The Lottery Paradox

(& The St Petersburg Paradox too!)

Selmer Bringsjord & Mike Giancola

AHR?

Rensselaer AI and Reasoning Lab

The Paradoxes: Our Coverage, & RPI Context

The Paradoxes: Our Coverage, & RPI Context

The Liar: You're on your own :). Presented 10/21/19.

The Paradoxes: Our Coverage, & RPI Context

The Liar: You're on your own :). Presented 10/21/19.

The Barber: Flat-out solved, period (via ZFC foundation for math); take *Intro to Logic*. Presented 10/21/19.

The Paradoxes: Our Coverage, & RPI Context

The Liar: You're on your own :). Presented 10/21/19.

The Barber: Flat-out solved, period (via ZFC foundation for math); take *Intro to Logic*. Presented 10/21/19.

[**The Knowability Paradox**: Active area of sponsored r&d for RAIR Lab. Will be in a documentary film on AI based in part on a visit to RAIR Lab. Offered 10/21/19 (w/ optional content.)]

The Paradoxes: Our Coverage, & RPI Context

The Liar: You're on your own :). Presented 10/21/19.

The Barber: Flat-out solved, period (via ZFC foundation for math); take *Intro to Logic*. Presented 10/21/19.

[**The Knowability Paradox**: Active area of sponsored r&d for RAIR Lab. Will be in a documentary film on AI based in part on a visit to RAIR Lab. Offered 10/21/19 (w/ optional content.)]

Newcomb's Problem: Active area of *on-spec* r&d for RAIR Lab. Gist of Selmer's proposed solution conveyed (10/28/19).

The Paradoxes: Our Coverage, & RPI Context

The Liar: You're on your own :). Presented 10/21/19.

The Barber: Flat-out solved, period (via ZFC foundation for math); take *Intro to Logic*. Presented 10/21/19.

[**The Knowability Paradox**: Active area of sponsored r&d for RAIR Lab. Will be in a documentary film on AI based in part on a visit to RAIR Lab. Offered 10/21/19 (w/ optional content.)]

Newcomb's Problem: Active area of *on-spec* r&d for RAIR Lab. Gist of Selmer's proposed solution conveyed (10/28/19).

The Lottery Paradox: Solved by RAIR Lab! Today's class (10/31/19). And the St. Petersburg Paradox too?

The Paradoxes: Our Coverage, & RPI Context

The Liar: You're on your own :). Presented 10/21/19.

The Barber: Flat-out solved, period (via ZFC foundation for math); take *Intro to Logic*. Presented 10/21/19.

[**The Knowability Paradox**: Active area of sponsored r&d for RAIR Lab. Will be in a documentary film on AI based in part on a visit to RAIR Lab. Offered 10/21/19 (w/ optional content.)]

Newcomb's Problem: Active area of *on-spec* r&d for RAIR Lab. Gist of Selmer's proposed solution conveyed (10/28/19).

Today

The Lottery Paradox: Solved by RAIR Lab! Today's class (10/31/19). And the St. Petersburg Paradox too?

Today

The Paradoxes: Our Coverage, & RPI Context

The Liar: You're on your own :). Presented 10/21/19.

The Barber: Flat-out solved, period (via ZFC foundation for math); take *Intro to Logic*. Presented 10/21/19.

[**The Knowability Paradox**: Active area of sponsored r&d for RAIR Lab. Will be in a documentary film on AI based in part on a visit to RAIR Lab. Offered 10/21/19 (w/ optional content.)]

Newcomb's Problem: Active area of *on-spec* r&d for RAIR Lab. Gist of Selmer's proposed solution conveyed (10/28/19).

Today

The Lottery Paradox: Solved by RAIR Lab! Today's class (10/31/19). And the St. Petersburg Paradox too?

Today

The Paradoxes: Our Coverage, & RPI Context

The Liar: You're on your own :). Presented 10/21/19.

The Barber: Flat-out solved, period (via ZFC foundation for math); take *Intro to Logic*. Presented 10/21/19.

[**The Knowability Paradox**: Active area of sponsored r&d for RAIR Lab. Will be in a documentary film on AI based in part on a visit to RAIR Lab. Offered 10/21/19 (w/ optional content.)]

Newcomb's Problem: Active area of *on-spec* r&d for RAIR Lab. Gist of Selmer's proposed solution conveyed (10/28/19).

Today

The Lottery Paradox: Solved by RAIR Lab! Today's class (10/31/19). And the St. Petersburg Paradox too?

Today

The Paradoxes of Time Travel: Grandfather & "Looping Painter": Painter is an active research area, sponsored by Air Force; Naveen Sundar G., S Bringsjord. Covered 11/4/19.

Types of Paradoxes

- Deductive Paradoxes — corresponding to Area 1, going all the way back to our syllabus
- Inductive Paradoxes — corresponding to Area 2 (i.e. probability etc.)

Types of Paradoxes

- Deductive Paradoxes — corresponding to Area 1, going all the way back to our syllabus
- Inductive Paradoxes — corresponding to Area 2 (i.e. probability etc.)

Today (x 2)

Inductive-Reasoning Example from Pollock — for Peek Ahead

Imagine the following:

Keith tells you that the morning news predicts rain in Troy today. However, Alvin tells you that the same news report predicted sunshine.

Imagine the following:

Keith tells you that the morning news predicts rain in Troy today. However, Alvin tells you that the same news report predicted sunshine.

Without any other source of information, it would be irrational to place belief in either Keith's or Alvin's statements.

Imagine the following:

Keith tells you that the morning news predicts rain in Troy today. However, Alvin tells you that the same news report predicted sunshine.

Without any other source of information, it would be irrational to place belief in either Keith's or Alvin's statements.

Further, suppose you happened to watch the noon news report, and that report predicted rain. Then you should believe that it will rain despite your knowledge of Alvin's argument.

Cast in *Inference Graphs* for Pollock's OSCAR

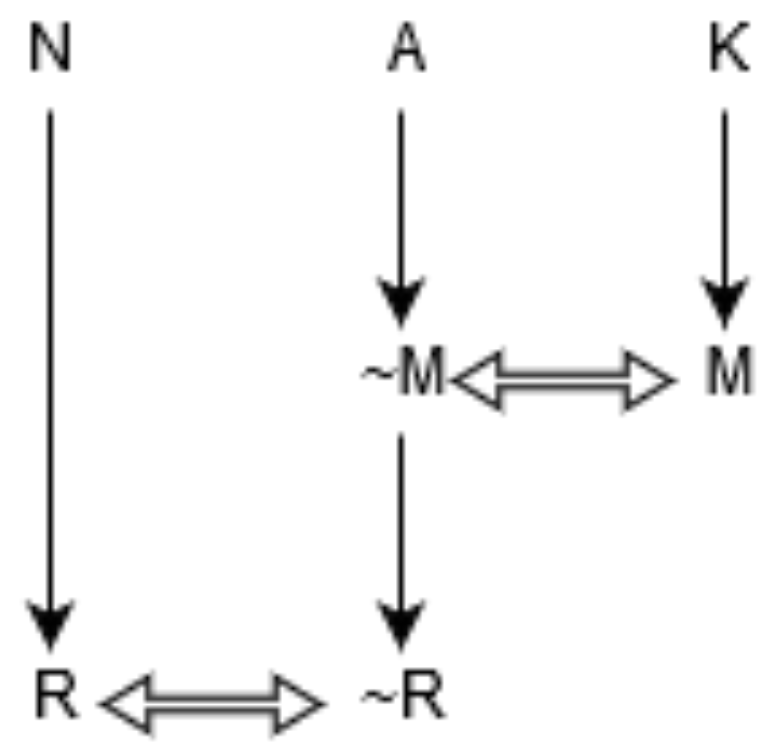
K- Keith says that M

A- Alvin says that ~M

M- The morning news said that R

R- It is going to rain this afternoon

N- The noon news says that R



All such can be absorbed into and refined in our inductive logics and our automated inductive reasoners for argument adjudication.

Relevant Inference Schemata

(simplified)

$$\frac{\mathbf{K}\phi}{\phi} \quad I_1$$

$$\frac{\mathbf{S}(\mathfrak{a}_1, \phi, \mathfrak{a}_2), \quad \chi(\mathfrak{a}_1)}{\mathbf{B}^2(\mathfrak{a}_2, \phi)} \quad I_2 \qquad\qquad \frac{\mathbf{S}(\mathfrak{a}_1, \phi, \mathfrak{a}_2), \quad \chi'(\mathfrak{a}_1)}{\mathbf{B}^3(\mathfrak{a}_2, \phi)} \quad I_{2'}$$

$$\frac{\mathbf{B}^2(\mathfrak{a}, \phi, t), \quad \mathbf{B}^2(\mathfrak{a}, \neg\phi, t)}{\neg\mathbf{B}^2(\mathfrak{a}, \phi, t) \ \wedge \ \neg\mathbf{B}^2(\mathfrak{a}, \neg\phi, t)} \quad \text{Clash Principle}$$

χ, χ' : preconditions

$$\mathbf{K}(you, \mathbf{S}(keith, \mathbf{S}(m, rain))) \quad \mathbf{K}(you, \mathbf{S}(alvin, \mathbf{S}(m, \neg rain)))$$

$$\mathbf{K}(you, \mathbf{S}(keith, \mathbf{S}(m, rain))) \qquad \mathbf{K}(you, \mathbf{S}(alvin, \mathbf{S}(m, \neg rain)))$$

$$\Big\downarrow \text{ via } I_1 \qquad\qquad \Big\downarrow \text{ via } I_1$$

$$\mathbf{K}(you, \mathbf{S}(keith, \mathbf{S}(m, rain))) \xrightarrow{\text{via } I_1} \mathbf{S}(keith, \mathbf{S}(m, rain))$$

$$\mathbf{K}(you, \mathbf{S}(alvin, \mathbf{S}(m, \neg rain))) \xrightarrow{\text{via } I_1} \mathbf{S}(alvin, \mathbf{S}(m, \neg rain))$$

$$\begin{tikzcd} \mathbf{K}(you, \mathbf{S}(keith, \mathbf{S}(m, rain))) \arrow[d, "\text{via } I_1"] \\ \mathbf{S}(keith, \mathbf{S}(m, rain)) \arrow[d, "\text{via } I_2"] \\ {} \end{tikzcd} \qquad \begin{tikzcd} \mathbf{K}(you, \mathbf{S}(alvin, \mathbf{S}(m, \neg rain))) \arrow[d, "\text{via } I_1"] \\ \mathbf{S}(alvin, \mathbf{S}(m, \neg rain)) \arrow[d, "\text{via } I_2"] \\ {} \end{tikzcd}$$

$$
\begin{array}{ccc}
\mathbf{K}(you, \mathbf{S}(keith, \mathbf{S}(m, rain))) & & \mathbf{K}(you, \mathbf{S}(alvin, \mathbf{S}(m, \neg rain))) \\
\big\downarrow \text{ via } I_1 & & \big\downarrow \text{ via } I_1 \\
\mathbf{S}(keith, \mathbf{S}(m, rain)) & & \mathbf{S}(alvin, \mathbf{S}(m, \neg rain)) \\
\big\downarrow \text{ via } I_2 & & \big\downarrow \text{ via } I_2 \\
\mathbf{B}^2(you, \mathbf{S}(m, rain)) & \wedge & \mathbf{B}^2(you, \mathbf{S}(m, \neg rain))
\end{array}
$$

$$\begin{array}{ccc}
\mathbf{K}(you, \mathbf{S}(keith, \mathbf{S}(m, rain))) & & \mathbf{K}(you, \mathbf{S}(alvin, \mathbf{S}(m, \neg rain))) \\
\Big\downarrow \text{ via } I_1 & & \Big\downarrow \text{ via } I_1 \\
\mathbf{S}(keith, \mathbf{S}(m, rain)) & & \mathbf{S}(alvin, \mathbf{S}(m, \neg rain)) \\
\Big\downarrow \text{ via } I_2 & & \Big\downarrow \text{ via } I_2 \\
\mathbf{B}^2(you, \mathbf{S}(m, rain)) & \wedge & \mathbf{B}^2(you, \mathbf{S}(m, \neg rain)) \\
 & \Big\downarrow & \\
 & \textbf{Clash !} &
\end{array}$$

$$
\begin{array}{ccc}
\mathbf{K}(you, \mathbf{S}(keith, \mathbf{S}(m, rain))) & & \mathbf{K}(you, \mathbf{S}(alvin, \mathbf{S}(m, \neg rain))) \\
\downarrow \text{ via } I_1 & & \downarrow \text{ via } I_1 \\
\mathbf{S}(keith, \mathbf{S}(m, rain)) & & \mathbf{S}(alvin, \mathbf{S}(m, \neg rain)) \\
\downarrow \text{ via } I_2 & & \downarrow \text{ via } I_2 \\
\mathbf{B}^2(you, \mathbf{S}(m, rain)) & \wedge & \mathbf{B}^2(you, \mathbf{S}(m, \neg rain)) \\
& \downarrow &
\end{array}
$$

Clash !

$$
\neg \mathbf{B}^2(you, \mathbf{S}(m, rain)) \quad \wedge \quad \neg \mathbf{B}^2(you, \mathbf{S}(m, \neg rain))
$$

$$\begin{array}{ccc}
\mathbf{K}(you, \mathbf{S}(keith, \mathbf{S}(m, rain))) & & \mathbf{K}(you, \mathbf{S}(alvin, \mathbf{S}(m, \neg rain))) \\
\downarrow \text{ via } I_1 & & \downarrow \text{ via } I_1 \\
\mathbf{S}(keith, \mathbf{S}(m, rain)) & & \mathbf{S}(alvin, \mathbf{S}(m, \neg rain)) \\
\downarrow \text{ via } I_2 & & \downarrow \text{ via } I_2 \\
\mathbf{B}^2(you, \mathbf{S}(m, rain)) & \wedge & \mathbf{B}^2(you, \mathbf{S}(m, \neg rain)) \\
 & \downarrow &
\end{array}$$

Clash !

$$\neg\mathbf{B}^2(you, \mathbf{S}(m, rain)) \quad \wedge \quad \neg\mathbf{B}^2(you, \mathbf{S}(m, \neg rain))$$

$$\mathbf{K}(you, \mathbf{S}(noonnews, rain))$$

$$\begin{array}{ccc}
\mathbf{K}(you, \mathbf{S}(keith, \mathbf{S}(m, rain))) & & \mathbf{K}(you, \mathbf{S}(alvin, \mathbf{S}(m, \neg rain))) \\
\downarrow \text{via } I_1 & & \downarrow \text{via } I_1 \\
\mathbf{S}(keith, \mathbf{S}(m, rain)) & & \mathbf{S}(alvin, \mathbf{S}(m, \neg rain)) \\
\downarrow \text{via } I_2 & & \downarrow \text{via } I_2 \\
\mathbf{B}^2(you, \mathbf{S}(m, rain)) & \wedge & \mathbf{B}^2(you, \mathbf{S}(m, \neg rain)) \\
& \downarrow &
\end{array}$$

Clash !

$$\neg\mathbf{B}^2(you, \mathbf{S}(m, rain)) \quad \wedge \quad \neg\mathbf{B}^2(you, \mathbf{S}(m, \neg rain))$$

$$\begin{array}{c}
\mathbf{K}(you, \mathbf{S}(noonnews, rain)) \\
\downarrow \text{via } I_1
\end{array}$$

$$
\begin{array}{ccc}
\mathbf{K}(you, \mathbf{S}(keith, \mathbf{S}(m, rain))) & & \mathbf{K}(you, \mathbf{S}(alvin, \mathbf{S}(m, \neg rain))) \\
\downarrow \text{ via } I_1 & & \downarrow \text{ via } I_1 \\
\mathbf{S}(keith, \mathbf{S}(m, rain)) & & \mathbf{S}(alvin, \mathbf{S}(m, \neg rain)) \\
\downarrow \text{ via } I_2 & & \downarrow \text{ via } I_2 \\
\mathbf{B}^2(you, \mathbf{S}(m, rain)) & \wedge & \mathbf{B}^2(you, \mathbf{S}(m, \neg rain)) \\
 & \downarrow &
\end{array}
$$

Clash !

$$
\neg \mathbf{B}^2(you, \mathbf{S}(m, rain)) \quad \wedge \quad \neg \mathbf{B}^2(you, \mathbf{S}(m, \neg rain))
$$

$$
\begin{array}{c}
\mathbf{K}(you, \mathbf{S}(noonnews, rain)) \\
\downarrow \text{ via } I_1 \\
\mathbf{S}(noonnews, rain)
\end{array}
$$

$$\begin{array}{ccc}
\mathbf{K}(you, \mathbf{S}(keith, \mathbf{S}(m, rain))) & & \mathbf{K}(you, \mathbf{S}(alvin, \mathbf{S}(m, \neg rain))) \\
\Big\downarrow \text{ via } I_1 & & \Big\downarrow \text{ via } I_1 \\
\mathbf{S}(keith, \mathbf{S}(m, rain)) & & \mathbf{S}(alvin, \mathbf{S}(m, \neg rain)) \\
\Big\downarrow \text{ via } I_2 & & \Big\downarrow \text{ via } I_2 \\
\mathbf{B}^2(you, \mathbf{S}(m, rain)) & \wedge & \mathbf{B}^2(you, \mathbf{S}(m, \neg rain)) \\
& \Big\downarrow &
\end{array}$$

Clash !

$$\neg\mathbf{B}^2(you, \mathbf{S}(m, rain)) \quad \wedge \quad \neg\mathbf{B}^2(you, \mathbf{S}(m, \neg rain))$$

$$\begin{array}{c}
\mathbf{K}(you, \mathbf{S}(noonnews, rain)) \\
\Big\downarrow \text{ via } I_1 \\
\mathbf{S}(noonnews, rain) \\
\Big\downarrow \text{ via } I_{2'}
\end{array}$$

$$\begin{array}{ccc}
\mathbf{K}(\mathit{you}, \mathbf{S}(\mathit{keith}, \mathbf{S}(m, \mathit{rain}))) & & \mathbf{K}(\mathit{you}, \mathbf{S}(\mathit{alvin}, \mathbf{S}(m, \neg \mathit{rain}))) \\
\big\downarrow \text{ via } I_1 & & \big\downarrow \text{ via } I_1 \\
\mathbf{S}(\mathit{keith}, \mathbf{S}(m, \mathit{rain})) & & \mathbf{S}(\mathit{alvin}, \mathbf{S}(m, \neg \mathit{rain})) \\
\big\downarrow \text{ via } I_2 & & \big\downarrow \text{ via } I_2 \\
\mathbf{B}^2(\mathit{you}, \mathbf{S}(m, \mathit{rain})) & \wedge & \mathbf{B}^2(\mathit{you}, \mathbf{S}(m, \neg \mathit{rain}))
\end{array}$$

$$\big\downarrow$$

Clash !

$$\neg \mathbf{B}^2(\mathit{you}, \mathbf{S}(m, \mathit{rain})) \quad \wedge \quad \neg \mathbf{B}^2(\mathit{you}, \mathbf{S}(m, \neg \mathit{rain}))$$

$$\begin{array}{c}
\mathbf{K}(\mathit{you}, \mathbf{S}(\mathit{noonnews}, \mathit{rain})) \\
\big\downarrow \text{ via } I_1 \\
\mathbf{S}(\mathit{noonnews}, \mathit{rain}) \\
\big\downarrow \text{ via } I_{2'} \\
\mathbf{B}^3(\mathit{you}, \mathit{rain})
\end{array}$$

The Lottery Paradox

The Lottery Paradox

- A perfectly rational person can never believe P and believe ¬P at the same time.

