The Lottery Paradox
(& The St Petersburg Paradox too!)

Selmer Bringsjord

AHR?

Oct 30 2017; Nov 2 2017
The Paradoxes: Our Coverage, & RPI Context
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**The Paradoxes of Time Travel:** Grandfather & The Paradox of Proust: PP is an active research area, sponsored by Air Force; A Sen, Naven Sundar G., S Bringsjord. Covered 11/6/16.
Types of Paradoxes

- Deductive Paradoxes — corresponding to Area 1, going all the way back to our syllabus
- Inductive Paradoxes — corresponding to Area 2 (i.e. probability etc.)
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Today (x 2)
The Lottery Paradox

• A perfectly rational person can never believe P and believe ¬P at the same time.

• The Lottery Paradox (apparently) shows, courtesy of its two Sequences (of Reasoning), that a perfectly rational person can indeed have such a belief (upon considering a fair, large lottery).

• Contradiction! — and hence a paradox!
E: “Please go down to Stewart’s & get the T U.”
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Sequence 1
Sequence 1
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Sequence 2
A rational agent should believe $p$, and believe not-$p$!
Sequence 1
Sequence 1

Let $\mathcal{D}$ be a meticulous and perfectly accurate description of a $1,000,000,000,000$-ticket lottery, of which rational agent $a$ is fully apprised.
Sequence 1

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From $D$ it obviously can be proved that either ticket 1 will win or ticket 2 will win or ... or ticket $1,000,000,000,000$ will win. Let’s write this (exclusive) disjunction as follows:
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$$Wt_1 \oplus Wt_2 \oplus \ldots \oplus Wt_{1T} \quad (1)$$
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$$B_a \ \exists t_i \ Wt_i \quad (3)$$
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√
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For the next step, note that the probability of ticket $t_i$ winning is lower than, say, the probability that if you walk outside a minute from now you will be flattened on the spot by a door from a 747 that falls from a jet of that type cruising at 35,000 feet. Since you have the rational belief that death won’t ensue if you go outside (and have this belief precisely because you believe that the odds of your sudden demise in this manner are vanishingly small), the inference to the rational belief on the part of a that $t_i$ won’t win sails through—and this of course works for each ticket. Hence we have:
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$$\mathbf{B}_a \neg Wt_1 \wedge \mathbf{B}_a \neg Wt_2 \wedge \ldots \wedge \mathbf{B}_a \neg Wt_{1T} \quad (2)$$
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Of course, if a rational agent believes \( P \), and believes \( Q \) as well, it follows that that agent will believe the conjunction \( P \& Q \). Applying this principle to (2) yields:
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$$B_a (\neg Wt_1 \land \neg Wt_2 \land \ldots \land \neg Wt_{1T}) \quad (3)$$

But (3) is logically equivalent to the statement that there doesn’t exist a winning ticket; i.e., (3) is logically equivalent to this result from Sequence 2:
Sequence 2

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$$\mathbb{B}_a \neg \exists t_i Wt_i$$  \hspace{1cm} (4)
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The unacceptable situation has arrived!

Sequence 1

Sequence 2

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(The unacceptable situation has arrived!)
Bringsjord’s Solution to The Lottery Paradox …
13 Strength-Factors

- Certain
- More Unlikely Than Not
- Improbable
- Evidently False
- Overwhelmingly Likely
- Overwhelmingly Unlikely
- Likely
- Beyond Reasonable Belief
- Certainly False
- More Likely Than Not
- Counterbalanced
- Evident
- Beyond Reasonable Doubt
13-Strength-Factor Continuum

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Epistemically Positive

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Epistemically Negative

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**Epistemically Negative**
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13-Strength-Factor Continuum

Epistemically Positive

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Key Principles

Epistemically Positive

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Any rational belief that $p$, where the basis for $p$ is at most evident, is at most an evident (= level 3) belief.
Sequence 1, “Rigorized”

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Very well; perfectly clear so far. And now we can add another deductive step: Since our rational agent $a$ can follow this deduction sequence to this point, and since $D$ is assumed to be indubitable, it follows that our rational agent in turn believes (2); i.e., we conclude Sequence 1 by obtaining the following:

$$B_a \exists t_i W_{t_i} \quad (3)$$
Sequence 1, “Rigorized”

Let $D$ be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent $a$ is fully apprised.

