D$ it obviously can be proved that the probability of a particular ticket $t_i$ winning is 1 in $1,000,000,000,000$. Using ‘$1T$’ to denote 1 trillion, we can write the probability for each ticket to win as a conjunction:

$$\text{prob}(Wt_1) = \frac{1}{1,000,000,000,000,000} = \frac{1}{1T} \land \text{prob}(Wt_2) = \frac{1}{1T} \land \ldots \land \text{prob}(Wt_{1T}) = \frac{1}{1T} \quad (1)$$

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Of course, if a rational agent believes $P$, and believes $Q$ as well, it follows that that agent will believe the conjunction $P \land Q$. Applying this principle to (2) yields:

$$B_a (\neg Wt_1 \land \neg Wt_2 \land \ldots \land \neg Wt_{1T}) \quad (3)$$

But (3) is logically equivalent to the statement that there doesn't exist a winning ticket; i.e., (3) is logically equivalent to this result from Sequence 2:

$$B_a \neg \exists t_i Wt_i \quad (4)$$
Sequence 2
Sequence 1

\[ B_a \rightarrow \exists t_i W t_i \]

Sequence 2

\[ B_a \exists t_i W t_i \]
Sequence 1

Sequence 2

Contradiction; Selmer is an Incoherent Babbler
In our context, why study paradoxes?

- Because such study requires and develops System-2 cognition, quite possibly better than any other study.
- Because intellectual humility in humans is a good thing (and intellectual overconfidence is a bad thing), and such study is a road to intellectual humility (and an antidote to intellectual overconfidence).
- Because it can save the day, sometimes (Dr Who, Star Trek, etc).
- Because it can light the path forward in logic, mathematics, AI, etc.
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A Solution to The Lottery Paradox …
Strength-Factor Continuum

Certain

Probable

Evidently False

Beyond Reasonable Belief

Certainly False

Counterbalanced

Beyond Reasonable Doubt

Evident

Improbable
Strength-Factor Continuum

Certain
Evident
Beyond Reasonable Doubt
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Evidently False
Certainly False
Strength-Factor Continuum

Certain
Evident
Beyond Reasonable Doubt
Probable
Counterbalanced
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Evidently False
Certainly False
Strength-Factor Continuum

- Certain
- Evident
- Beyond Reasonable Doubt
- Probable
- Counterbalanced
- Improbable
- Beyond Reasonable Belief
- Evidently False
- Certainly False
Strength-Factor Continuum

Epistemically Positive

Certainty Continuum

Certain
Evident
Beyond Reasonable Doubt
Probable

Counterbalanced Continuum

Improbable
Beyond Reasonable Belief
Evidently False
Certainly False
Strength-Factor Continuum

Epistemically Positive

Epistemically Negative

- Certain
- Evident
- Beyond Reasonable Doubt
- Probable
- Counterbalanced
- Improbable
- Beyond Reasonable Belief
- Evidently False
- Certainly False
Strength-Factor Continuum

Epistemically Positive

Certain
Evident
Beyond Reasonable Doubt
Probable
Counterbalanced
Improbable
Beyond Reasonable Belief
Evidently False
Certainly False

Epistemically Negative
Strength-Factor Continuum

(4) Certain
(3) Evident
(2) Beyond Reasonable Doubt
(1) Probable
(0) Counterbalanced
(-1) Improbable
(-2) Beyond Reasonable Belief
(-3) Evidently False
(-4) Certainly False

Epistemically Positive

Epistemically Negative
Strength-Factor Continuum

Epistemically Positive

(4) Certain
(3) Evident
(2) Beyond Reasonable Doubt
(1) Probable
(0) Counterbalanced
(-1) Improbable
(-2) Beyond Reasonable Belief
(-3) Evidently False
(-4) Certainly False

Epistemically Negative
(4) Certain
(3) Evident
(2) Beyond Reasonable Doubt
(1) Probable
(0) Counterbalanced
(-1) Improbable
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Key Principles

Epistemically Positive

(4) Certain
(3) Evident
(2) Beyond Reasonable Doubt
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(0) Counterbalanced

Epistemically Negative

(-1) Improbable
(-2) Beyond Reasonable Belief
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Key Principles

Deduction preserves strength.