The Lottery Paradox

- A perfectly rational person can never believe P and believe $\neg P$ at the same time.
- The Lottery Paradox (apparently) shows, courtesy of its two Sequences (of Reasoning), that a perfectly rational person can indeed have such a belief (upon considering a fair, large lottery).

The Lottery Paradox

- A perfectly rational person can never believe P and believe ¬P at the same time.
- The Lottery Paradox (apparently) shows, courtesy of its two Sequences (of Reasoning), that a perfectly rational person can indeed have such a belief (upon considering a fair, large lottery).
- Contradiction! — and hence a paradox!

Stewart's Shops

WE ARE CLOSER TO YOU!

E: “Please go down to Stewart’s & get the T U.”

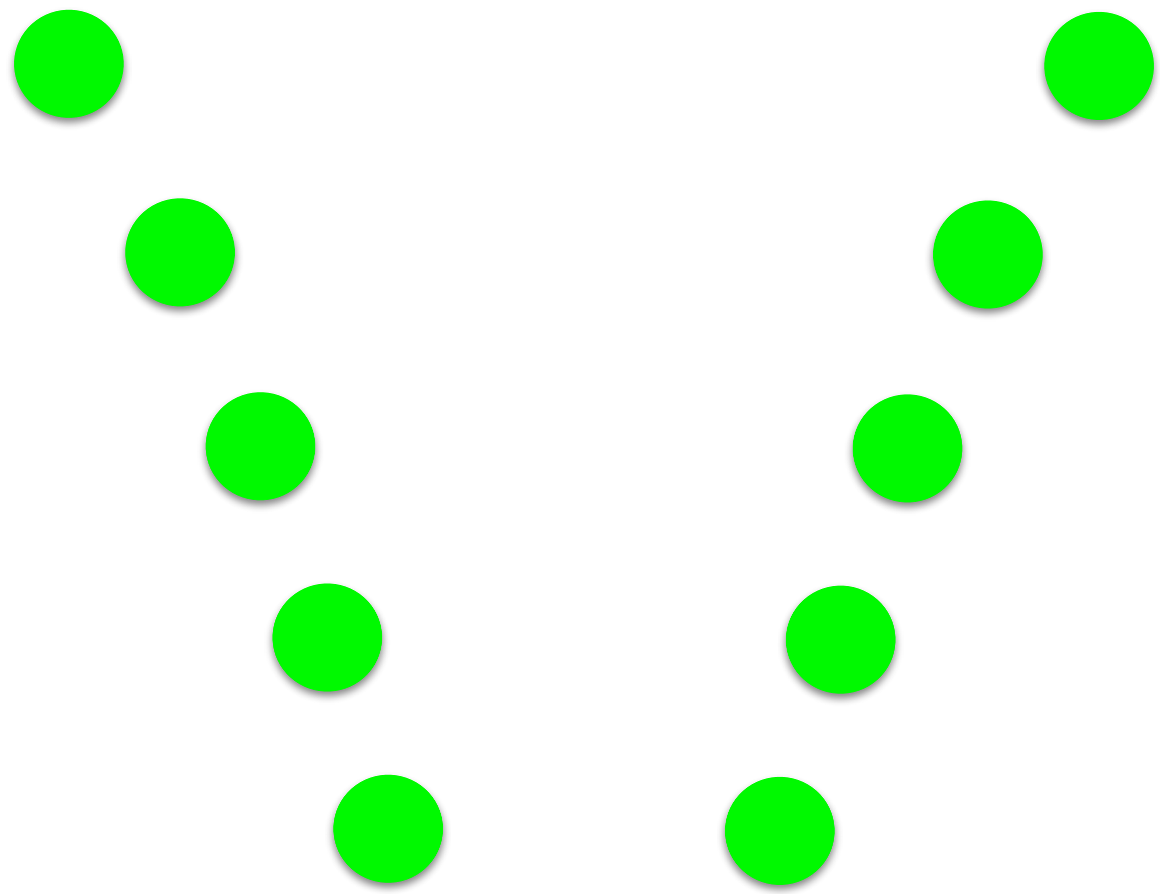
E: "Please go down to Stewart's & get the T U."

S: "I'm sorry, E, I'm afraid I can't do that.
It would be irrational."

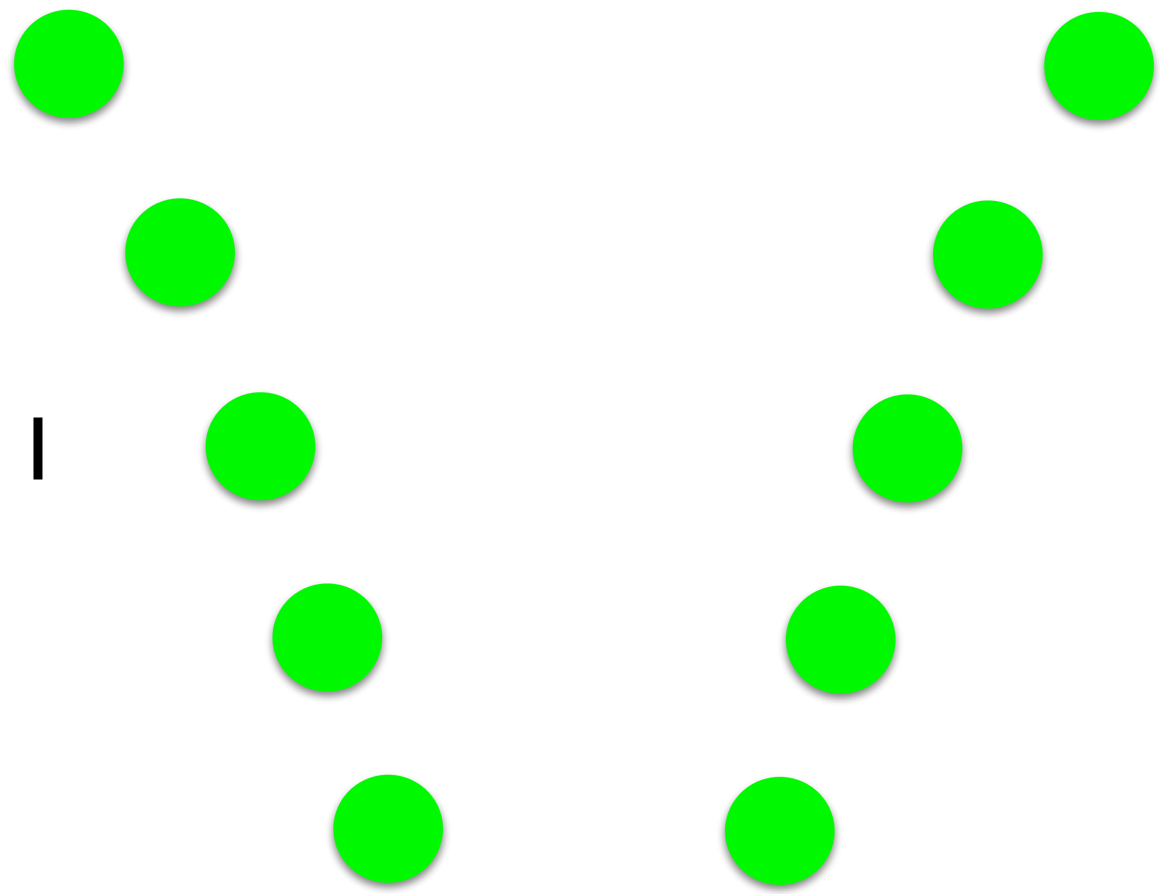
E: “Please go down to Stewart’s & get the T U.”

S: “I’m sorry, E, I’m afraid I can’t do that.
It would be irrational.”