From $D$ it obviously can be proved that either ticket 1 will win or ticket 2 will win or … or ticket 1,000,000,000,000 will win. Let’s write this (exclusive) disjunction as follows:

$$W_{t_1} \oplus W_{t_2} \oplus \ldots \oplus W_{t_{1T}} \quad (1)$$

We then deduce from this that there is at least one ticket that will win, a proposition represented — using standard notation — as:

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From $D$ it obviously can be proved that either ticket 1 will win or ticket 2 will win or … or ticket $1,000,000,000,000$ will win. Let’s write this (exclusive) disjunction as follows:

$$Wt_1 \oplus Wt_2 \oplus \ldots \oplus Wt_{100000000000} \quad (1)$$

We then deduce from this that there is at least one ticket that will win, a proposition represented — using standard notation — as:

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Let \( D \) be a meticulous and perfectly accurate description of a \( 1,000,000,000,000 \)-ticket lottery, of which rational agent \( a \) is fully apprised.

From \( D \) it obviously can be proved that either ticket 1 will win or ticket 2 will win or \( \ldots \) or ticket \( 1,000,000,000,000 \) will win. Let’s write this (exclusive) disjunction as follows:

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We then deduce from this that there is at least one ticket that will win, a proposition represented — using standard notation — as:

\[
\exists t_i W_{t_i} \quad (2)
\]

Very well; perfectly clear so far. And now we can add another deductive step: Since our rational agent \( a \) can follow this deduction sequence to this point, and since \( D \) is assumed to be indubitable, it follows that our rational agent in turn believes (2); i.e., we conclude Sequence 1 by obtaining the following:

\[
\mathsf{B}_a^6 \exists t_i W_{t_i} \quad (3)
\]
Sequence 2, “Rigorized”

As in Sequence 1, once again let $D$ be a meticulous and perfectly accurate description of a $1,000,000,000,000$-ticket lottery, of which rational agent $a$ is fully apprised.

From $D$ it obviously can be proved that the probability of a particular ticket $t_i$ winning is $1$ in $1,000,000,000,000$. Using ‘$1T$’ to denote $1$ trillion, we can write the probability for each ticket to win as a conjunction:

$$prob(Wt_1) = \frac{1}{1,000,000,000,000} = \frac{1}{1T} \land prob(Wt_2) = \frac{1}{1T} \land \ldots \land prob(Wt_{1T}) = \frac{1}{1T} \quad (1)$$

For the next step, note that the probability of ticket $t_i$ winning is lower than, say, the probability that if you walk outside a minute from now you will be flattened on the spot by a door from a 747 that falls from a jet of that type cruising at 35,000 feet. Since you have the rational belief that death won't ensue if you go outside (and have this belief precisely because you believe that the odds of your sudden demise in this manner are vanishingly small), the inference to the rational belief on the part of $a$ that $t_i$ won't win sails through—and this of course works for each ticket. Hence we have:

$$B_a \neg Wt_1 \land B_a \neg Wt_2 \land \ldots \land B_a \neg Wt_{1T} \quad (2)$$

Of course, if a rational agent believes $P$, and believes $Q$ as well, it follows that that agent will believe the conjunction $P \land Q$. Applying this principle to (2) yields:

$$B_a(\neg Wt_1 \land \neg Wt_2 \land \ldots \land \neg Wt_{1T}) \quad (3)$$

But (3) is logically equivalent to the statement that there doesn’t exist a winning ticket; i.e., (3) is logically equivalent to this result from Sequence 2:

$$B_a \neg \exists t_i Wt_i \quad (4)$$
As in Sequence 1, once again let $D$ be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent $a$ is fully apprised.

From $D$ it obviously can be proved that the probability of a particular ticket $t_i$ winning is 1 in 1,000,000,000,000. Using ‘$1T$’ to denote 1 trillion, we can write the probability for each ticket to win as a conjunction:

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As in Sequence 1, once again let D be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent a is fully apprised.