Epistemically Positive

(4) Certain
(3) Evident
(2) Beyond Reasonable Doubt
(1) Probable
(0) Counterbalanced
(-1) Improbable
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Epistemically Negative
Key Principles

Deduction preserves strength.

Clashes are resolved in favor of higher strength.
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Any proposition $p$ such that $\text{prob}(p) < 1$ is at most evident.
Key Principles

**Epistemically Positive**

- (4) Certain
- (3) Evident
- (2) Beyond Reasonable Doubt
- (1) Probable

**Epistemically Negative**

- (0) Counterbalanced
- (-1) Improbable
- (-2) Beyond Reasonable Belief
- (-3) Evidently False
- (-4) Certainly False

Deduction preserves strength.

Clashes are resolved in favor of higher strength.

Any proposition $p$ such that $\text{prob}(p) < 1$ is at most evident.

Any rational belief that $p$, where the basis for $p$ is at most evident, is at most an evident (= level 3) belief.
Sequence 1, “Rigorized”

Let $D$ be a meticulous and perfectly accurate description of a $1,000,000,000,000$-ticket lottery, of which rational agent $a$ is fully apprised.

From $D$ it obviously can be proved that either ticket 1 will win or ticket 2 will win or … or ticket $1,000,000,000,000$ will win. Let’s write this (exclusive) disjunction as follows:

$$Wt_1 \oplus Wt_2 \oplus \ldots \oplus Wt_{1T} \quad (1)$$

We then deduce from this that there is at least one ticket that will win, a proposition represented — using standard notation — as:

$$\exists t_i Wt_i \quad (2)$$

Very well; perfectly clear so far. And now we can add another deductive step: Since our rational agent $a$ can follow this deduction sequence to this point, and since $D$ is assumed to be indubitable, it follows that our rational agent in turn believes (2); i.e., we conclude Sequence 1 by obtaining the following:

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Sequence 1, “Rigorized”

Let $D$ be a meticulous and perfectly accurate description of a $1,000,000,000,000$-ticket lottery, of which rational agent $a$ is fully apprised.

From $D$ it obviously can be proved that either ticket 1 will win or ticket 2 will win or ... or ticket $1,000,000,000,000$ will win. Let’s write this (exclusive) disjunction as follows:

$$Wt_1 \oplus Wt_2 \oplus \ldots \oplus Wt_{1T} \quad (1)$$

We then deduce from this that there is at least one ticket that will win, a proposition represented — using standard notation — as:

$$\exists t_i Wt_i \quad (2)$$

Very well; perfectly clear so far. And now we can add another deductive step: Since our rational agent $a$ can follow this deduction sequence to this point, and since $D$ is assumed to be indubitable, it follows that our rational agent in turn believes $(2)$; i.e., we conclude Sequence 1 by obtaining the following:

$$\mathbb{B}_a \exists t_i Wt_i \quad (3)$$
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From D it obviously can be proved that either ticket 1 will win or ticket 2 will win or ... or ticket 1,000,000,000,000 will win. Let’s write this (exclusive) disjunction as follows:

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$$B^4_a \exists t_i W_{t_i} \quad (3)$$
As in Sequence 1, once again let \( D \) be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent \( a \) is fully apprised.