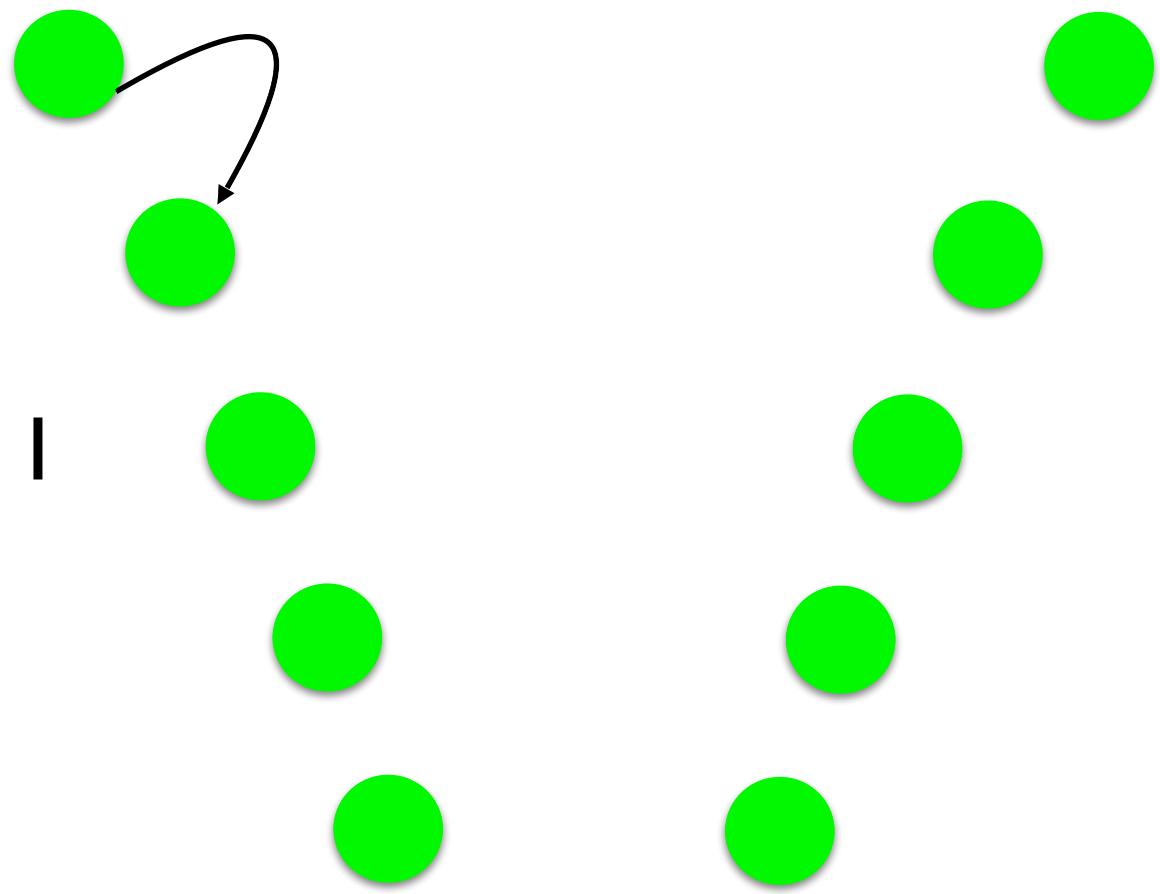
•••

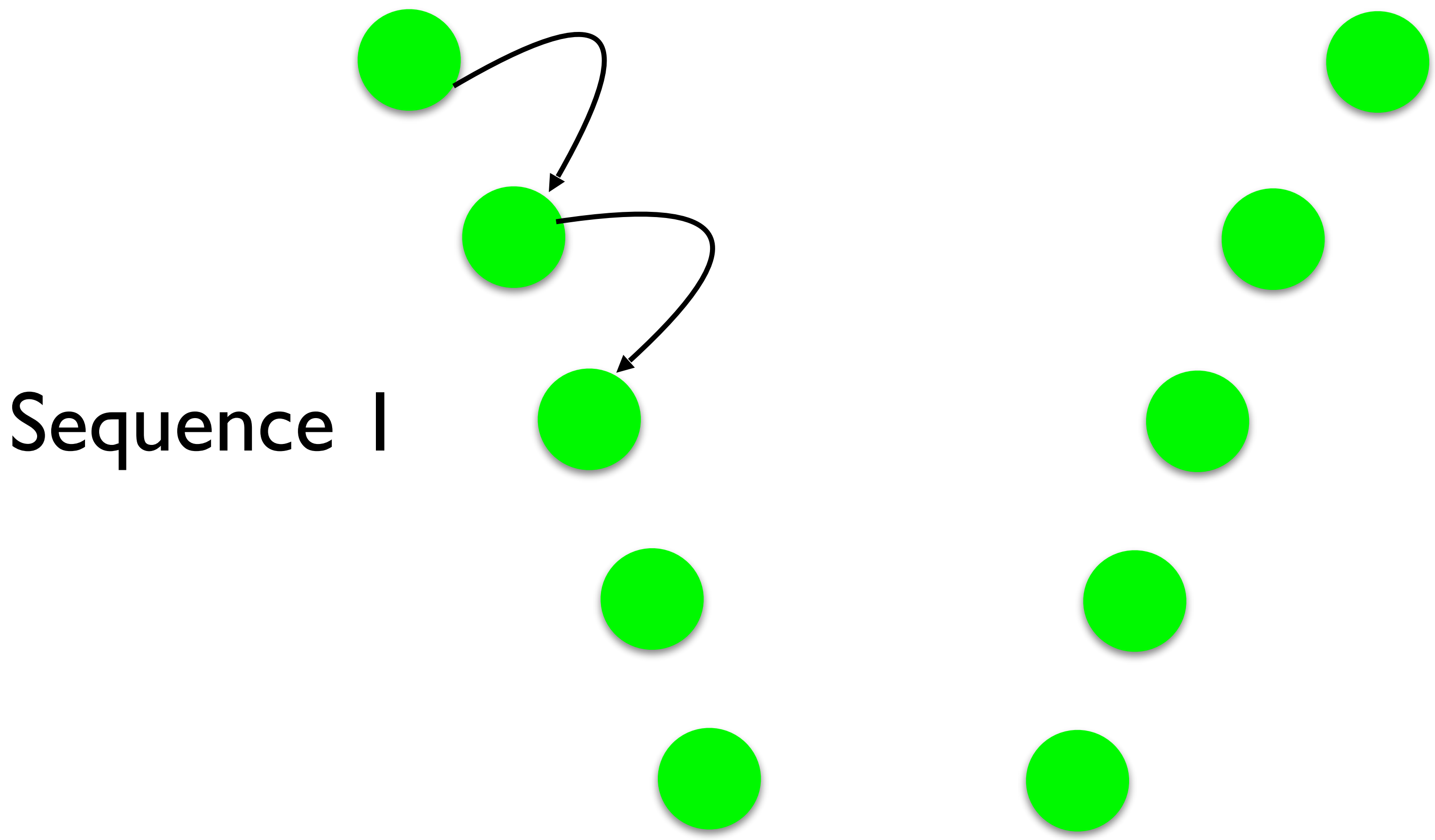


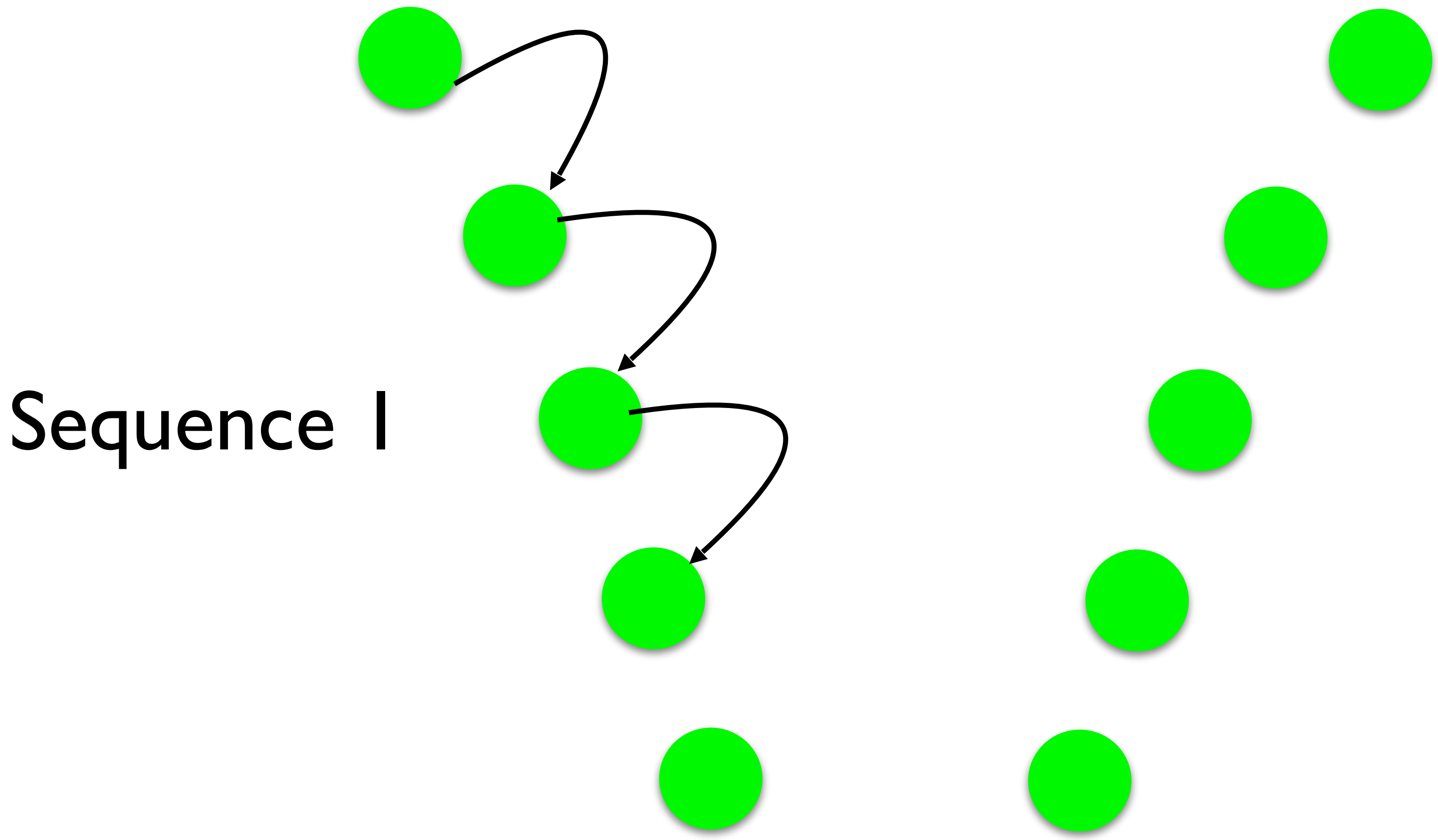
Sequence 1



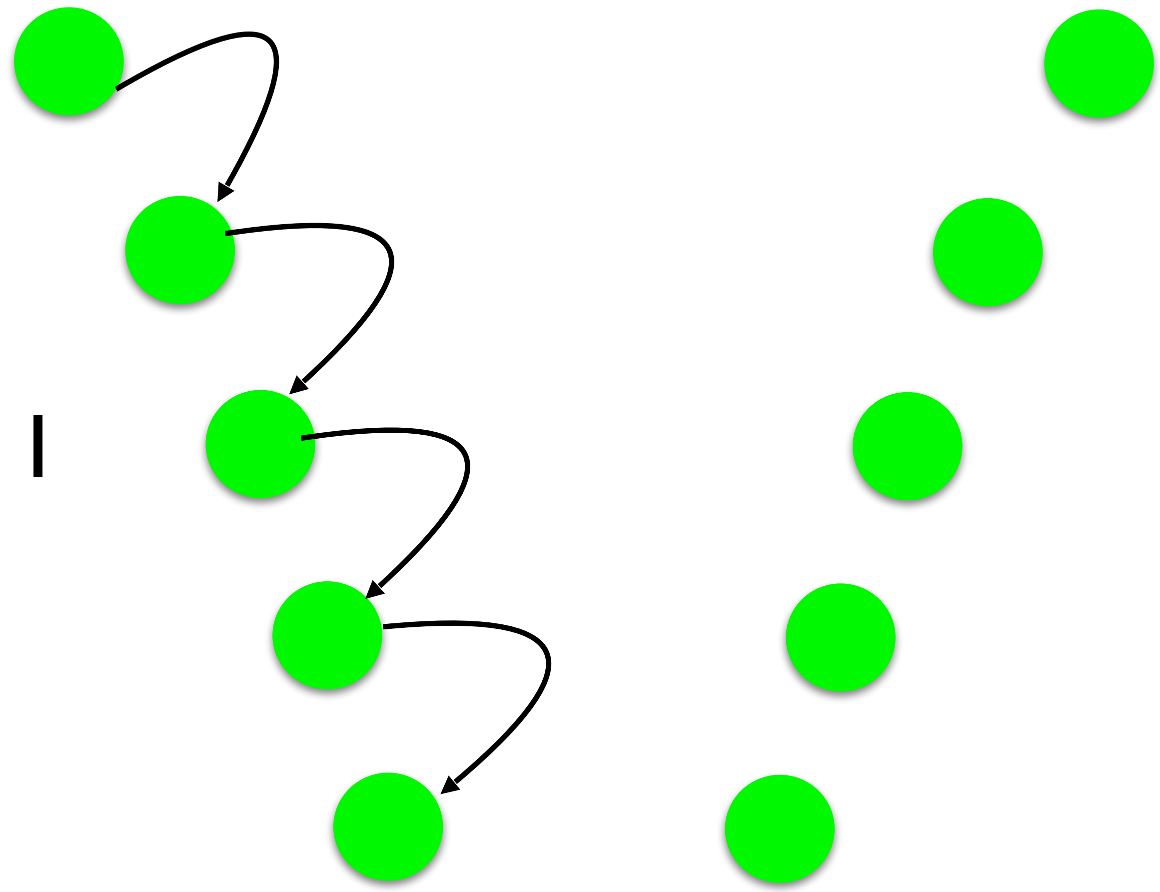
Sequence 1



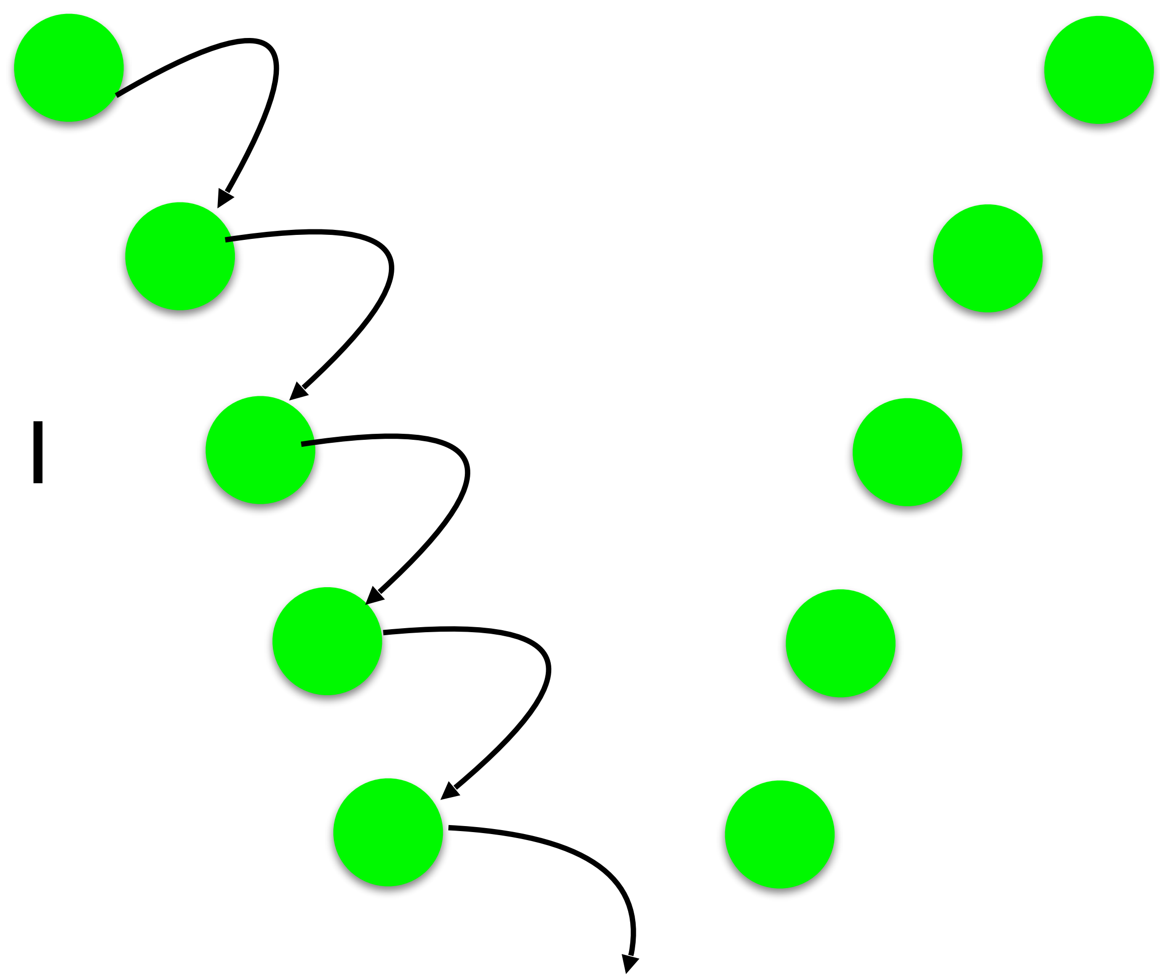

Sequence 1

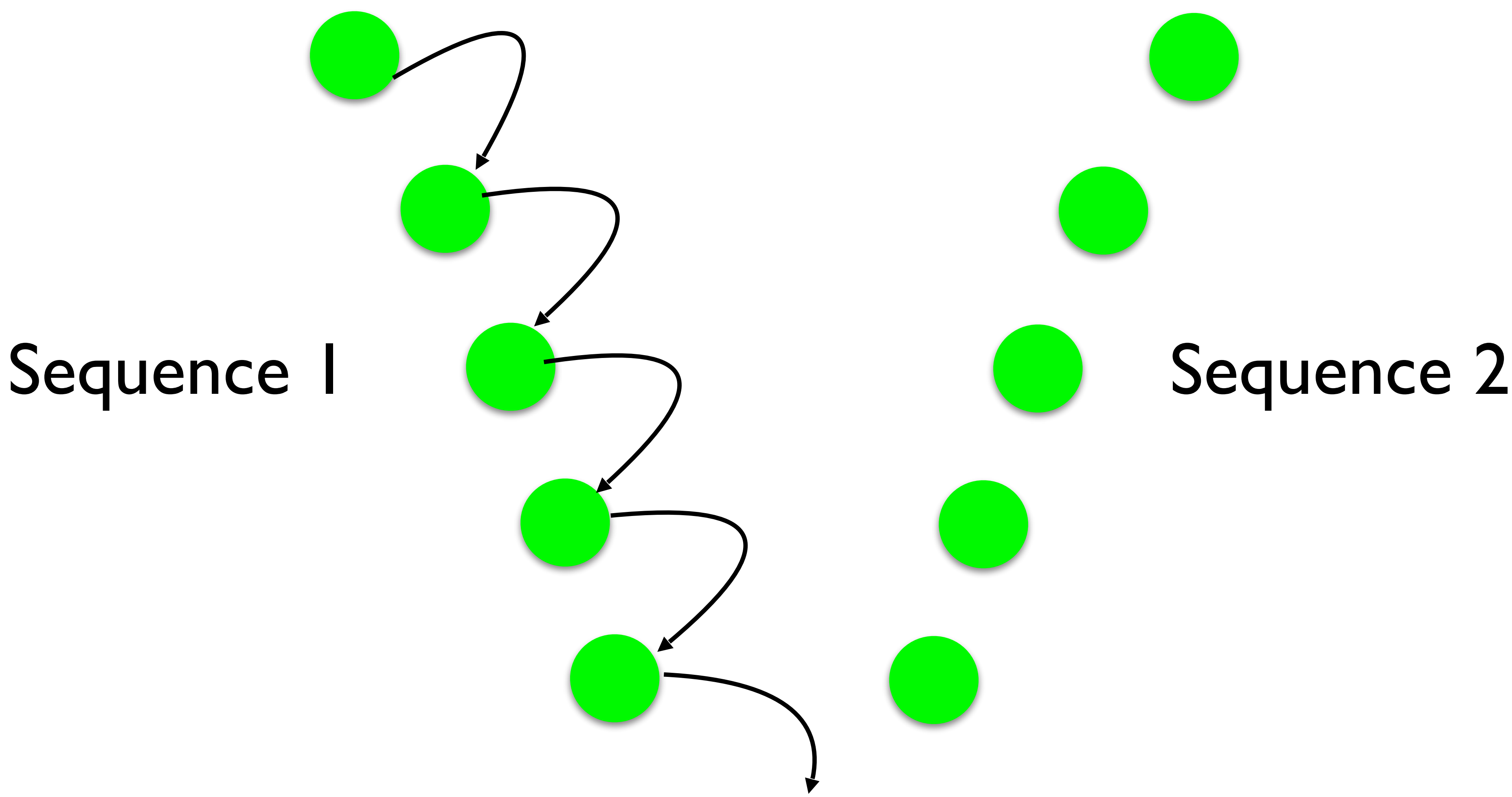

Sequence 1

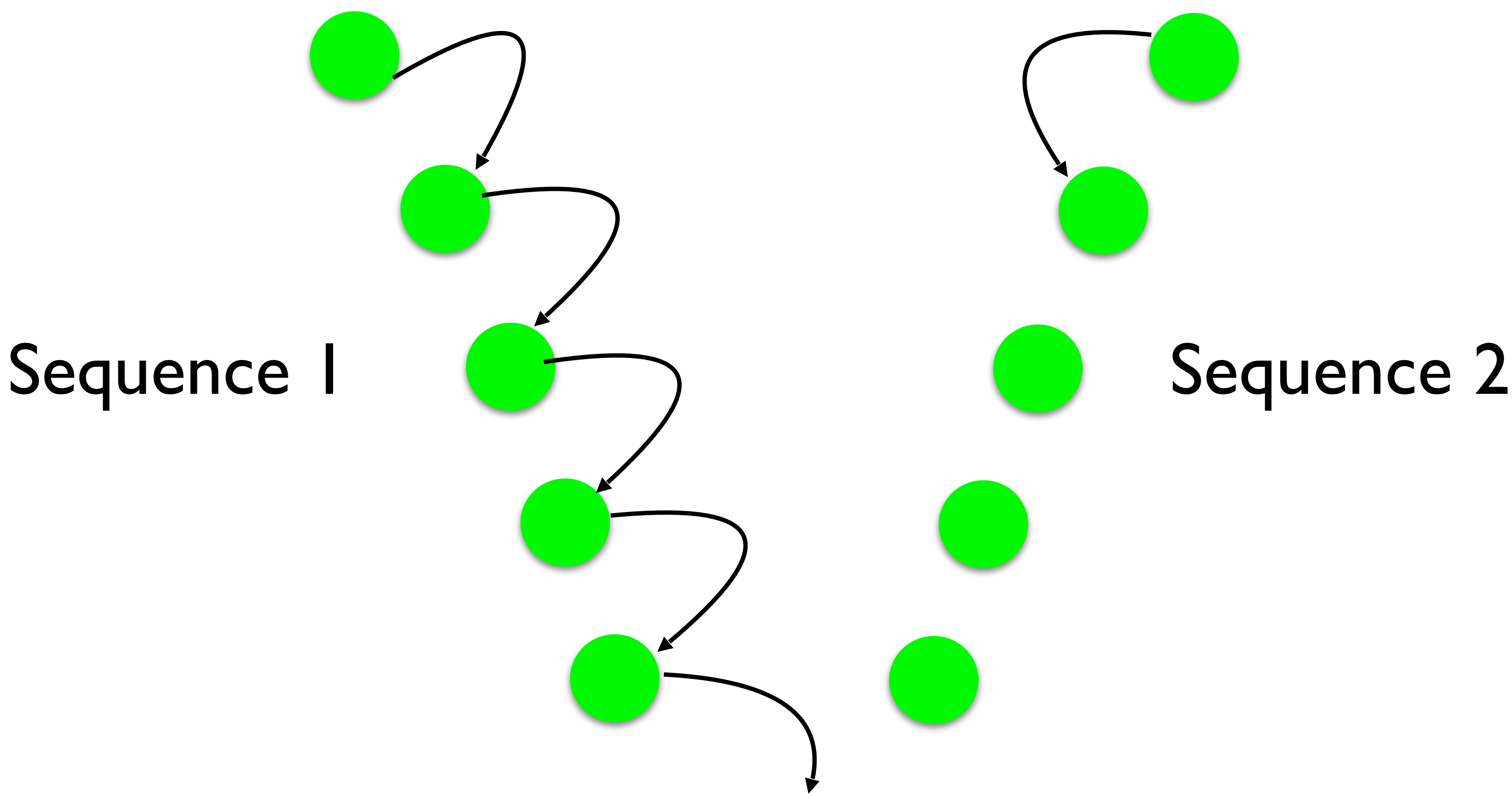
Sequence 1

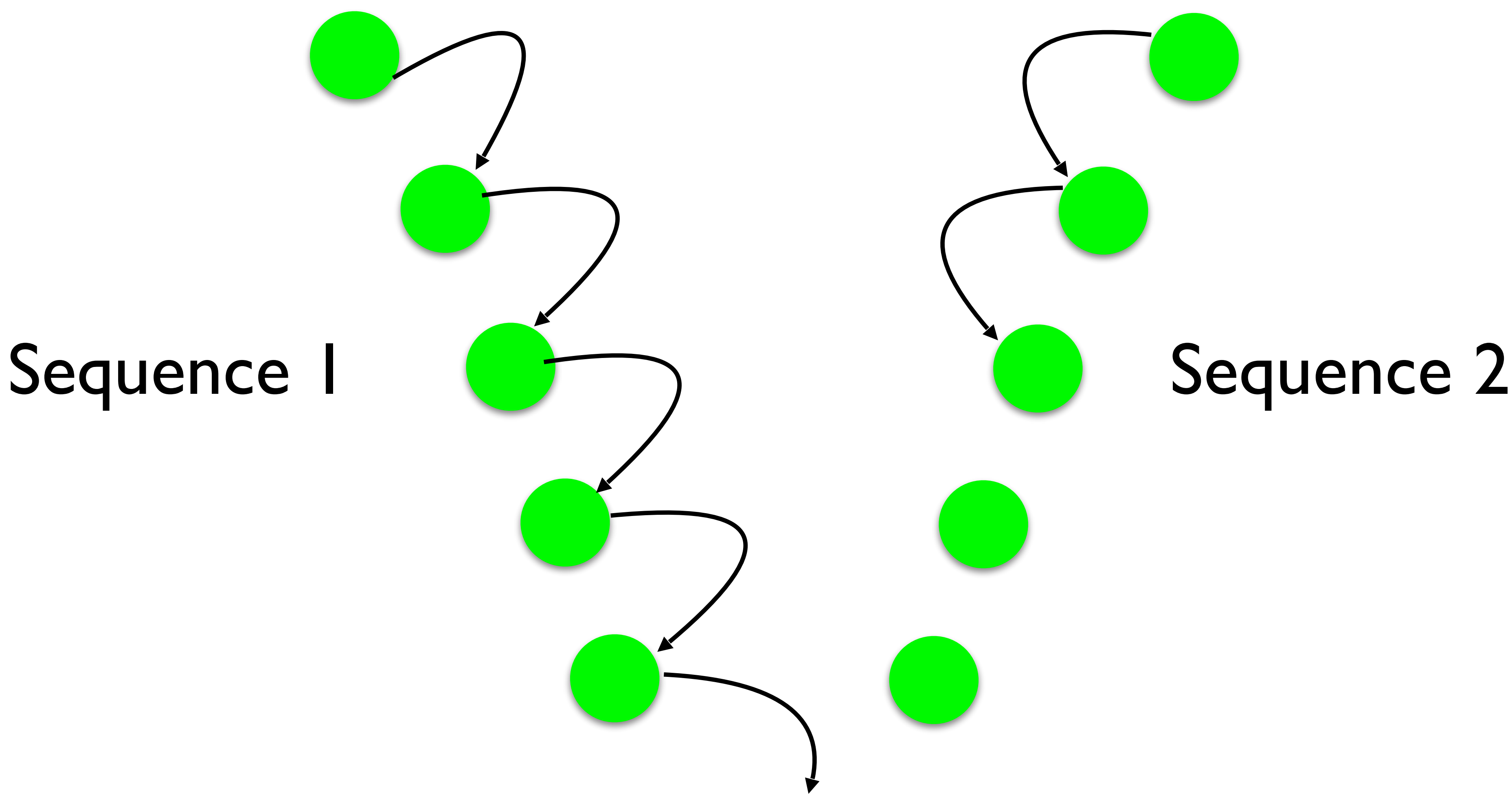


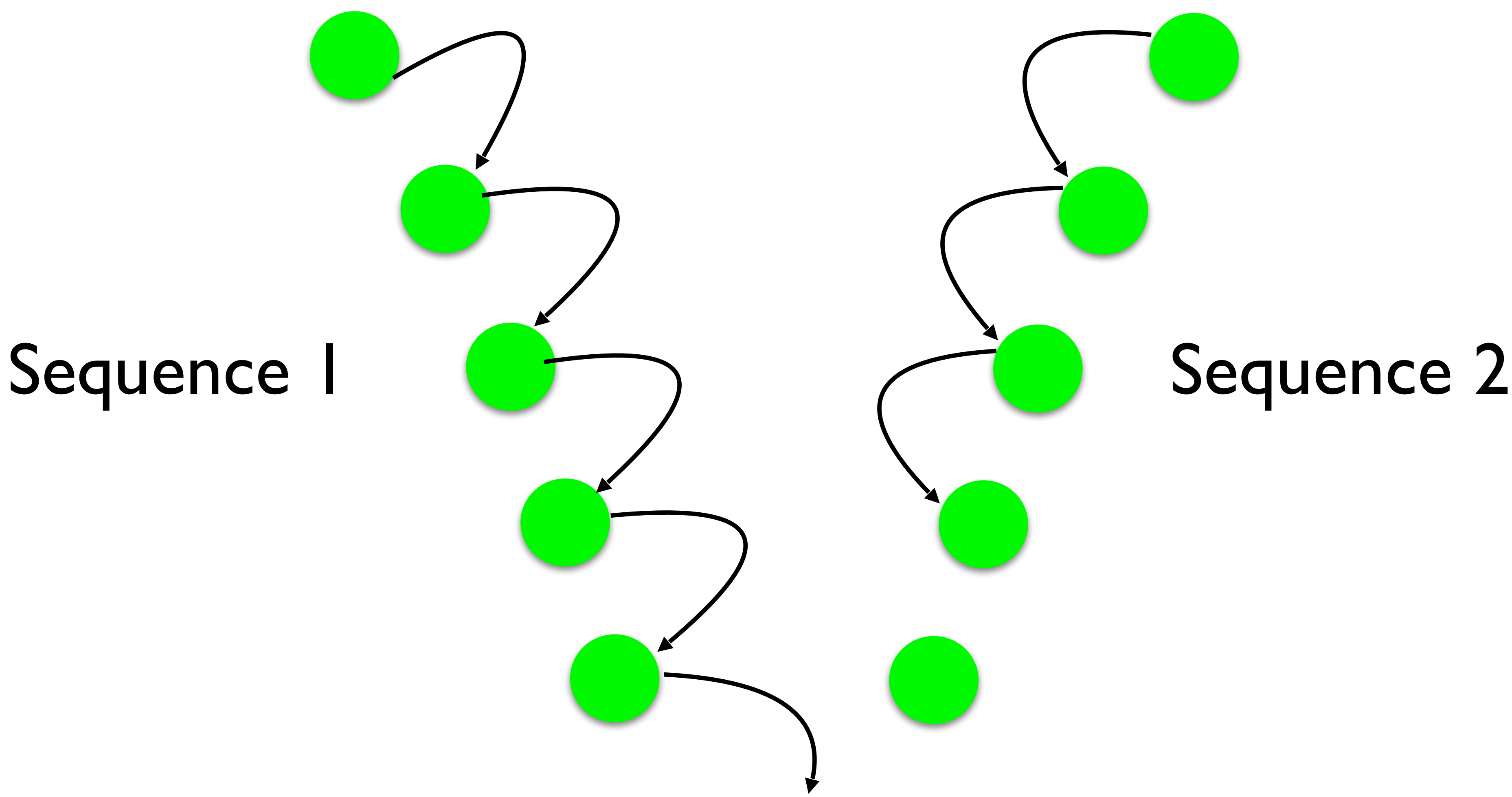
Sequence 1

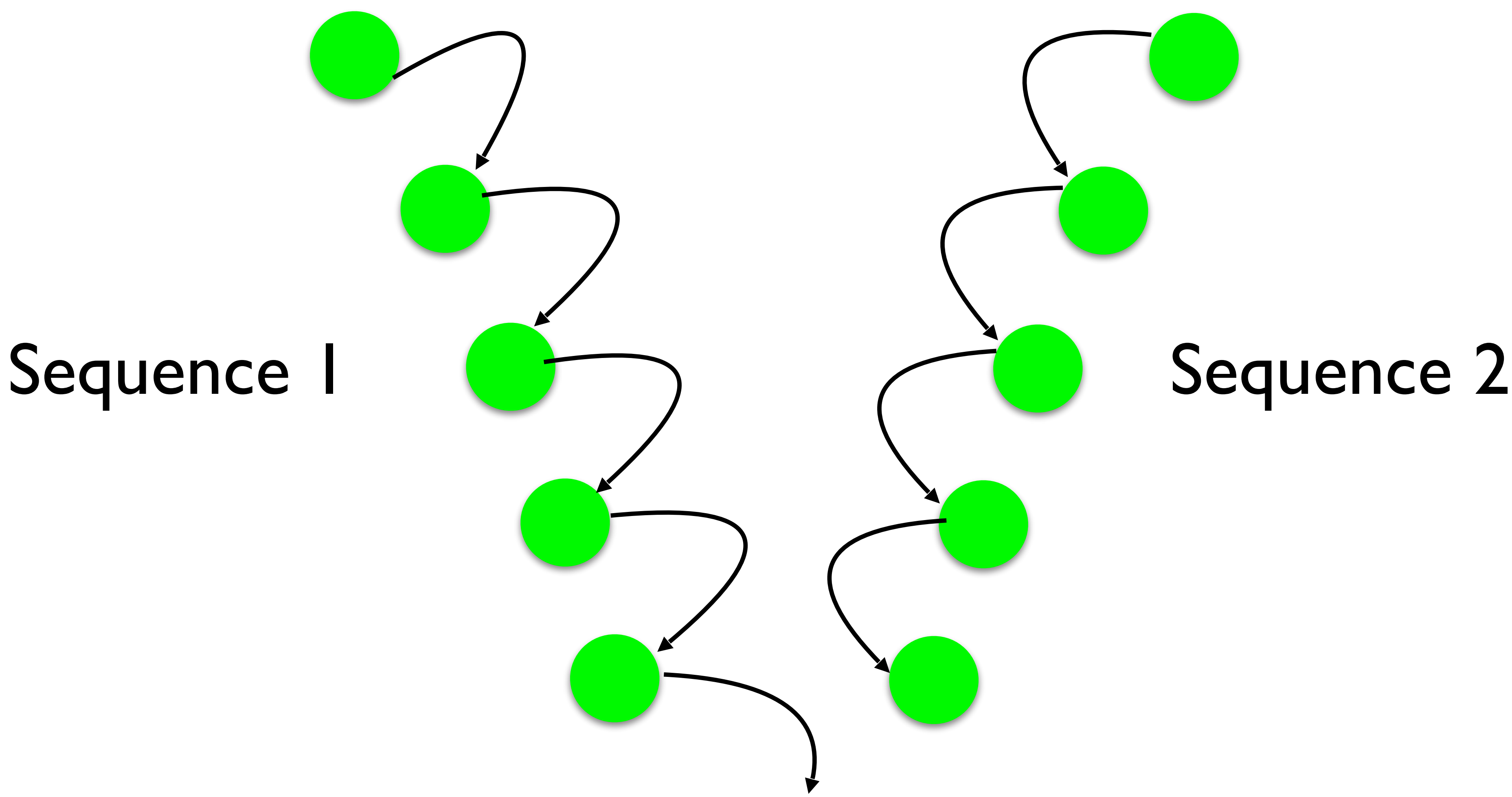


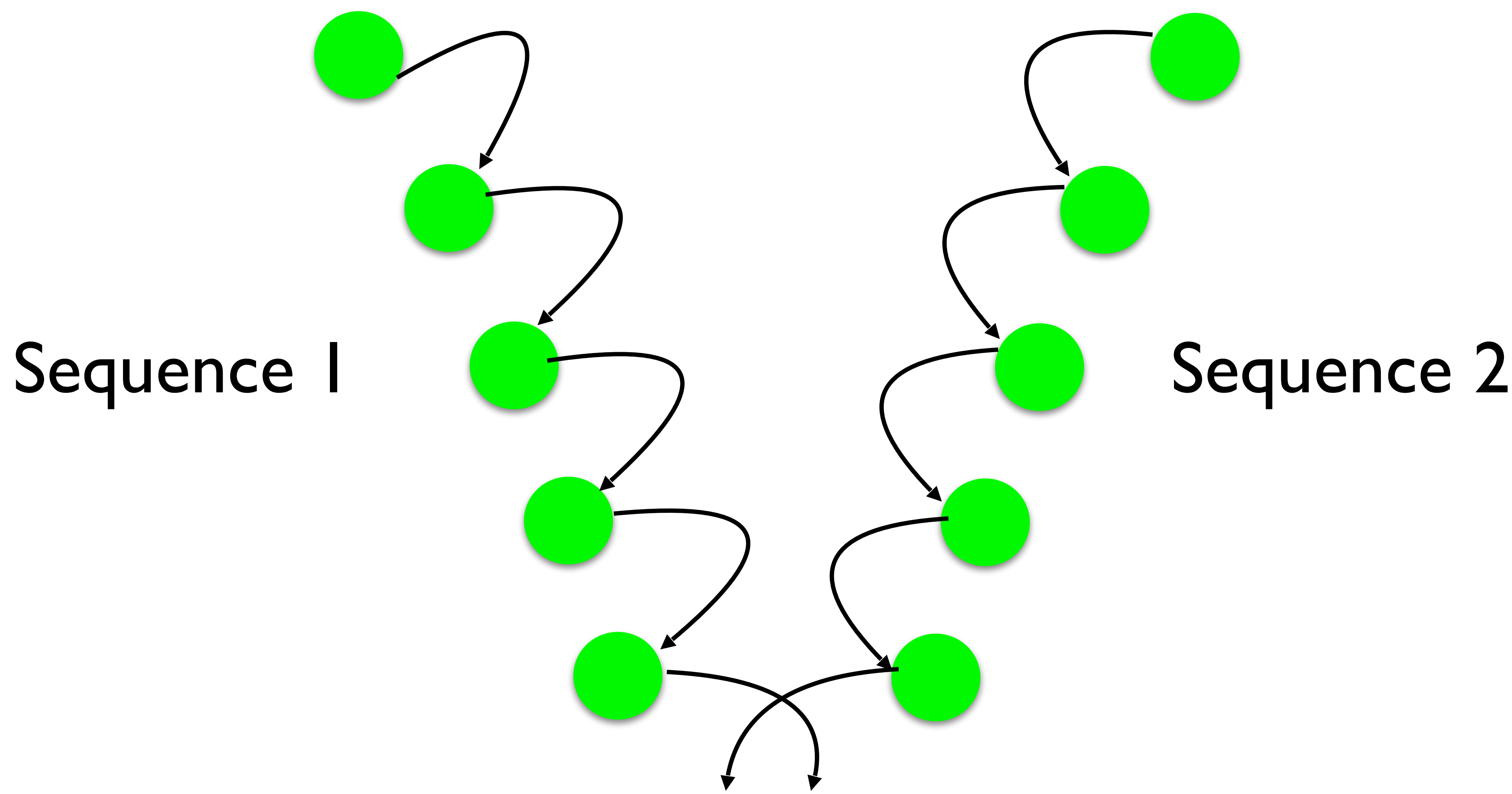

Sequence 1
Sequence 2

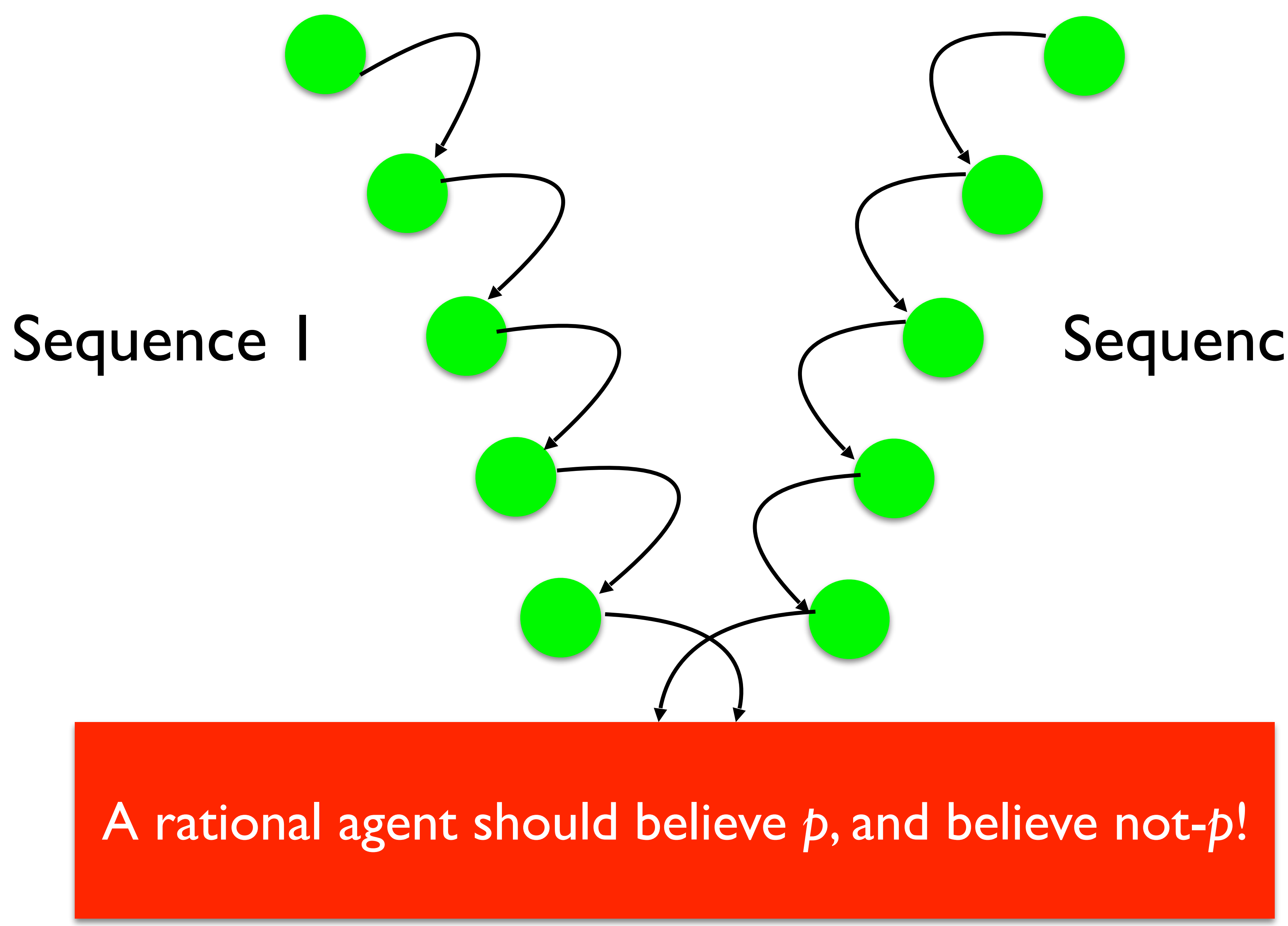

Sequence 1
Sequence 2


Sequence 1
Sequence 2


Sequence 1
Sequence 2


Sequence 1
Sequence 2


Sequence 1
Sequence 2


Sequence 1
Sequence 2
A rational agent should believe *p*, and believe not-*p*!

Sequence 1

Sequence 1

Let **D** be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent *a* is fully apprised.

Sequence 1

Let **D** be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent *a* is fully apprised.

From **D** it obviously can be proved that either ticket 1 will win or ticket 2 will win or … or ticket 1,000,000,000,000 will win. Let's write this (exclusive) disjunction as follows:

Sequence 1

Let **D** be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent *a* is fully apprised.

From **D** it obviously can be proved that either ticket 1 will win or ticket 2 will win or … or ticket 1,000,000,000,000 will win. Let's write this (exclusive) disjunction as follows:

$$Wt_1 \oplus Wt_2 \oplus \ldots \oplus Wt_{1T} \quad (1)$$

Sequence 1

Let **D** be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent *a* is fully apprised.

From **D** it obviously can be proved that either ticket 1 will win or ticket 2 will win or … or ticket 1,000,000,000,000 will win. Let's write this (exclusive) disjunction as follows:

$$Wt_1 \oplus Wt_2 \oplus \ldots \oplus Wt_{1T} \quad (1)$$

We then deduce from this that there is at least one ticket that will win, a proposition represented — using standard notation — as:

Sequence 1

Let **D** be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent *a* is fully apprised.

From **D** it obviously can be proved that either ticket 1 will win or ticket 2 will win or … or ticket 1,000,000,000,000 will win. Let's write this (exclusive) disjunction as follows:

$$Wt_1 \oplus Wt_2 \oplus \ldots \oplus Wt_{1T} \quad (1)$$

We then deduce from this that there is at least one ticket that will win, a proposition represented — using standard notation — as:

$$\exists t_i Wt_i \quad (2)$$

Sequence 1

Let **D** be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent *a* is fully apprised.

From **D** it obviously can be proved that either ticket 1 will win or ticket 2 will win or … or ticket 1,000,000,000,000 will win. Let's write this (exclusive) disjunction as follows:

$$Wt_1 \oplus Wt_2 \oplus \ldots \oplus Wt_{1T} \quad (1)$$

We then deduce from this that there is at least one ticket that will win, a proposition represented — using standard notation — as:

$$\exists t_i W t_i \quad (2)$$

Very well; perfectly clear so far. And now we can add another deductive step: Since our rational agent *a* can follow this deduction sequence to this point, and since **D** is assumed to be indubitable, it follows that our rational agent in turn believes (2); i.e., we conclude Sequence 1 by obtaining the following:

Sequence 1

Let **D** be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent *a* is fully apprised.

From **D** it obviously can be proved that either ticket 1 will win or ticket 2 will win or … or ticket 1,000,000,000,000 will win. Let's write this (exclusive) disjunction as follows:

$$Wt_1 \oplus Wt_2 \oplus \ldots \oplus Wt_{1T} \quad (1)$$

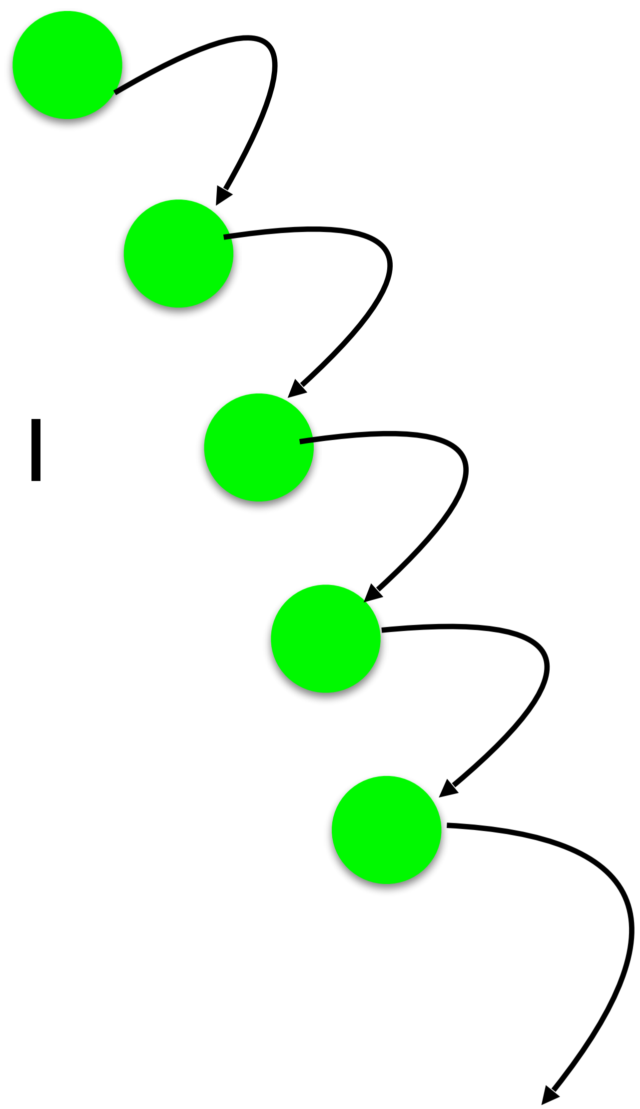
We then deduce from this that there is at least one ticket that will win, a proposition represented — using standard notation — as:

$$\exists t_i W t_i \quad (2)$$

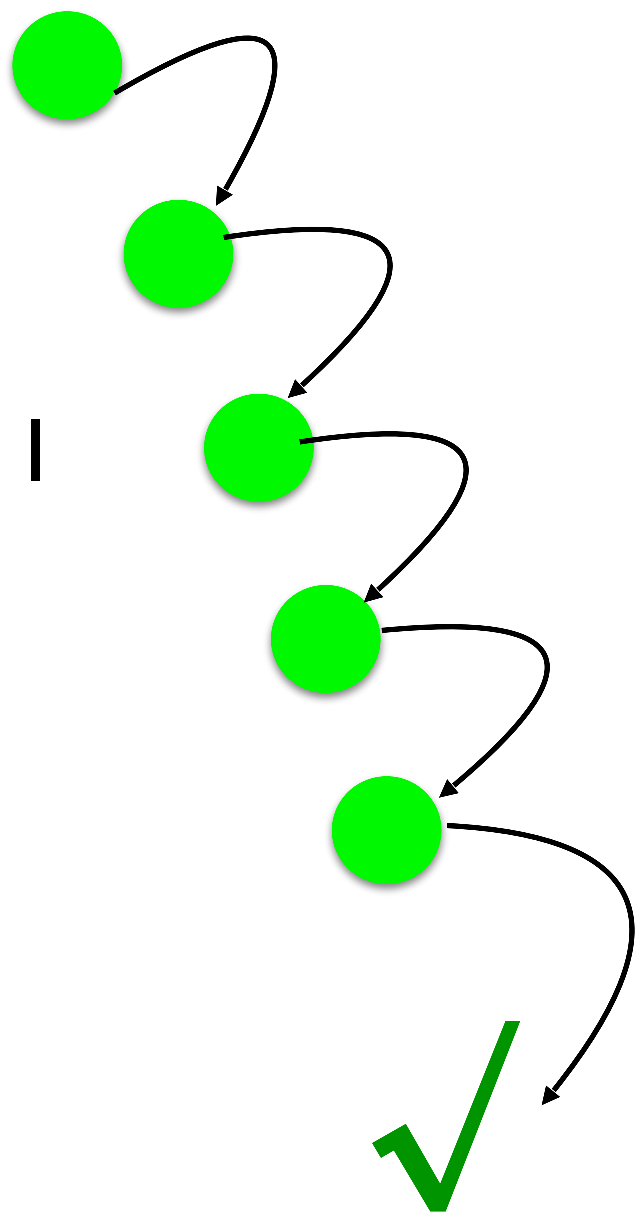
Very well; perfectly clear so far. And now we can add another deductive step: Since our rational agent *a* can follow this deduction sequence to this point, and since **D** is assumed to be indubitable, it follows that our rational agent in turn believes (2); i.e., we conclude Sequence 1 by obtaining the following:

$$\mathbf{B}_a \exists t_i W t_i \quad (3)$$

Sequence 1



Sequence 1



Sequence 2

Sequence 2

As in Sequence 1, once again let **D** be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent *a* is fully apprised.