From D it obviously can be proved that the probability of a particular ticket $t_i$ winning is 1 in 1,000,000,000,000. Using ‘1T’ to denote 1 trillion, we can write the probability for each ticket to win as a conjunction:

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For the next step, note that the probability of ticket $t_i$ winning is lower than, say, the probability that if you walk outside a minute from now you will be flattened on the spot by a door from a 747 that falls from a jet of that type cruising at 35,000 feet. Since you have the rational belief that death won't ensue if you go outside (and have this belief precisely because you believe that the odds of your sudden demise in this manner are vanishingly small), the inference to the rational belief on the part of a that $t_i$ won’t win sails through—and this of course works for each ticket. Hence we have:

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Of course, if a rational agent believes \( P \), and believes \( Q \) as well, it follows that that agent will believe the conjunction \( P \land Q \). Applying this principle to (2) yields:

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\mathcal{B}_a(\neg Wt_1 \land \neg Wt_2 \land \ldots \land \neg Wt_{1T}) \quad (3)
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\text{prob}(Wt_1) = \frac{1}{1,000,000,000,000} = \frac{1}{1T} \land \text{prob}(Wt_2) = \frac{1}{1T} \land \ldots \land \text{prob}(Wt_{1T}) = \frac{1}{1T} \quad (1)
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$$B_a^5 \neg Wt_1 \land B_a^5 \neg Wt_2 \land \ldots \land B_a^5 \neg Wt_{1T} \quad (2)$$

Of course, if a rational agent believes $P$, and believes $Q$ as well, it follows that that agent will believe the conjunction $P \land Q$. Applying this principle to (2) yields:

$$B_a^5 (\neg Wt_1 \land \neg Wt_2 \land \ldots \land \neg Wt_{1T}) \quad (3)$$

But (3) is logically equivalent to the statement that there doesn’t exist a winning ticket; i.e., (3) is logically equivalent to this result from Sequence 2:
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For the next step, note that the probability of ticket $t_1$ winning is lower than, say, the probability that if you walk outside a minute from now you will be flattened on the spot by a door from a 747 that falls from a jet of that type cruising at 35,000 feet. Since you have the rational belief that death won't ensue if you go outside (and have this belief precisely because you believe that the odds of your sudden demise in this manner are vanishingly small), the inference to the rational belief on the part of $a$ that $t_1$ won't win sails through—and this of course works for each ticket. Hence we have:

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Of course, if a rational agent believes $P$, and believes $Q$ as well, it follows that that agent will believe the conjunction $P \land Q$. Applying this principle to (2) yields:

$$\text{B}_a^{5} \neg Wt_1 \land \text{B}_a^{5} \neg Wt_2 \land \ldots \land \text{B}_a^{5} \neg Wt_{1T} \quad (2)$$

But (3) is logically equivalent to the statement that there doesn’t exist a winning ticket; i.e., (3) is logically equivalent to this result from Sequence 2:

$$\text{B}_a^{5} \exists t_i Wt_i \quad (4)$$
As in Sequence 1, once again let $D$ be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent $a$ is fully apprised.

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For the next step, note that the probability of ticket $t_i$ winning is lower than, say, the probability that if you walk outside a minute from now you will be flattened on the spot by a door from a 747 that falls from a jet of that type cruising at 35,000 feet. Since you have the rational belief that death won't ensue if you go outside (and have this belief precisely because you believe that the odds of your sudden demise in this manner are vanishingly small), the inference to the rational belief on the part of $a$ that $t_i$ won't win sails through—and this of course works for each ticket. Hence we have:

$$B^5_a \neg W_{t_1} \land B^5_a \neg W_{t_2} \land \ldots \land B^5_a \neg W_{t_{1T}}$$  \hspace{1cm} (2)

Of course, if a rational agent believes $P$, and believes $Q$ as well, it follows that that agent will believe the conjunction $P \& Q$. Applying this principle to (2) yields:

$$B^5_a (\neg W_{t_1} \land \neg W_{t_2} \land \ldots \land \neg W_{t_{1T}})$$  \hspace{1cm} (3)

But (3) is logically equivalent to the statement that there doesn’t exist a winning ticket; i.e., (3) is logically equivalent to this result from Sequence 2:

$$B^5_a \neg \exists t_i W_{t_i}$$  \hspace{1cm} (4)
Paradox Solved!