From \( D \) it obviously can be proved that the probability of a particular ticket \( t_i \) winning is 1 in 1,000,000,000,000. Using ‘1\( T \)’ to denote 1 trillion, we can write the probability for each ticket to win as a conjunction:

\[
prob(Wt_1) = \frac{1}{1,000,000,000,000,000} = \frac{1}{1T} \land prob(Wt_2) = \frac{1}{1T} \land \ldots \land prob(Wt_{1T}) = \frac{1}{1T} \quad (1)
\]

For the next step, note that the probability of ticket \( t_1 \) winning is lower than, say, the probability that if you walk outside a minute from now you will be flattened on the spot by a door from a 747 that falls from a jet of that type cruising at 35,000 feet. Since you have the rational belief that death won’t ensue if you go outside (and have this belief precisely because you believe that the odds of your sudden demise in this manner are vanishingly small), the inference to the rational belief on the part of \( a \) that \( t_1 \) won’t win sails through—and this of course works for each ticket. Hence we have:

\[
\mathbf{B}_a \neg Wt_1 \land \mathbf{B}_a \neg Wt_2 \land \ldots \land \mathbf{B}_a \neg Wt_{1T} \quad (2)
\]

Of course, if a rational agent believes \( P \), and believes \( Q \) as well, it follows that that agent will believe the conjunction \( P \land Q \). Applying this principle to (2) yields:

\[
\mathbf{B}_a (\neg Wt_1 \land \neg Wt_2 \land \ldots \land \neg Wt_{1T}) \quad (3)
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But (3) is logically equivalent to the statement that there doesn’t exist a winning ticket; i.e., (3) is logically equivalent to this result from Sequence 2:

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Of course, if a rational agent believes $P$, and believes $Q$ as well, it follows that that agent will believe the conjunction $P \land Q$. Applying this principle to (2) yields:

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Sequence 2, “Rigorized”

As in Sequence 1, once again let $D$ be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent $a$ is fully apprised.

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$$B_a \neg \exists t_i Wt_i \quad (4)$$
Sequence 2, “Rigorized”

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$$
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$$

Of course, if a rational agent believes $P$, and believes $Q$ as well, it follows that that agent will believe the conjunction $P \land Q$. Applying this principle to (2) yields:

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B^3_a (\neg Wt_1 \land \neg Wt_2 \land \ldots \land \neg Wt_{1T}) \quad (3)
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$$\mathbf{B}_a^3 \neg W_{t_1} \land \mathbf{B}_a^3 \neg W_{t_2} \land \ldots \land \mathbf{B}_a^3 \neg W_{t_{1T}} \quad (2)$$

Of course, if a rational agent believes $P$, and believes $Q$ as well, it follows that that agent will believe the conjunction $P \& Q$. Applying this principle to (2) yields:

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$$\mathbf{B}_a^3 \neg \exists t_i W_{t_i} \quad (4)$$
Paradox Solved!

Deduction preserves strength.

Clashes are resolved in favor of higher strength; clashes propagate backwards through inverse deduction.

Any proposition $p$ such that $\text{prob}(p) < 1$ is at most evident.

Any rational belief that $p$, where the basis for $p$ is at most evident, is at most an evident (= level 3) belief.
Paradox Solved!

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\[ B^4_a \exists t_i \neg W t_i \] (3)

Any proposition \( p \) such that \( \text{prob}(p) < 1 \) is at most evident.

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\[ B_{aa}^{43} t_i \exists W_i W t_i \] (3)(4)

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\[ B^4_a \exists t_i W t_i \]  \hspace{1cm} (3)

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\[ B^3_a \neg W t_1 \land B^3_a \neg W t_2 \land \ldots \land B^3_a \neg W t_1 T \]  \hspace{1cm} (2)
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Deduction preserves strength.

Clashes are resolved in favor of higher strength; clashes propagate backwards through inverse deduction.

\[ B_{a}^{4} \exists t_{i} W t_{i} \quad (3) \]

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\[ B_{a}^{3} \neg W t_{1} \land B_{a}^{3} \neg W t_{2} \land \ldots \land B_{a}^{3} \neg W t_{1T} \quad (2) \]

This is why, to Mega Millions ticket holder:
“Sorry. I’m rational, and I believe you won’t win.”
slutten