Sequence 2

As in Sequence 1, once again let **D** be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent *a* is fully apprised.

From **D** it obviously can be proved that the probability of a particular ticket t_i winning is 1 in 1,000,000,000,000. Using '1T' to denote 1 trillion, we can write the probability for each ticket to win as a conjunction:

Sequence 2

As in Sequence 1, once again let **D** be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent *a* is fully apprised.

From **D** it obviously can be proved that the probability of a particular ticket t_i winning is 1 in 1,000,000,000,000. Using '1T' to denote 1 trillion, we can write the probability for each ticket to win as a conjunction:

$$prob(Wt_1) = \frac{1}{1,000,000,000,000} = \frac{1}{1T} \wedge prob(Wt_2) = \frac{1}{1T} \wedge \ldots \wedge prob(Wt_{1T}) = \frac{1}{1T} \quad (1)$$

Sequence 2

As in Sequence 1, once again let **D** be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent *a* is fully apprised.

From **D** it obviously can be proved that the probability of a particular ticket t_i winning is 1 in 1,000,000,000,000. Using '1T' to denote 1 trillion, we can write the probability for each ticket to win as a conjunction:

$$prob(Wt_1) = \frac{1}{1,000,000,000,000} = \frac{1}{1T} \wedge prob(Wt_2) = \frac{1}{1T} \wedge \ldots \wedge prob(Wt_{1T}) = \frac{1}{1T} \quad (1)$$

For the next step, note that the probability of ticket t_l winning is lower than, say, the probability that if you walk outside a minute from now you will be flattened on the spot by a door from a 747 that falls from a jet of that type cruising at 35,000 feet. Since you have the rational belief that death won't ensue if you go outside (and have this belief precisely because you believe that the odds of your sudden demise in this manner are vanishingly small), the inference to the rational belief on the part of *a* that t_l won't win sails through—and this of course works for each ticket. Hence we have:

Sequence 2

As in Sequence 1, once again let **D** be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent *a* is fully apprised.

From **D** it obviously can be proved that the probability of a particular ticket t_i winning is 1 in 1,000,000,000,000. Using '1T' to denote 1 trillion, we can write the probability for each ticket to win as a conjunction:

$$prob(Wt_1) = \frac{1}{1,000,000,000,000} = \frac{1}{1T} \wedge prob(Wt_2) = \frac{1}{1T} \wedge \ldots \wedge prob(Wt_{1T}) = \frac{1}{1T} \quad (1)$$

For the next step, note that the probability of ticket t_I winning is lower than, say, the probability that if you walk outside a minute from now you will be flattened on the spot by a door from a 747 that falls from a jet of that type cruising at 35,000 feet. Since you have the rational belief that death won't ensue if you go outside (and have this belief precisely because you believe that the odds of your sudden demise in this manner are vanishingly small), the inference to the rational belief on the part of *a* that t_I won't win sails through—and this of course works for each ticket. Hence we have:

$$\mathbf{B}_a \neg Wt_1 \wedge \mathbf{B}_a \neg Wt_2 \wedge \ldots \wedge \mathbf{B}_a \neg Wt_{1T} \quad (2)$$

Sequence 2

As in Sequence 1, once again let **D** be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent *a* is fully apprised.

From **D** it obviously can be proved that the probability of a particular ticket t_i winning is 1 in 1,000,000,000,000. Using '1T' to denote 1 trillion, we can write the probability for each ticket to win as a conjunction:

$$prob(Wt_1) = \frac{1}{1,000,000,000,000} = \frac{1}{1T} \wedge prob(Wt_2) = \frac{1}{1T} \wedge \ldots \wedge prob(Wt_{1T}) = \frac{1}{1T} \quad (1)$$

For the next step, note that the probability of ticket t_l winning is lower than, say, the probability that if you walk outside a minute from now you will be flattened on the spot by a door from a 747 that falls from a jet of that type cruising at 35,000 feet. Since you have the rational belief that death won't ensue if you go outside (and have this belief precisely because you believe that the odds of your sudden demise in this manner are vanishingly small), the inference to the rational belief on the part of *a* that t_l won't win sails through—and this of course works for each ticket. Hence we have:

$$\mathbf{B}_a \neg Wt_1 \wedge \mathbf{B}_a \neg Wt_2 \wedge \ldots \wedge \mathbf{B}_a \neg Wt_{1T} \quad (2)$$

Of course, if a rational agent believes *P*, and believes *Q* as well, it follows that that agent will believe the conjunction *P* & *Q*. Applying this principle to (2) yields:

Sequence 2

As in Sequence 1, once again let **D** be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent *a* is fully apprised.

From **D** it obviously can be proved that the probability of a particular ticket t_i winning is 1 in 1,000,000,000,000. Using '1T' to denote 1 trillion, we can write the probability for each ticket to win as a conjunction:

$$prob(Wt_1) = \frac{1}{1,000,000,000,000} = \frac{1}{1T} \wedge prob(Wt_2) = \frac{1}{1T} \wedge \ldots \wedge prob(Wt_{1T}) = \frac{1}{1T} \quad (1)$$

For the next step, note that the probability of ticket t_l winning is lower than, say, the probability that if you walk outside a minute from now you will be flattened on the spot by a door from a 747 that falls from a jet of that type cruising at 35,000 feet. Since you have the rational belief that death won't ensue if you go outside (and have this belief precisely because you believe that the odds of your sudden demise in this manner are vanishingly small), the inference to the rational belief on the part of *a* that t_l won't win sails through—and this of course works for each ticket. Hence we have:

$$\mathbf{B}_a \neg Wt_1 \wedge \mathbf{B}_a \neg Wt_2 \wedge \ldots \wedge \mathbf{B}_a \neg Wt_{1T} \quad (2)$$

Of course, if a rational agent believes *P*, and believes *Q* as well, it follows that that agent will believe the conjunction *P* & *Q*. Applying this principle to (2) yields:

$$\mathbf{B}_a(\neg Wt_1 \wedge \neg Wt_2 \wedge \ldots \wedge \neg Wt_{1T}) \quad (3)$$

Sequence 2

As in Sequence 1, once again let **D** be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent *a* is fully apprised.

From **D** it obviously can be proved that the probability of a particular ticket t_i winning is 1 in 1,000,000,000,000. Using '1T' to denote 1 trillion, we can write the probability for each ticket to win as a conjunction:

$$prob(Wt_1) = \frac{1}{1,000,000,000,000} = \frac{1}{1T} \land prob(Wt_2) = \frac{1}{1T} \land \ldots \land prob(Wt_{1T}) = \frac{1}{1T} \quad (1)$$

For the next step, note that the probability of ticket t_l winning is lower than, say, the probability that if you walk outside a minute from now you will be flattened on the spot by a door from a 747 that falls from a jet of that type cruising at 35,000 feet. Since you have the rational belief that death won't ensue if you go outside (and have this belief precisely because you believe that the odds of your sudden demise in this manner are vanishingly small), the inference to the rational belief on the part of *a* that t_l won't win sails through—and this of course works for each ticket. Hence we have:

$$\mathbf{B}_a \neg Wt_1 \land \mathbf{B}_a \neg Wt_2 \land \ldots \land \mathbf{B}_a \neg Wt_{1T} \quad (2)$$

Of course, if a rational agent believes *P*, and believes *Q* as well, it follows that that agent will believe the conjunction *P* & *Q*. Applying this principle to (2) yields:

$$\mathbf{B}_a(\neg Wt_1 \land \neg Wt_2 \land \ldots \land \neg Wt_{1T}) \quad (3)$$

But (3) is logically equivalent to the statement that there doesn't exist a winning ticket; i.e., (3) is logically equivalent to this result from Sequence 2:

Sequence 2

As in Sequence 1, once again let **D** be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent *a* is fully apprised.

From **D** it obviously can be proved that the probability of a particular ticket t_i winning is 1 in 1,000,000,000,000. Using '1T' to denote 1 trillion, we can write the probability for each ticket to win as a conjunction:

$$prob(Wt_1) = \frac{1}{1,000,000,000,000} = \frac{1}{1T} \wedge prob(Wt_2) = \frac{1}{1T} \wedge \ldots \wedge prob(Wt_{1T}) = \frac{1}{1T} \quad (1)$$

For the next step, note that the probability of ticket t_l winning is lower than, say, the probability that if you walk outside a minute from now you will be flattened on the spot by a door from a 747 that falls from a jet of that type cruising at 35,000 feet. Since you have the rational belief that death won't ensue if you go outside (and have this belief precisely because you believe that the odds of your sudden demise in this manner are vanishingly small), the inference to the rational belief on the part of *a* that t_l won't win sails through—and this of course works for each ticket. Hence we have:

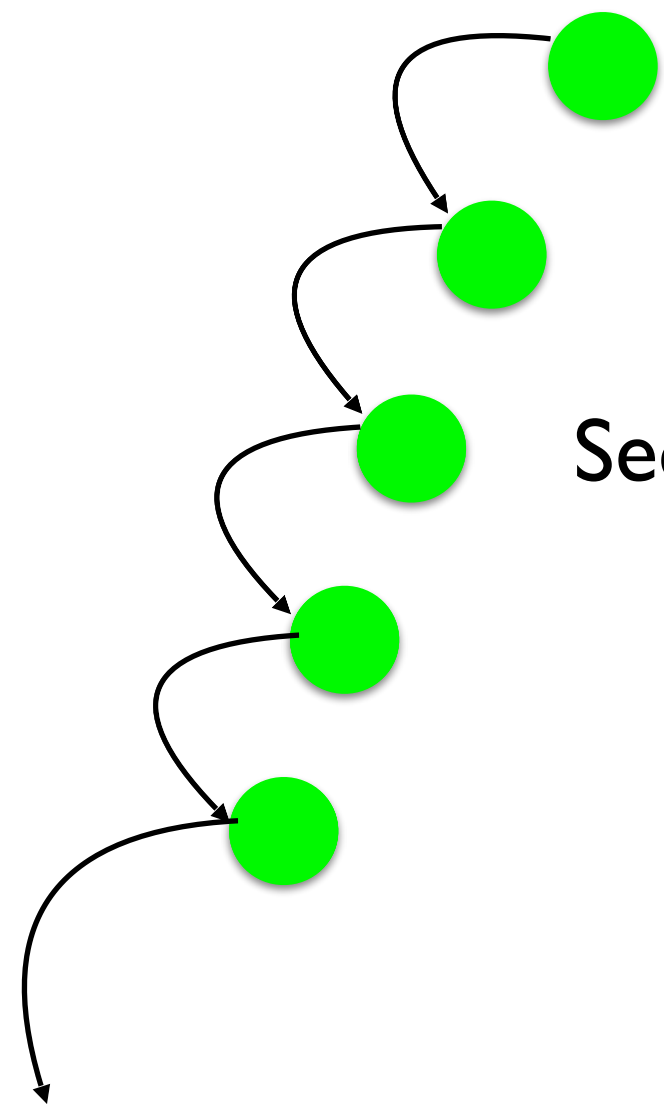
$$\mathbf{B}_a \neg Wt_1 \wedge \mathbf{B}_a \neg Wt_2 \wedge \ldots \wedge \mathbf{B}_a \neg Wt_{1T} \quad (2)$$

Of course, if a rational agent believes *P*, and believes *Q* as well, it follows that that agent will believe the conjunction *P* & *Q*. Applying this principle to (2) yields:

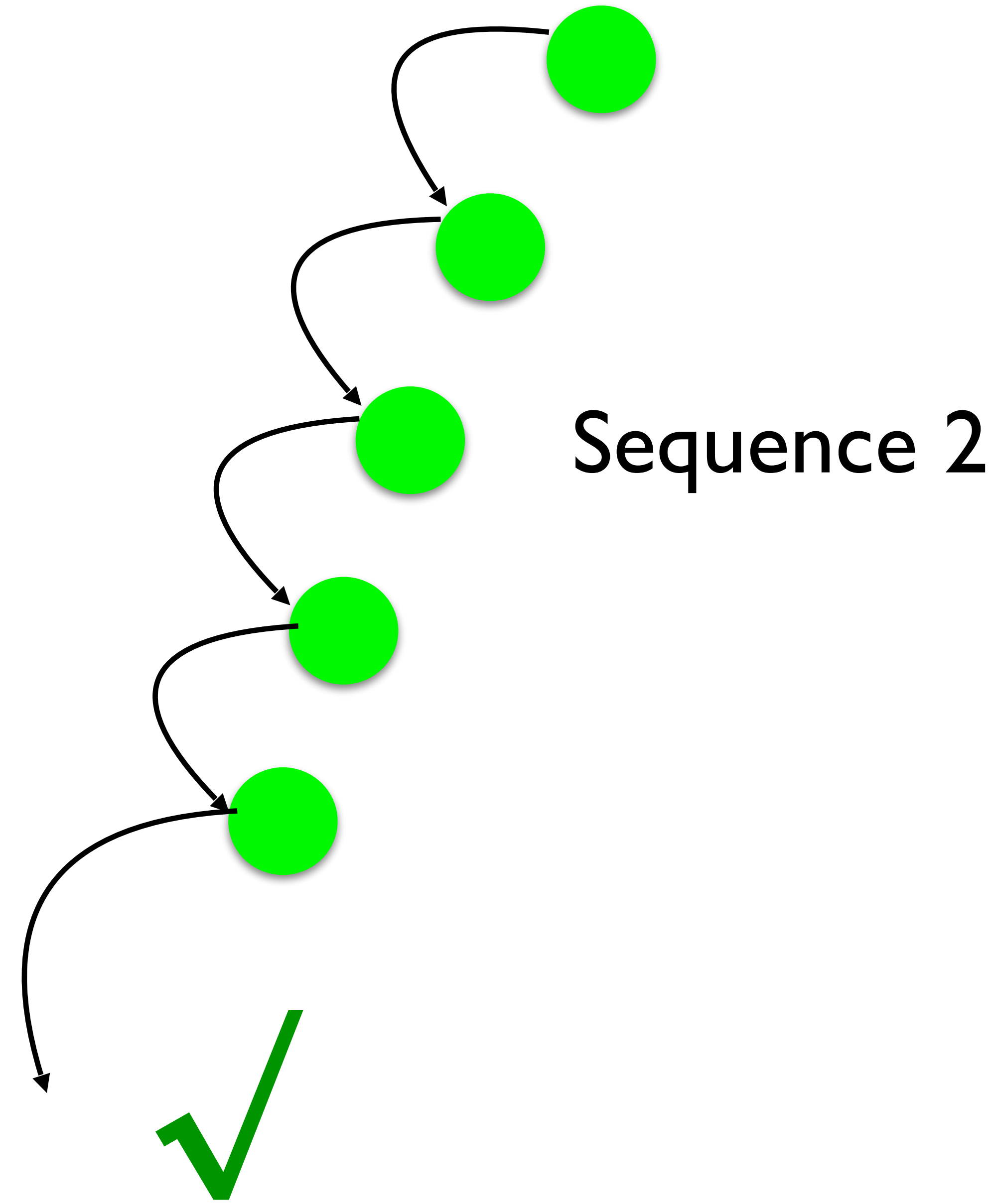
$$\mathbf{B}_a(\neg Wt_1 \wedge \neg Wt_2 \wedge \ldots \wedge \neg Wt_{1T}) \quad (3)$$

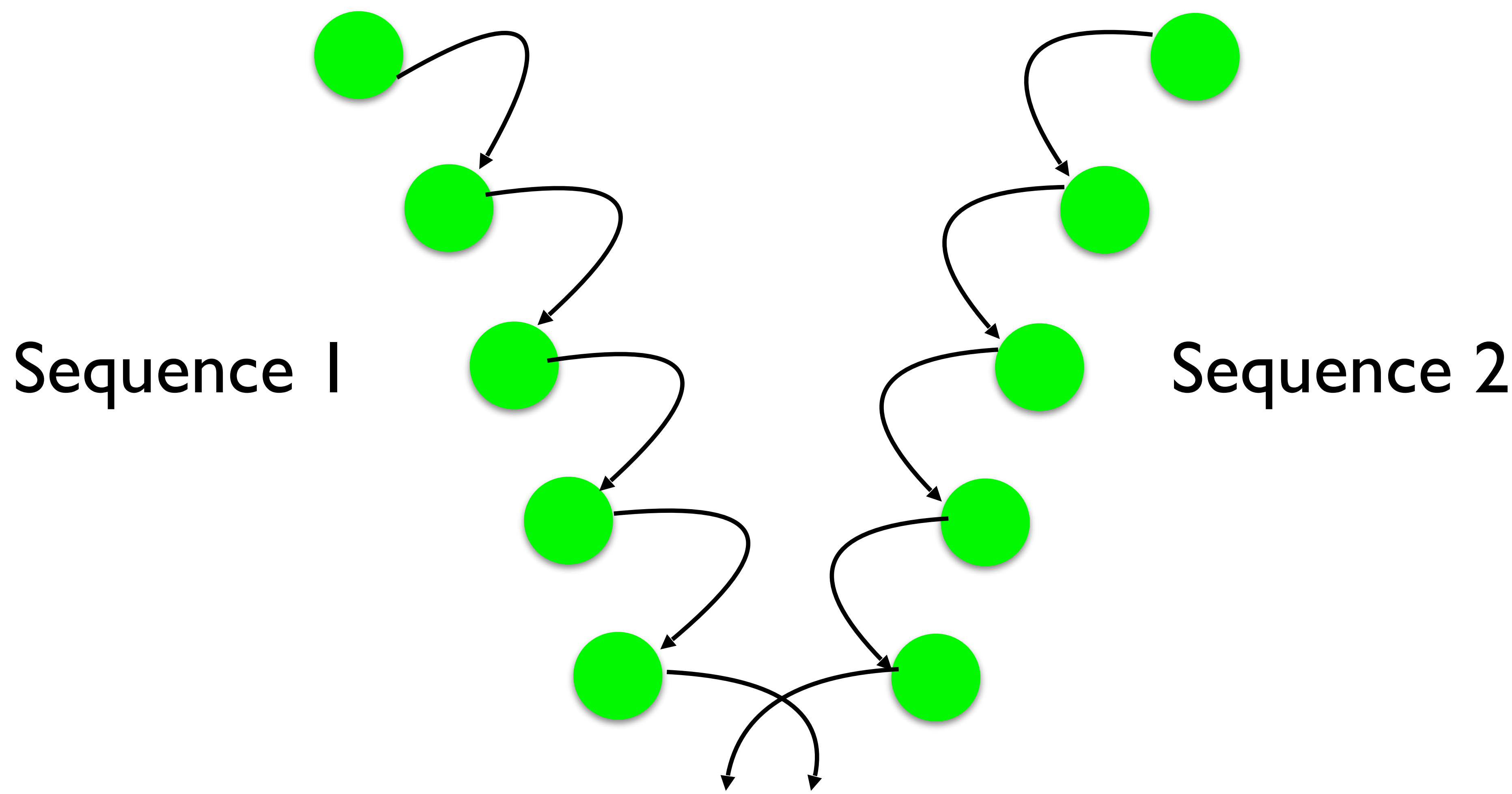
But (3) is logically equivalent to the statement that there doesn't exist a winning ticket; i.e., (3) is logically equivalent to this result from Sequence 2:

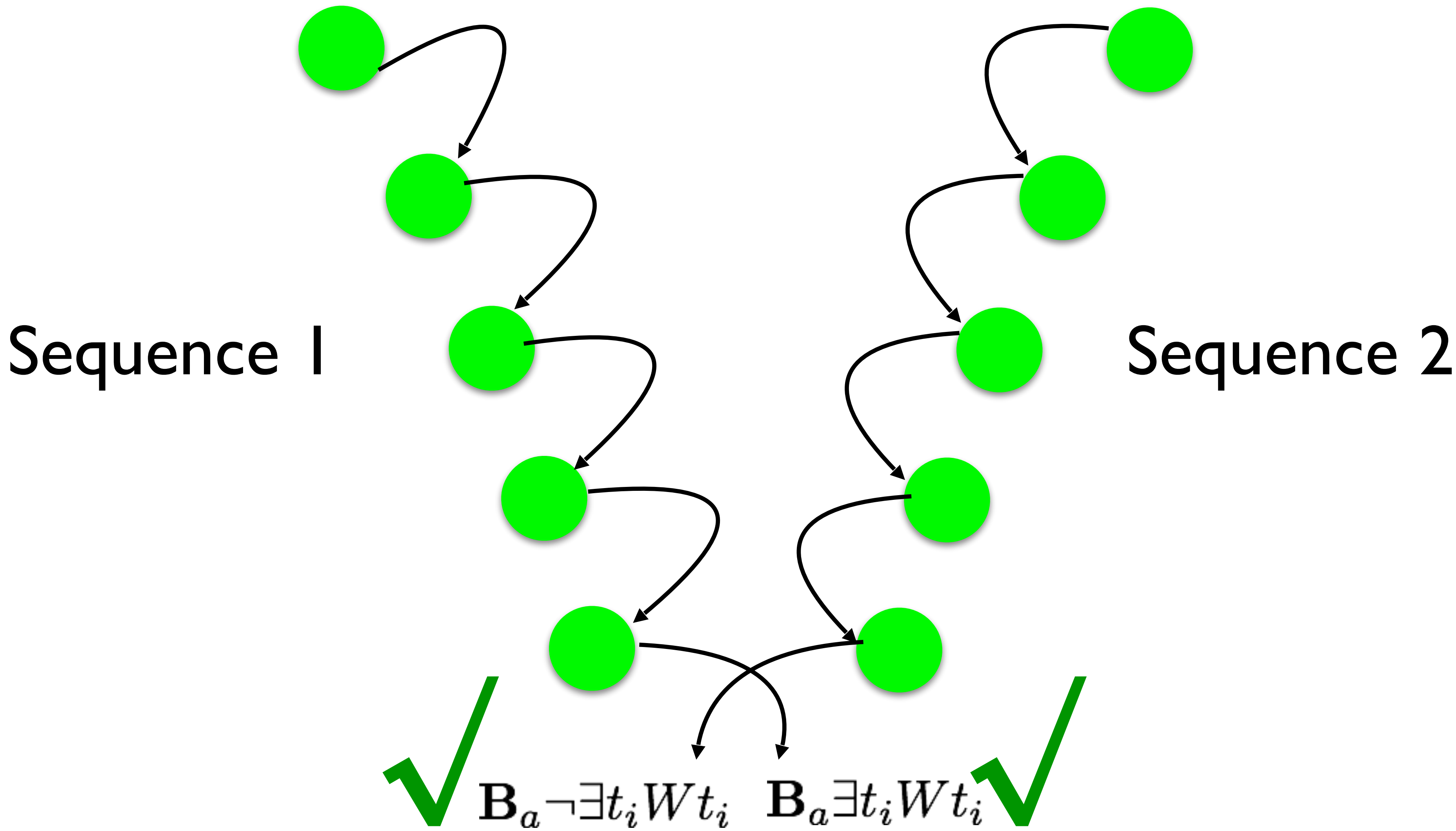
$$\mathbf{B}_a \neg \exists t_i W t_i \quad (4)$$

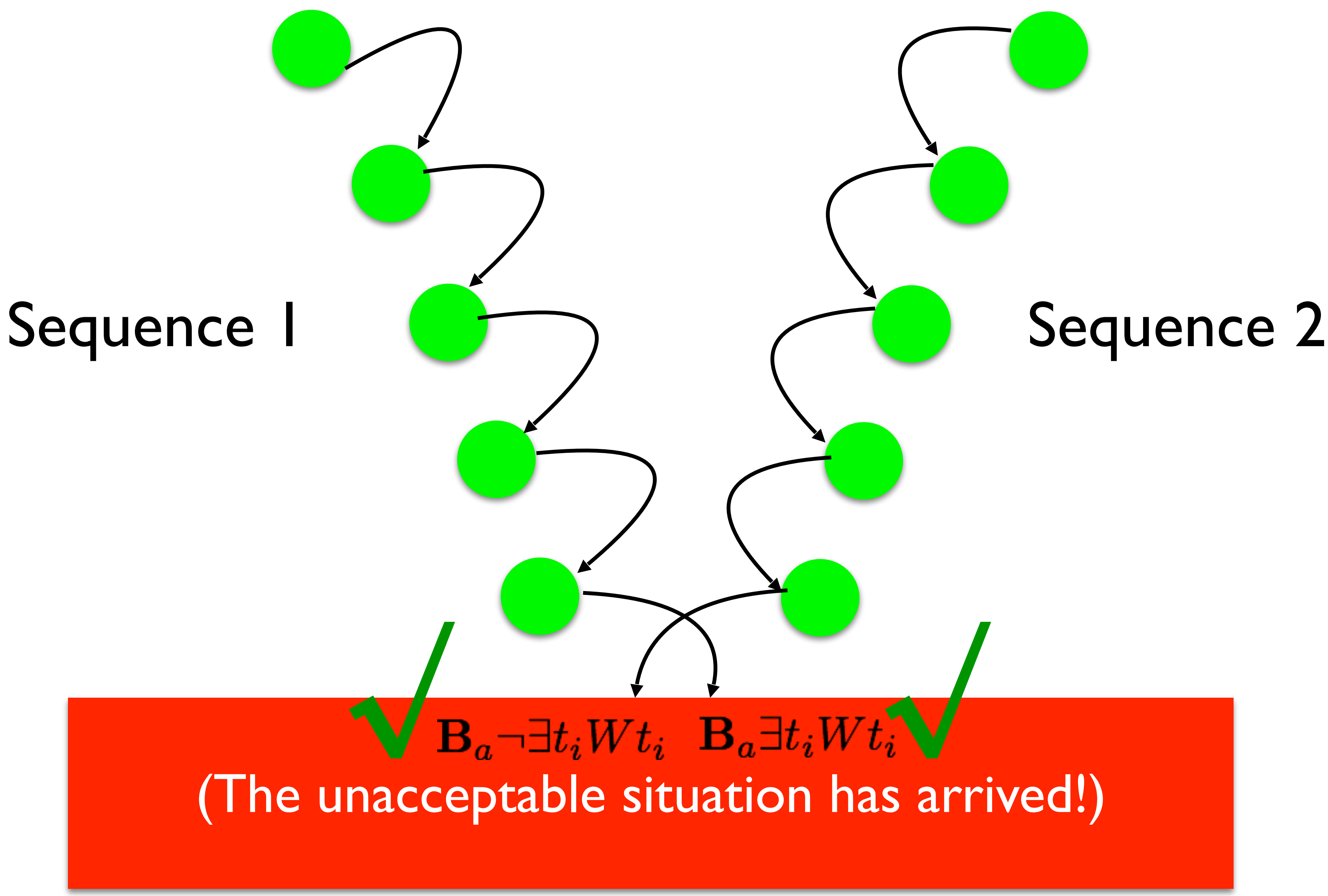


Sequence 2


Sequence 2


Sequence 1
Sequence 2


Sequence 1
Sequence 2
$\mathbf{B}_a \neg \exists t_i W t_i$
$\mathbf{B}_a \exists t_i W t_i$


Sequence 1
Sequence 2
$\mathbf{B}_a \neg \exists t_i W t_i$
$\mathbf{B}_a \exists t_i W t_i$
(The unacceptable situation has arrived!)

Bringsjord's Solution to The Lottery Paradox …

(ACS; 13 likelihood values)

13 Strength-Factors

More Unlikely Than Not

Certain

Improbable

Evidently False

Overwhelmingly Likely

Overwhelmingly Unlikely

Likely

Beyond Reasonable Belief

Certainly False

More Likely Than Not

Counterbalanced

Evident

Beyond Reasonable Doubt

$MV_{13}CC$

13-Strength-Factor Continuum

Certain

Evident

Overwhelmingly Likely

Beyond Reasonable Doubt

Likely

More Likely Than Not

Counterbalanced

More Unlikely Than Not

Unlikely

Beyond Reasonable Belief

Overwhelmingly Unlikely

Evidently False

Certainly False

13-Strength-Factor Continuum

Certain

Evident

Overwhelmingly Likely

Beyond Reasonable Doubt

Likely

More Likely Than Not

Counterbalanced

More Unlikely Than Not

Unlikely

Beyond Reasonable Belief

Overwhelmingly Unlikely

Evidently False

Certainly False

13-Strength-Factor Continuum

Certain

Evident

Overwhelmingly Likely

Beyond Reasonable Doubt

Likely

More Likely Than Not

Counterbalanced

More Unlikely Than Not

Unlikely

Beyond Reasonable Belief

Overwhelmingly Unlikely

Evidently False

Certainly False

13-Strength-Factor Continuum

Epistemically Positive

Certain

Evident

Overwhelmingly Likely

Beyond Reasonable Doubt

Likely

More Likely Than Not

Counterbalanced

More Unlikely Than Not

Unlikely

Beyond Reasonable Belief

Overwhelmingly Unlikely

Evidently False

Certainly False

13-Strength-Factor Continuum

Epistemically Positive

Certain

Evident

Overwhelmingly Likely

Beyond Reasonable Doubt

Likely

More Likely Than Not

Counterbalanced

More Unlikely Than Not

Unlikely

Beyond Reasonable Belief

Overwhelmingly Unlikely

Evidently False

Certainly False

Epistemically Negative

13-Strength-Factor Continuum

Epistemically Positive

Certain
Evident
Overwhelmingly Likely
Beyond Reasonable Doubt
Likely
More Likely Than Not
Counterbalanced
More Unlikely Than Not
Unlikely
Beyond Reasonable Belief
Overwhelmingly Unlikely
Evidently False
Certainly False

Epistemically Negative

13-Strength-Factor Continuum

Epistemically Positive

(6)	Certain
(5)	Evident
(4)	Overwhelmingly Likely
(3)	Beyond Reasonable Doubt
(2)	Likely
(1)	More Likely Than Not
(0)	Counterbalanced
(-1)	More Unlikely Than Not
(-2)	Unlikely
(-3)	Beyond Reasonable Belief
(-4)	Overwhelmingly Unlikely
(-5)	Evidently False
(-6)	Certainly False

Epistemically Negative

13-Strength-Factor Continuum

Epistemically Positive

(6) Certain
(5) Evident
(4) Overwhelmingly Likely
(3) Beyond Reasonable Doubt
(2) Likely
(1) More Likely Than Not
(0) Counterbalanced
(-1) More Unlikely Than Not
(-2) Unlikely
(-3) Overwhelmingly Unlikely
(-4) Beyond Reasonable Belief
(-5) Evidently False
(-6) Certainly False

Epistemically Negative

13-Strength-Factor Continuum

Key Principles

Epistemically Positive

(6) Certain
(5) Evident
(4) Overwhelmingly Likely
(3) Beyond Reasonable Doubt
(2) Likely
(1) More Likely Than Not
(0) Counterbalanced
(-1) More Unlikely Than Not
(-2) Unlikely
(-3) Overwhelmingly Unlikely
(-4) Beyond Reasonable Belief
(-5) Evidently False
(-6) Certainly False

Epistemically Negative

Key Principles

Deduction preserves strength.

13-Strength-Factor Continuum

Epistemically Positive

(6) Certain
(5) Evident
(4) Overwhelmingly Likely
(3) Beyond Reasonable Doubt
(2) Likely
(1) More Likely Than Not
(0) Counterbalanced
(-1) More Unlikely Than Not
(-2) Unlikely
(-3) Overwhelmingly Unlikely
(-4) Beyond Reasonable Belief
(-5) Evidently False
(-6) Certainly False

Epistemically Negative

Key Principles

Deduction preserves strength.

Clashes are resolved in favor of higher strength; clashes propagate backwards through inverse deduction.

13-Strength-Factor Continuum

Epistemically Positive

(6) Certain

(5) Evident

(4) Overwhelmingly Likely

(3) Beyond Reasonable Doubt

(2) Likely

(1) More Likely Than Not

(0) Counterbalanced

(-1) More Unlikely Than Not

(-2) Unlikely

(-3) Overwhelmingly Unlikely

(-4) Beyond Reasonable Belief

(-5) Evidently False

(-6) Certainly False

Epistemically Negative

Key Principles

Deduction preserves strength.

Clashes are resolved in favor of higher strength; clashes propagate backwards through inverse deduction.

Any proposition *p* such that *prob*(*p*) < 1 is at most evident.

13-Strength-Factor Continuum

Epistemically Positive

(6) Certain
(5) Evident
(4) Overwhelmingly Likely
(3) Beyond Reasonable Doubt
(2) Likely
(1) More Likely Than Not
(0) Counterbalanced
(-1) More Unlikely Than Not
(-2) Unlikely
(-3) Overwhelmingly Unlikely
(-4) Beyond Reasonable Belief
(-5) Evidently False
(-6) Certainly False

Epistemically Negative

Key Principles

Deduction preserves strength.

Clashes are resolved in favor of higher strength; clashes propagate backwards through inverse deduction.

Any proposition *p* such that *prob*(*p*) < 1 is at most evident.

Any rational belief that *p*, where the basis for *p* is at most evident, is at most an evident (= level 3) belief.

Sequence 1, "Rigorized"

Let **D** be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent *a* is fully apprised.

From **D** it obviously can be proved that either ticket 1 will win or ticket 2 will win or … or ticket 1,000,000,000,000 will win. Let's write this (exclusive) disjunction as follows:

$$Wt_1 \oplus Wt_2 \oplus \ldots \oplus Wt_{1T} \quad (1)$$

We then deduce from this that there is at least one ticket that will win, a proposition represented — using standard notation — as:

$$\exists t_i W t_i \quad (2)$$

Very well; perfectly clear so far. And now we can add another deductive step: Since our rational agent *a* can follow this deduction sequence to this point, and since **D** is assumed to be indubitable, it follows that our rational agent in turn believes (2); i.e., we conclude Sequence 1 by obtaining the following:

$$\mathbf{B}_a \exists t_i W t_i \quad (3)$$

Sequence 1, "Rigorized"

6 Let **D** be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent *a* is fully apprised.

From **D** it obviously can be proved that either ticket 1 will win or ticket 2 will win or … or ticket 1,000,000,000,000 will win. Let's write this (exclusive) disjunction as follows:

$$Wt_1 \oplus Wt_2 \oplus \ldots \oplus Wt_{1T} \quad (1)$$

We then deduce from this that there is at least one ticket that will win, a proposition represented — using standard notation — as:

$$\exists t_i Wt_i \quad (2)$$

Very well; perfectly clear so far. And now we can add another deductive step: Since our rational agent *a* can follow this deduction sequence to this point, and since **D** is assumed to be indubitable, it follows that our rational agent in turn believes (2); i.e., we conclude Sequence 1 by obtaining the following:

$$\mathbf{B}_a \exists t_i Wt_i \quad (3)$$

Sequence 1, “Rigorized”

6 Let **D** be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent *a* is fully apprised.

From **D** it obviously can be proved that either ticket 1 will win or ticket 2 will win or … or ticket 1,000,000,000,000 will win. Let’s write this (exclusive) disjunction as follows:

6 $$Wt_1 \oplus Wt_2 \oplus \ldots \oplus Wt_{1T} \quad (1)$$

We then deduce from this that there is at least one ticket that will win, a proposition represented — using standard notation — as:

$$\exists t_i W t_i \quad (2)$$

Very well; perfectly clear so far. And now we can add another deductive step: Since our rational agent *a* can follow this deduction sequence to this point, and since **D** is assumed to be indubitable, it follows that our rational agent in turn believes (2); i.e., we conclude Sequence 1 by obtaining the following:

$$\mathbf{B}_a \exists t_i W t_i \quad (3)$$

Sequence 1, "Rigorized"

6 Let **D** be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent *a* is fully apprised.

From **D** it obviously can be proved that either ticket 1 will win or ticket 2 will win or … or ticket 1,000,000,000,000 will win. Let's write this (exclusive) disjunction as follows:

6 $$Wt_1 \oplus Wt_2 \oplus \ldots \oplus Wt_{1T} \quad (1)$$

We then deduce from this that there is at least one ticket that will win, a proposition represented — using standard notation — as:

6 $$\exists t_i W t_i \quad (2)$$

Very well; perfectly clear so far. And now we can add another deductive step: Since our rational agent *a* can follow this deduction sequence to this point, and since **D** is assumed to be indubitable, it follows that our rational agent in turn believes (2); i.e., we conclude Sequence 1 by obtaining the following:

$$\mathbf{B}_a \exists t_i W t_i \quad (3)$$

Sequence 1, "Rigorized"

6 Let **D** be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent *a* is fully apprised.

From **D** it obviously can be proved that either ticket 1 will win or ticket 2 will win or … or ticket 1,000,000,000,000 will win. Let's write this (exclusive) disjunction as follows:

6 $$Wt_1 \oplus Wt_2 \oplus \ldots \oplus Wt_{1T} \quad (1)$$

We then deduce from this that there is at least one ticket that will win, a proposition represented — using standard notation — as:

6 $$\exists t_i Wt_i \quad (2)$$

Very well; perfectly clear so far. And now we can add another deductive step: Since our rational agent *a* can follow this deduction sequence to this point, and since **D** is assumed to be indubitable, it follows that our rational agent in turn believes (2); i.e., we conclude Sequence 1 by obtaining the following:

6 $$\mathbf{B}_a \exists t_i Wt_i \quad (3)$$

Sequence 1, "Rigorized"

6 Let **D** be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent *a* is fully apprised.

From **D** it obviously can be proved that either ticket 1 will win or ticket 2 will win or … or ticket 1,000,000,000,000 will win. Let's write this (exclusive) disjunction as follows:

6 $$Wt_1 \oplus Wt_2 \oplus \ldots \oplus Wt_{1T} \quad (1)$$

We then deduce from this that there is at least one ticket that will win, a proposition represented — using standard notation — as:

6 $$\exists t_i W t_i \quad (2)$$

Very well; perfectly clear so far. And now we can add another deductive step: Since our rational agent *a* can follow this deduction sequence to this point, and since **D** is assumed to be indubitable, it follows that our rational agent in turn believes (2); i.e., we conclude Sequence 1 by obtaining the following:

6

Sequence 1, “Rigorized”

6 Let **D** be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent *a* is fully apprised.

From **D** it obviously can be proved that either ticket 1 will win or ticket 2 will win or … or ticket 1,000,000,000,000 will win. Let’s write this (exclusive) disjunction as follows:

$$6 \quad Wt_1 \oplus Wt_2 \oplus \ldots \oplus Wt_{1T} \quad (1)$$

We then deduce from this that there is at least one ticket that will win, a proposition represented — using standard notation — as:

$$6 \quad \exists t_i W t_i \quad (2)$$

Very well; perfectly clear so far. And now we can add another deductive step: Since our rational agent *a* can follow this deduction sequence to this point, and since **D** is assumed to be indubitable, it follows that our rational agent in turn believes (2); i.e., we conclude Sequence 1 by obtaining the following:

$$6 \quad \mathbf{B}_a^6 \exists t_i W t_i \quad (3)$$

Sequence 2, "Rigorized"

As in Sequence 1, once again let **D** be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent *a* is fully apprised.