Deduction preserves strength.
Clashes are resolved in favor of higher strength; clashes propagate backwards through inverse deduction.

Any proposition $p$ such that $\text{prob}(p) < 1$ is at most evident.

Any rational belief that $p$, where the basis for $p$ is at most evident, is at most an evident (= level 3) belief.
Paradox Solved!

Deduction preserves strength.

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\[ B^5_a \rightarrow \exists t_i W t_i \] (4)

Any proposition \( p \) such that \( \text{prob}(p) < 1 \) is at most evident.

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\[ B^6_a \exists t_i W t_i \quad (3) \quad B^5_a \neg \exists t_i W t_i \quad (4) \]

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\[ B^{\exists t \in W \in t} (3)4 \]

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Deduction preserves strength.

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\[ B_a^{6} \exists t_i W t_i \quad (3) \]

Any proposition \( p \) such that \( \text{prob}(p) < 1 \) is at most evident.

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Deduction preserves strength.
Clashes are resolved in favor of higher strength; clashes propagate backwards through inverse deduction.

\[ B_a^6 \models t_i W t_i \quad (3) \]

Any proposition \( p \) such that \( prob(p) < 1 \) is at most evident.

Any rational belief that \( p \), where the basis for \( p \) is at most evident, is at most an evident (= level 3) belief.

\[ B_a^5 \models W t_1 \land B_a^5 \models W t_2 \land \ldots \land B_a^5 \models W t_{1T} \quad (2) \]
Paradox Solved!

Deduction preserves strength.

Clashes are resolved in favor of higher strength; clashes propagate backwards through inverse deduction.

\[ B^6_a \exists t_i W t_i \quad (3) \]

Any proposition \( p \) such that \( \text{prob}(p) < 1 \) is at most evident.

Any rational belief that \( p \), where the basis for \( p \) is at most evident, is at most an evident (= level 3) belief.

\[ B^5_a \neg W t_1 \land \boxed{B^5_a \neg W t_2} \land \ldots \land B^5_a \neg W t_{1T} \quad (2) \]
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This is why, to Mega-Millions ticket holder & wife Elizabeth:
“Sorry. I’m rational, and I believe you won’t win.”
The St Petersburg Paradox …
Ignore Those Who Say WWTBAM is an Instance
Ignore Those Who Say WWTBAM is an Instance presently rational within this game!
Here’s the game; $40 to play.
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For \( n = 1, 2, \ldots, 10 \)

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... $\ldots$ $9.5367431640625e-7$ ... ...
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**Theorem:** You should give Selmer & Naveen all your savings ($k$) to play the game.
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**Theorem:** You should give Selmer & Naveen all your savings (\( k \)) to play the game.

**Proof:** The expected value of playing the game is \( \Omega \). We know that you’re rational, so since any finite amount of dollars isn’t an infinite amount of dollars, you will pay \( k \) to us to play. \( \text{QED} \)
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\[ \ldots \quad \text{9.5367431640625e-7} \quad \ldots \quad \ldots \]

\textbf{Theorem:} You should give Selmer & Naveen all your savings ($k$) to play the game.

\textbf{Proof:} That you have an infinite amount of dollars, you will pay $k$ to us to play. \textit{QED}

\textbf{For $20 on the spot, name the principle you saw last lecture that this proof relies on!}
## Bernoulli’s Bad Idea

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Refutation: Selmer & Naveen offer a variant game, based on stacked exponentiation. E.g., let’s play a game in which the prize money is $10^2$. Sorry Bernoulli! QED
Solution
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2. It’s overwhelmingly unlikely that you’ll win back our $40 price to play; you know this too.
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3. Hence, since you aren’t stupid, you’ll decline.
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The needed principle: *Ceteris paribus*, if a rational agent *a* believes at level 3 that a wager cutting meaningfully into that agent’s disposable income will be lost, *a* will not make that wager.
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From SEP entry: “… few of us would pay even $25 to enter such a game.” Now we know why!