From **D** it obviously can be proved that the probability of a particular ticket t_i winning is 1 in 1,000,000,000,000. Using '1T' to denote 1 trillion, we can write the probability for each ticket to win as a conjunction:

$$prob(Wt_1) = \frac{1}{1,000,000,000,000} = \frac{1}{1T} \wedge prob(Wt_2) = \frac{1}{1T} \wedge \ldots \wedge prob(Wt_{1T}) = \frac{1}{1T} \quad (1)$$

For the next step, note that the probability of ticket t_I winning is lower than, say, the probability that if you walk outside a minute from now you will be flattened on the spot by a door from a 747 that falls from a jet of that type cruising at 35,000 feet. Since you have the rational belief that death won't ensue if you go outside (and have this belief precisely because you believe that the odds of your sudden demise in this manner are vanishingly small), the inference to the rational belief on the part of *a* that t_I won't win sails through—and this of course works for each ticket. Hence we have:

$$\mathbf{B}_a \neg Wt_1 \wedge \mathbf{B}_a \neg Wt_2 \wedge \ldots \wedge \mathbf{B}_a \neg Wt_{1T} \quad (2)$$

Of course, if a rational agent believes *P*, and believes *Q* as well, it follows that that agent will believe the conjunction *P* & *Q*. Applying this principle to (2) yields:

$$\mathbf{B}_a(\neg Wt_1 \wedge \neg Wt_2 \wedge \ldots \wedge \neg Wt_{1T}) \quad (3)$$

But (3) is logically equivalent to the statement that there doesn't exist a winning ticket; i.e., (3) is logically equivalent to this result from Sequence 2:

$$\mathbf{B}_a \neg \exists t_i Wt_i \quad (4)$$

Sequence 2, "Rigorized"

6 As in Sequence 1, once again let **D** be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent *a* is fully apprised.

From **D** it obviously can be proved that the probability of a particular ticket t_i winning is 1 in 1,000,000,000,000. Using '1T' to denote 1 trillion, we can write the probability for each ticket to win as a conjunction:

$$prob(Wt_1) = \frac{1}{1,000,000,000,000} = \frac{1}{1T} \wedge prob(Wt_2) = \frac{1}{1T} \wedge \ldots \wedge prob(Wt_{1T}) = \frac{1}{1T} \quad (1)$$

For the next step, note that the probability of ticket t_I winning is lower than, say, the probability that if you walk outside a minute from now you will be flattened on the spot by a door from a 747 that falls from a jet of that type cruising at 35,000 feet. Since you have the rational belief that death won't ensue if you go outside (and have this belief precisely because you believe that the odds of your sudden demise in this manner are vanishingly small), the inference to the rational belief on the part of *a* that t_I won't win sails through—and this of course works for each ticket. Hence we have:

$$\mathbf{B}_a \neg W t_1 \wedge \mathbf{B}_a \neg W t_2 \wedge \ldots \wedge \mathbf{B}_a \neg W t_{1T} \quad (2)$$

Of course, if a rational agent believes *P*, and believes *Q* as well, it follows that that agent will believe the conjunction *P* & *Q*. Applying this principle to (2) yields:

$$\mathbf{B}_a (\neg W t_1 \wedge \neg W t_2 \wedge \ldots \wedge \neg W t_{1T}) \quad (3)$$

But (3) is logically equivalent to the statement that there doesn't exist a winning ticket; i.e., (3) is logically equivalent to this result from Sequence 2:

$$\mathbf{B}_a \neg \exists t_i W t_i \quad (4)$$

Sequence 2, "Rigorized"

6 As in Sequence 1, once again let **D** be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent *a* is fully apprised.

From **D** it obviously can be proved that the probability of a particular ticket t_i winning is 1 in 1,000,000,000,000. Using '1T' to denote 1 trillion, we can write the probability for each ticket to win as a conjunction:

6 $$prob(Wt_1) = \frac{1}{1,000,000,000,000} = \frac{1}{1T} \land prob(Wt_2) = \frac{1}{1T} \land \ldots \land prob(Wt_{1T}) = \frac{1}{1T} \quad (1)$$

For the next step, note that the probability of ticket t_l winning is lower than, say, the probability that if you walk outside a minute from now you will be flattened on the spot by a door from a 747 that falls from a jet of that type cruising at 35,000 feet. Since you have the rational belief that death won't ensue if you go outside (and have this belief precisely because you believe that the odds of your sudden demise in this manner are vanishingly small), the inference to the rational belief on the part of *a* that t_l won't win sails through—and this of course works for each ticket. Hence we have:

$$\mathbf{B}_a \neg Wt_1 \land \mathbf{B}_a \neg Wt_2 \land \ldots \land \mathbf{B}_a \neg Wt_{1T} \quad (2)$$

Of course, if a rational agent believes *P*, and believes *Q* as well, it follows that that agent will believe the conjunction *P* & *Q*. Applying this principle to (2) yields:

$$\mathbf{B}_a (\neg Wt_1 \land \neg Wt_2 \land \ldots \land \neg Wt_{1T}) \quad (3)$$

But (3) is logically equivalent to the statement that there doesn't exist a winning ticket; i.e., (3) is logically equivalent to this result from Sequence 2:

$$\mathbf{B}_a \neg \exists t_i Wt_i \quad (4)$$

Sequence 2, “Rigorized”

6 As in Sequence 1, once again let **D** be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent *a* is fully apprised.

From **D** it obviously can be proved that the probability of a particular ticket t_i winning is 1 in 1,000,000,000,000. Using ‘1T’ to denote 1 trillion, we can write the probability for each ticket to win as a conjunction:

6 $$prob(Wt_1) = \frac{1}{1,000,000,000,000} = \frac{1}{1T} \wedge prob(Wt_2) = \frac{1}{1T} \wedge \ldots \wedge prob(Wt_{1T}) = \frac{1}{1T} \quad (1)$$

For the next step, note that the probability of ticket t_1 winning is lower than, say, the probability that if you walk outside a minute from now you will be flattened on the spot by a door from a 747 that falls from a jet of that type cruising at 35,000 feet. Since you have the rational belief that death won't ensue if you go outside (and have this belief precisely because you believe that the odds of your sudden demise in this manner are vanishingly small), the inference to the rational belief on the part of *a* that t_1 won’t win sails through—and this of course works for each ticket. Hence we have:

Of course, if a rational agent believes *P*, and believes *Q* as well, it follows that that agent will believe the conjunction *P* & *Q*. Applying this principle to (2) yields:

$$\mathbf{B}_a(\neg Wt_1 \wedge \neg Wt_2 \wedge \ldots \wedge \neg Wt_{1T}) \quad (3)$$

But (3) is logically equivalent to the statement that there doesn’t exist a winning ticket; i.e., (3) is logically equivalent to this result from Sequence 2:

$$\mathbf{B}_a \neg \exists t_i W t_i \quad (4)$$

Sequence 2, "Rigorized"

6 As in Sequence 1, once again let **D** be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent *a* is fully apprised.

From **D** it obviously can be proved that the probability of a particular ticket t_i winning is 1 in 1,000,000,000,000. Using '1T' to denote 1 trillion, we can write the probability for each ticket to win as a conjunction:

6 $$prob(Wt_1) = \frac{1}{1,000,000,000,000} = \frac{1}{1T} \wedge prob(Wt_2) = \frac{1}{1T} \wedge \ldots \wedge prob(Wt_{1T}) = \frac{1}{1T} \quad (1)$$

For the next step, note that the probability of ticket t_1 winning is lower than, say, the probability that if you walk outside a minute from now you will be flattened on the spot by a door from a 747 that falls from a jet of that type cruising at 35,000 feet. Since you have the rational belief that death won't ensue if you go outside (and have this belief precisely because you believe that the odds of your sudden demise in this manner are vanishingly small), the inference to the rational belief on the part of *a* that t_1 won't win sails through—and this of course works for each ticket. Hence we have:

$$\mathbf{B}_a^5 \neg Wt_1 \wedge \mathbf{B}_a^5 \neg Wt_2 \wedge \ldots \wedge \mathbf{B}_a^5 \neg Wt_{1T} \quad (2)$$

Of course, if a rational agent believes *P*, and believes *Q* as well, it follows that that agent will believe the conjunction *P* & *Q*. Applying this principle to (2) yields:

$$\mathbf{B}_a(\neg Wt_1 \wedge \neg Wt_2 \wedge \ldots \wedge \neg Wt_{1T}) \quad (3)$$

But (3) is logically equivalent to the statement that there doesn't exist a winning ticket; i.e., (3) is logically equivalent to this result from Sequence 2:

$$\mathbf{B}_a \neg \exists t_i W t_i \quad (4)$$

Sequence 2, "Rigorized"

6 As in Sequence 1, once again let **D** be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent *a* is fully apprised.

From **D** it obviously can be proved that the probability of a particular ticket t_i winning is 1 in 1,000,000,000,000. Using '1T' to denote 1 trillion, we can write the probability for each ticket to win as a conjunction:

6 $$prob(Wt_1) = \frac{1}{1,000,000,000,000} = \frac{1}{1T} \wedge prob(Wt_2) = \frac{1}{1T} \wedge \ldots \wedge prob(Wt_{1T}) = \frac{1}{1T} \quad (1)$$

For the next step, note that the probability of ticket t_1 winning is lower than, say, the probability that if you walk outside a minute from now you will be flattened on the spot by a door from a 747 that falls from a jet of that type cruising at 35,000 feet. Since you have the rational belief that death won't ensue if you go outside (and have this belief precisely because you believe that the odds of your sudden demise in this manner are vanishingly small), the inference to the rational belief on the part of *a* that t_1 won't win sails through—and this of course works for each ticket. Hence we have:

$$\mathbf{B}_a^5 \neg Wt_1 \wedge \mathbf{B}_a^5 \neg Wt_2 \wedge \ldots \wedge \mathbf{B}_a^5 \neg Wt_{1T} \quad (2)$$

Of course, if a rational agent believes *P*, and believes *Q* as well, it follows that that agent will believe the conjunction *P* & *Q*. Applying this principle to (2) yields:

But (3) is logically equivalent to the statement that there doesn't exist a winning ticket; i.e., (3) is logically equivalent to this result from Sequence 2:

$$\mathbf{B}_a \neg \exists t_i W t_i \quad (4)$$

Sequence 2, "Rigorized"

6 As in Sequence 1, once again let **D** be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent *a* is fully apprised.

From **D** it obviously can be proved that the probability of a particular ticket t_i winning is 1 in 1,000,000,000,000. Using '1T' to denote 1 trillion, we can write the probability for each ticket to win as a conjunction:

6 $$prob(Wt_1) = \frac{1}{1,000,000,000,000} = \frac{1}{1T} \wedge prob(Wt_2) = \frac{1}{1T} \wedge \ldots \wedge prob(Wt_{1T}) = \frac{1}{1T} \quad (1)$$

For the next step, note that the probability of ticket t_1 winning is lower than, say, the probability that if you walk outside a minute from now you will be flattened on the spot by a door from a 747 that falls from a jet of that type cruising at 35,000 feet. Since you have the rational belief that death won't ensue if you go outside (and have this belief precisely because you believe that the odds of your sudden demise in this manner are vanishingly small), the inference to the rational belief on the part of *a* that t_1 won't win sails through—and this of course works for each ticket. Hence we have:

$$\mathbf{B}_a^5 \neg Wt_1 \wedge \mathbf{B}_a^5 \neg Wt_2 \wedge \ldots \wedge \mathbf{B}_a^5 \neg Wt_{1T} \quad (2)$$

Of course, if a rational agent believes *P*, and believes *Q* as well, it follows that that agent will believe the conjunction *P* & *Q*. Applying this principle to (2) yields:

$$\mathbf{B}_a^5 (\neg Wt_1 \wedge \neg Wt_2 \wedge \ldots \wedge \neg Wt_{1T}) \quad (3)$$

But (3) is logically equivalent to the statement that there doesn't exist a winning ticket; i.e., (3) is logically equivalent to this result from Sequence 2:

$$\mathbf{B}_a \neg \exists t_i Wt_i \quad (4)$$

Sequence 2, "Rigorized"

6 As in Sequence 1, once again let **D** be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent *a* is fully apprised.

From **D** it obviously can be proved that the probability of a particular ticket t_i winning is 1 in 1,000,000,000,000. Using '1T' to denote 1 trillion, we can write the probability for each ticket to win as a conjunction:

6 $$prob(Wt_1) = \frac{1}{1,000,000,000,000} = \frac{1}{1T} \wedge prob(Wt_2) = \frac{1}{1T} \wedge \ldots \wedge prob(Wt_{1T}) = \frac{1}{1T} \quad (1)$$

For the next step, note that the probability of ticket t_l winning is lower than, say, the probability that if you walk outside a minute from now you will be flattened on the spot by a door from a 747 that falls from a jet of that type cruising at 35,000 feet. Since you have the rational belief that death won't ensue if you go outside (and have this belief precisely because you believe that the odds of your sudden demise in this manner are vanishingly small), the inference to the rational belief on the part of *a* that t_l won't win sails through—and this of course works for each ticket. Hence we have:

$$\mathbf{B}_a^5 \neg Wt_1 \wedge \mathbf{B}_a^5 \neg Wt_2 \wedge \ldots \wedge \mathbf{B}_a^5 \neg Wt_{1T} \quad (2)$$

Of course, if a rational agent believes *P*, and believes *Q* as well, it follows that that agent will believe the conjunction *P* & *Q*. Applying this principle to (2) yields:

$$\mathbf{B}_a^5 (\neg Wt_1 \wedge \neg Wt_2 \wedge \ldots \wedge \neg Wt_{1T}) \quad (3)$$

But (3) is logically equivalent to the statement that there doesn't exist a winning ticket; i.e., (3) is logically equivalent to this result from Sequence 2:

Sequence 2, "Rigorized"

6 As in Sequence 1, once again let **D** be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent *a* is fully apprised.

From **D** it obviously can be proved that the probability of a particular ticket t_i winning is 1 in 1,000,000,000,000. Using '1T' to denote 1 trillion, we can write the probability for each ticket to win as a conjunction:

6 $$prob(Wt_1) = \frac{1}{1,000,000,000,000} = \frac{1}{1T} \wedge prob(Wt_2) = \frac{1}{1T} \wedge \ldots \wedge prob(Wt_{1T}) = \frac{1}{1T} \quad (1)$$

For the next step, note that the probability of ticket t_1 winning is lower than, say, the probability that if you walk outside a minute from now you will be flattened on the spot by a door from a 747 that falls from a jet of that type cruising at 35,000 feet. Since you have the rational belief that death won't ensue if you go outside (and have this belief precisely because you believe that the odds of your sudden demise in this manner are vanishingly small), the inference to the rational belief on the part of *a* that t_1 won't win sails through—and this of course works for each ticket. Hence we have:

$$\mathbf{B}_a^5 \neg Wt_1 \wedge \mathbf{B}_a^5 \neg Wt_2 \wedge \ldots \wedge \mathbf{B}_a^5 \neg Wt_{1T} \quad (2)$$

Of course, if a rational agent believes *P*, and believes *Q* as well, it follows that that agent will believe the conjunction *P* & *Q*. Applying this principle to (2) yields:

$$\mathbf{B}_a^5 (\neg Wt_1 \wedge \neg Wt_2 \wedge \ldots \wedge \neg Wt_{1T}) \quad (3)$$

But (3) is logically equivalent to the statement that there doesn't exist a winning ticket; i.e., (3) is logically equivalent to this result from Sequence 2:

$$\mathbf{B}_a^5 \neg \exists t_i W t_i \quad (4)$$

Sequence 2, "Rigorized"

6 As in Sequence 1, once again let **D** be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent *a* is fully apprised.

From **D** it obviously can be proved that the probability of a particular ticket t_i winning is 1 in 1,000,000,000,000. Using '1T' to denote 1 trillion, we can write the probability for each ticket to win as a conjunction:

6 $$prob(Wt_1) = \frac{1}{1,000,000,000,000} = \frac{1}{1T} \wedge prob(Wt_2) = \frac{1}{1T} \wedge \ldots \wedge prob(Wt_{1T}) = \frac{1}{1T} \quad (1)$$

For the next step, note that the probability of ticket t_l winning is lower than, say, the probability that if you walk outside a minute from now you will be flattened on the spot by a door from a 747 that falls from a jet of that type cruising at 35,000 feet. Since you have the rational belief that death won't ensue if you go outside (and have this belief precisely because you believe that the odds of your sudden demise in this manner are vanishingly small), the inference to the rational belief on the part of *a* that t_l won't win sails through—and this of course works for each ticket. Hence we have:

$$\mathbf{B}_a^5 \neg Wt_1 \wedge \mathbf{B}_a^5 \neg Wt_2 \wedge \ldots \wedge \mathbf{B}_a^5 \neg Wt_{1T} \quad (2)$$

Of course, if a rational agent believes *P*, and believes *Q* as well, it follows that that agent will believe the conjunction *P* & *Q*. Applying this principle to (2) yields:

$$\mathbf{B}_a^5 (\neg Wt_1 \wedge \neg Wt_2 \wedge \ldots \wedge \neg Wt_{1T}) \quad (3)$$

But (3) is logically equivalent to the statement that there doesn't exist a winning ticket; i.e., (3) is logically equivalent to this result from Sequence 2:

$$\mathbf{B}_a^5 \neg \exists t_i W t_i \quad (4)$$

Paradox Solved!

Deduction preserves strength.

Clashes are resolved in favor of higher strength; clashes propagate backwards through inverse deduction.

Any proposition *p* such that *prob*(*p*) < 1 is at most evident.

Any rational belief that *p*, where the basis for *p* is at most evident, is at most an evident (= level 3) belief.

Paradox Solved!

Deduction preserves strength.

Clashes are resolved in favor of higher strength; clashes propagate backwards through inverse deduction.

$$\mathrm{B}_a^5 \neg \exists t_i W t_i \quad (4)$$

Any proposition *p* such that *prob*(*p*) < 1 is at most evident.

Any rational belief that *p*, where the basis for *p* is at most evident, is at most an evident (= level 3) belief.

Paradox Solved!

Deduction preserves strength.

Clashes are resolved in favor of higher strength; clashes propagate backwards through inverse deduction.

$$\mathrm{B}_a^6 \exists t_i W t_i \quad (3) \qquad\qquad \mathrm{B}_a^5 \neg \exists t_i W t_i \quad (4)$$

Any proposition *p* such that *prob*(*p*) < 1 is at most evident.

Any rational belief that *p*, where the basis for *p* is at most evident, is at most an evident (= level 3) belief.

Paradox Solved!

Deduction preserves strength.

Clashes are resolved in favor of higher strength; clashes propagate backwards through inverse deduction.

$\mathbf{B}_a^6 \exists t_i W t_i \quad (3)$ $\mathbf{B}_a^5 \neg \exists t_i W t_i \quad (4)$

Any proposition *p* such that *prob*(*p*) < 1 is at most evident.

Any rational belief that *p*, where the basis for *p* is at most evident, is at most an evident (= level 3) belief.

Paradox Solved!

Deduction preserves strength.

Clashes are resolved in favor of higher strength; clashes propagate backwards through inverse deduction.

$$\mathbf{B}^5_a \exists t_i W t_i \quad (3)$$
$$\mathbf{B}^6_a \exists t_i W t_i \quad (4)$$

Any proposition *p* such that *prob*(*p*) < 1 is at most evident.

Any rational belief that *p*, where the basis for *p* is at most evident, is at most an evident (= level 3) belief.

Paradox Solved!

Deduction preserves strength.

Clashes are resolved in favor of higher strength; clashes propagate backwards through inverse deduction.

$$\mathrm{B}_a^6 \exists t_i W t_i \quad (3)$$

Any proposition *p* such that *prob*(*p*) < 1 is at most evident.

Any rational belief that *p*, where the basis for *p* is at most evident, is at most an evident (= level 3) belief.

Paradox Solved!

Deduction preserves strength.

Clashes are resolved in favor of higher strength; clashes propagate backwards through inverse deduction.

$$\mathbf{B}_a^6 \exists t_i W t_i \quad (3)$$

Any proposition *p* such that *prob*(*p*) < 1 is at most evident.

Any rational belief that *p*, where the basis for *p* is at most evident, is at most an evident (= level 3) belief.

$$\mathbf{B}_a^5 \neg W t_1 \wedge \mathbf{B}_a^5 \neg W t_2 \wedge \ldots \wedge \mathbf{B}_a^5 \neg W t_{1T} \quad (2)$$

Paradox Solved!

Deduction preserves strength.

Clashes are resolved in favor of higher strength; clashes propagate backwards through inverse deduction.

$$\mathbf{B}_a^6 \exists t_i W t_i \quad (3)$$

Any proposition *p* such that *prob*(*p*) < 1 is at most evident.

Any rational belief that *p*, where the basis for *p* is at most evident, is at most an evident (= level 3) belief.

$$\mathbf{B}_a^5 \neg W t_1 \wedge \boxed{\mathbf{B}_a^5 \neg W t_2} \wedge \ldots \wedge \mathbf{B}_a^5 \neg W t_{1T} \quad (2)$$

Paradox Solved!

Deduction preserves strength.

Clashes are resolved in favor of higher strength; clashes propagate backwards through inverse deduction.

$$\mathbf{B}_a^6 \exists t_i W t_i \quad (3)$$

Any proposition *p* such that *prob*(*p*) < 1 is at most evident.

Any rational belief that *p*, where the basis for *p* is at most evident, is at most an evident (= level 3) belief.

$$\mathbf{B}_a^5 \neg W t_1 \wedge \boxed{\mathbf{B}_a^5 \neg W t_2} \wedge \ldots \wedge \mathbf{B}_a^5 \neg W t_{1T} \quad (2)$$

This is why, to Mega-Millions ticket holder & wife Elizabeth:
"Sorry. I'm rational, and I believe you won't win."

To be clear about the effects of the first principle:

$$\vdash \mathbf{B}^4_a \neg\exists x W x \wedge \mathbf{B}^4_a \exists x W x!$$

$$\vdash \mathbf{B}^3_a \neg\exists x W x \wedge \mathbf{B}^3_a \exists x W x!$$

$$\vdash \mathbf{B}^2_a \neg\exists x W x \wedge \mathbf{B}^2_a \exists x W x!$$

$$\vdash \mathbf{B}^1_a \neg\exists x W x \wedge \mathbf{B}^1_a \exists x W x!$$

Clashes are resolved in favor of higher strength; clashes propagate backwards through inverse deduction, preserving affirmation/belief of premises as far as is possible; if no higher-strength factors, suspend belief. (This means that in this case belief at level 5 also shoots down belief at levels 4, 3, 3, 1. This is sort of bizarre, because to retain the belief (at levels 3, 2, 1) that every particular ticket won't win, the step that gets to believing the existential formula is blocked. Pollock doesn't have steps in his "arguments." Our agents thus ends up believing at all levels that some ticket will win, and believing at all levels 4 and down to 1, of each particular ticket, that it won't win.)

The St Petersburg Paradox …

Ignore Those Who Say WWTBAM is an Instance

Ignore Those Who Say WWTBAM is an Instance

presently rational within this game!

Here’s the game; $30 to play.

Here's the game; $30 to play.

Here’s the game; $30 to play.

Here’s the game; $30 to play.

Here’s the game; $30 to play.

$2

Here's the game; $30 to play.

$2

Here's the game; $30 to play.

$2

$4

Here's the game; $30 to play.

$2

$4

Here’s the game; $30 to play.

$2

$4

$8

Here's the game; $30 to play.

$2

$4

$8

Here’s the game; $30 to play.

$2

$4

$8

$10

Here's the game; $30 to play.

For $n = 1, 2, \ldots, 10$

n	*prob(n)*	Prize	Expected Payoff
1	1/2	$2	$1
2	1/4	$4	$1
3	1/8	$8	$1
4	1/16	$16	$1
5	1/32	$32	$1
6	1/64	$64	$1
7	1/128	$128	$1
8	1/256	$256	$1
9	1/512	$512	$1
10	1/1024	$1,024	$1
…	…	…	…

For $n = 1, 2, \ldots, 10$

n	*prob(n)*	Prize	Expected Payoff
1	1/2	\$2	\$1
2	1/4	\$4	\$1
3	1/8	\$8	\$1
4	1/16	\$16	\$1
5	1/32	\$32	\$1
6	1/64	\$64	\$1
7	1/128	\$128	\$1
8	1/256	\$256	\$1
9	1/512	\$512	\$1
10	1/1024	\$1,024	\$1
...	9.5367431640625e-7	...	...

For $n = 1, 2, \ldots, 10$

n	$p$rob(n)	Prize	Expected Payoff
1	1/2	$2	$1
2	1/4	$4	$1
3	1/8	$8	$1
4	1/16	$16	$1
5	1/32	$32	$1
6	1/64	$64	$1
7	1/128	$128	$1
8	1/256	$256	$1
9	1/512	$512	$1
10	1/1024	$1,024	$1
...	9.5367431640625e-7	...	...

Theorem: You should give Selmer & Naveen all your savings ($*k*) to play the game.

For $n = 1, 2, \ldots, 10$

n	$p$rob(n)	Prize	Expected Payoff
1	1/2	\$2	\$1
2	1/4	\$4	\$1
3	1/8	\$8	\$1
4	1/16	\$16	\$1
5	1/32	\$32	\$1
6	1/64	\$64	\$1
7	1/128	\$128	\$1
8	1/256	\$256	\$1
9	1/512	\$512	\$1
10	1/1024	\$1,024	\$1
...	9.5367431640625e-7	...	...

Theorem: You should give Selmer & Naveen all your savings (\$$k$) to play the game.

Proof: The expected value of playing the game is \$$\Omega$. We know that you're rational, so since any finite amount of dollars isn't an infinite amount of dollars, you will pay \$$k$ to us to play. **QED**

For $n = 1, 2, \ldots, 10$

n	$prob(n)$	Prize	Expected Payoff
1	1/2	\$2	\$1
2	1/4	\$4	\$1
3	1/8	\$8	\$1
4	1/16	\$16	\$1
5	1/32	\$32	\$1
6	1/64	\$64	\$1
7	1/128	\$128	\$1
8	1/256	\$256	\$1
9	1/512	\$512	\$1
10	1/1024	\$1,024	\$1
…	9.5367431640625e-7	…	…

Theorem: … the game.

Pr… v

that …

infinite amount of dollars, you will pay \$$k$ to us to play. **QED**

For \$20 on the spot (& \$20 to me if you're wrong), raise your hand to name the principle you saw last lecture that this proof relies on!

The Optimality Principle

When choosing between alternative actions a_1 and a_2, rationality dictates choosing that action that maximizes expected value, computed by multiplying the value of each outcome that can result from each action by the probability that it will occur, adding the results together, and selecting the action associated with the higher utility.

The Optimality Principle

The Optimality Principle

Sorry, one-boxers!

Bernoulli's Bad Idea

n	*prob(n)*	Prize	Utiles	Expected Utility
1	1/2	$2	0.301	0.1505
2	1/4	$4	0.602	0.1505
3	1/8	$8	0.903	0.1129
4	1/16	$16	1.204	0.0753
5	1/32	$32	1.505	0.0470
6	1/64	$64	1.806	0.0282
7	1/128	$128	2.107	0.0165
8	1/256	$256	2.408	0.0094
9	1/512	$512	2.709	0.0053
10	1/1024	$1,024	3.010	0.0029
...	...	...		...

Bernoulli's Bad Idea

Principle of Decreasing Marginal Utility (DMU): The utility of $\$\mathbf{k} = log_{10}(k)$.

n				ted Utility
1				505
2	1/4	$4	0.602	0.1505
3	1/8	$8	0.903	0.1129
4	1/16	$16	1.204	0.0753
5	1/32	$32	1.505	0.0470
6	1/64	$64	1.806	0.0282
7	1/128	$128	2.107	0.0165
8	1/256	$256	2.408	0.0094
9	1/512	$512	2.709	0.0053
10	1/1024	$1,024	3.010	0.0029
...	...	...		...

Bernoulli's Bad Idea

n	*prob(n)*	Prize	Utiles	Expected Utility
1	1/2	$2	0.301	0.1505
2	1/4	$4	0.602	0.1505
3	1/8	$8	0.903	0.1129
4	1/16	$16	1.204	0.0753
5	1/32	$32	1.505	0.0470
6	1/64	$64	1.806	0.0282
7	1/128	$128	2.107	0.0165
8	1/256	$256	2.408	0.0094
9	1/512	$512	2.709	0.0053
10	1/1024	$1,024	3.010	0.0029
...	...	...		...

Bernoulli's Bad Idea

n	*prob(n)*	Prize	Utiles	Expected Utility
1	1/2	$2	0.301	0.1505
2	1/4	$4	0.602	0.1505
3	1/8	$8	0.903	0.1129
4	1/16	$16	1.204	0.0753
5	1/32	$32	1.505	0.0470
6	1/64	$64	1.806	0.0282
7	1/128	$128	2.107	0.0165
8	1/256	$256	2.408	0.0094
9	1/512	$512	2.709	0.0053
10	1/1024	$1,024	3.010	0.0029
...	...	...		...

Refutation: Selmer & Naveen offer a variant game, based on stacked exponentiation. E.g., let's play a game in which the prize money is 10^{2^n}. Sorry Bernoulli! **QED**

There are *a lot* of ornate but unsuccessful proposed solutions.

The St. Petersburg Paradox: A Subjective Probability Solution

Hongbin Wang (hwang@tamhsc.edu)
Yanlong Sun (ysun@tamhsc.edu)
Jack W. Smith (jwsmith@tamhsc.edu)
Texas A&M University Health Science Center
2121 West Holcombe Blvd, Suite 1109, Houston, TX 77030 USA

Abstract

The St. Petersburg Paradox (SPP), where people are willing to pay only a modest amount for a lottery with infinite expected gain, has been a famous showcase of human (ir)rationality. Since inception multiple solutions have been proposed, including the influential expected utility theory. Criticisms remain due to the lack of a priori justification for the utility function. Here we report a new solution to the long-standing paradox, which focuses on the probability weighting component (rather than the value/utility component) in calculating the expected value of the game. We show that a new Additional Transition Time (AT) based measure, motivated by both physics and psychology, can naturally lead to a converging expected value and therefore solve the paradox.

Keywords: human judgment and decision making, probability, St. Petersburg Paradox,

Fate laughs at probabilities.
-- E. G. Bulwer-Lytton

Introduction

Suppose you are offered the following gamble:

- Toss a fair coin. If you get a head, you are paid \$1 and the game is over. Otherwise, toss again.
- If you get a head in the second tossing, you are paid \$2 and the game is over. Otherwise, toss again.
- If you get a head in the third tossing, you are paid \$4 and the game is over. Otherwise, toss again.
- … Game continues until you get a head. If you get a head in the nth tossing, you will be paid $\$2^{n-1}$.

How much are you willing to pay to play this gamble?

A simple calculation shows that the gamble's expected value, S, is infinite:

$$S = \$\sum_{n=1}^{\infty} p^n 2^{n-1} = \$\sum_{n=1}^{\infty} (\frac{1}{2})^n 2^{n-1} = \$(\frac{1}{2} + \frac{1}{2} + \ldots) \qquad \text{Eq. 1}$$

where n is the number of tosses to get the first head (i.e., after a steak of n-1 tails, one gets a head, and the game is over).

The question is, are you willing to pay any price for a right to play this game? Probably not. More than three hundred years ago, in 1713, Nicolas Bernoulli, a young Swiss mathematician, first proposed this problem and pointed out that a sensible person would only be willing to pay very little to play the game. This constitutes a contradiction, which nowadays is called the St. Petersburg Paradox (SPP).

A Little History

The SPP was so named after the eponymous Russian city, where Daniel Bernoulli, a mathematician and Nicholas Bernoulli's cousin, published his classical solution to the problem in 1738. However, the problem was initially proposed by Nicolas Bernoulli in 1713, who was clearly troubled by it. According to him, while the expectation of game gain was infinity, the player would be guaranteed to lose since it is "morally impossible" that one not achieve a head in a finite number of tossing.

In 1728, Gabriel Cramer, another Swiss mathematician, wrote to N. Bernoulli and suggested a solution. In Cramer's solution, money's quantity was replaced by its "moral value", representing the pleasure or sorrow money (or loss of money) could produce. In doing so Cramer showed the expectation would converge to less than \$3 if "one wishes to suppose that the moral value of goods was as the square root of the mathematical quantities".

N. Bernoulli was not entirely satisfied with this solution. In his reply to Cramer, N. Bernoulli wrote that the pleasure difference "does not demonstrate the true reason" for why one should not pay infinity to play the game. Even Cramer himself thought his square-root assumption about money and pleasure was not just.

Eventually in 1738, D. Bernoulli published his solution to the problem (Bernoulli, 1738). D. Bernoulli's solution was similar to Cramer's and based on the concept of utility, which measured the usefulness of values and was taken to be a logarithmic function of values. It was shown that while the expected value diverged the expected utility converged. D. Bernoulli's solution was seminal and extremely influential, and has since shaped the whole field of economics and of the psychology of decision making.

It was interesting to note that N. Bernoulli vigorously objected his cousin's approach. A series of communication showed that the two had engaged in serious arguments. To N. Bernoulli, the concept of utility, similar to the "moral value" of Cramer, was arbitrary and, to a certain extent, irrelevant. Rather, the concern here was to find a more general way to show if a game was fair, regardless of who was playing the game. "For example a game is considered fair, when the two players bet an equal sum on a game under equal conditions, although according to your theory, and by paying attention to their riches, the pleasure or the advantage of gain in the favorable case is not equal to the

Solution

Solution

1. It's irrational to bank on things that are *beyond reasonable belief*; you know this.

Solution

1. It’s irrational to bank on things that are *beyond reasonable belief*; you know this.
2. It’s *beyond reasonable belief* that you’ll win back our $30 price to play; you know this too.

Solution

1. It's irrational to bank on things that are *beyond reasonable belief*; you know this.
2. It's *beyond reasonable belief* that you'll win back our $30 price to play; you know this too.
3. Hence, since you are rational, you'll decline.

Solution

1. It's irrational to bank on things that are *beyond reasonable belief*; you know this.

2. It's *beyond reasonable belief* that you'll win back our $30 price to play; you know this too.

3. Hence, since you're rational, you'll decline.

Solution

1. It's irrational to bank on things that are *beyond reasonable belief*; you know this.

2. It's *beyond reasonable belief* that you'll win back our $30 price to play; you know this too.

3. Hence, since you're rational, you'll decline.

It's *beyond reasonable belief* that $32 will be won (an amount that would make a bet of $30 a nice wager), since the odds of that happening are 1/32.

Solution

1. It's irrational to bank on things that are *beyond reasonable belief*; you know this.

2. It's *beyond reasonable belief* that you'll win back our $30 price to play; you know this too.

3. Hence, since you're rational, you'll decline.

It's *beyond reasonable belief* that $32 will be won (an amount that would make a bet of $30 a nice wager), since the odds of that happening are 1/32.

Symmetrically put, a rational agent will believe at level 3 that $30 dollars will be lost. (Why?)

Solution

1. It's irrational to bank on things that are *beyond reasonable belief*; you know this.

2. It's *beyond reasonable belief* that you'll win back our $30 price to play; you know this too.

3. Hence, since you're rational, you'll decline.

It's *beyond reasonable belief* that $32 will be won (an amount that would make a bet of $30 a nice wager), since the odds of that happening are 1/32.

Symmetrically put, a rational agent will believe at level 3 that $30 dollars will be lost. (Why?)

The needed principle: *Ceteris paribus*, if a rational agent *a* believes at level 3 that a wager cutting meaningfully into that agent's disposable income will be lost, *a* will not make that wager.

Solution

1. It’s irrational to bank on things that are *beyond reasonable belief*; you know this.

2. It’s *beyond reasonable belief* that you’ll win back our $30 price to play; you know this too.

3. Hence, since you’re rational, you’ll decline.

It’s *beyond reasonable belief* that $32 will be won (an amount that would make a bet of $30 a nice wager), since the odds of that happening are 1/32.

Symmetrically put, a rational agent will believe at level 3 that $30 dollars will be lost. (Why?)

The needed principle: *Ceteris paribus*, if a rational agent *a* believes at level 3 that a wager cutting meaningfully into that agent’s disposable income will be lost, *a* will not make that wager.

From SEP entry: “… few of us would pay even $25 to enter such a game.” Now we know why!