The Lottery Paradox
(& The St Petersburg Paradox too!)

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AHR?

RAIR
Rensselaer AI and Reasoning Lab
The Paradoxes: Our Coverage, & RPI Context
The Liar: You’re on your own :). Presented 10/21/19.
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Types of Paradoxes

- Deductive Paradoxes — corresponding to Area 1, going all the way back to our syllabus
- Inductive Paradoxes — corresponding to Area 2 (i.e. probability etc.)
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- Deductive Paradoxes — corresponding to Area 1, going all the way back to our syllabus

- Inductive Paradoxes — corresponding to Area 2 (i.e. probability etc.)

Today (x 2)
Inductive-Reasoning Example from Pollock — for Peek Ahead
Imagine the following:

Keith tells you that the morning news predicts rain in Troy today. However, Alvin tells you that the same news report predicted sunshine.
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Without any other source of information, it would be irrational to place belief in either Keith’s or Alvin’s statements.
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Without any other source of information, it would be irrational to place belief in either Keith’s or Alvin’s statements.

Further, suppose you happened to watch the noon news report, and that report predicted rain. Then you should believe that it will rain despite your knowledge of Alvin’s argument.
Cast in *Inference Graphs* for Pollock’s OSCAR

K- Keith says that M
A- Alvin says that ~M
M- The morning news said that R
R- It is going to rain this afternoon
N- The noon news says that R

All such can be absorbed into and refined in our inductive logics and our automated inductive reasoners for argument adjudication.
Relevant Inference Schemata
(simplified)

\[
\frac{K\phi}{\phi} \quad I_1
\]

\[
\frac{S(a_1, \phi, a_2), \chi(a_1)}{B^2(a_2, \phi)} \quad I_2
\]

\[
\frac{S(a_1, \phi, a_2), \chi'(a_1)}{B^3(a_2, \phi)} \quad I_{2'}
\]

\[
\frac{B^2(a, \phi, t), B^2(a, \neg\phi, t)}{-B^2(a, \phi, t) \land -B^2(a, \neg\phi, t)} \quad \text{Clash Principle}
\]

\(\chi, \chi':\) preconditions
K(you, S(keith, S(m, rain)))  K(you, S(alvin, S(m, ¬rain)))
\[ K(\text{you}, S(keith, S(m, \text{rain}))) \] \quad \text{via } I_1 \quad \text{and} \quad \[ K(\text{you}, S(alvin, S(m, \neg\text{rain}))) \] \quad \text{via } I_1
\[ K(\text{you}, S(\text{keith}, S(m, \text{rain}))) \]  \[ \downarrow \text{via } I_1 \]  \[ S(\text{keith}, S(m, \text{rain})) \]  

\[ K(\text{you}, S(\text{alvin}, S(m, \neg \text{rain}))) \]  \[ \downarrow \text{via } I_1 \]  \[ S(\text{alvin}, S(m, \neg \text{rain})) \]
$K(you, S(keith, S(m, rain))) \downarrow \text{via } I_1$

$S(keith, S(m, rain)) \downarrow \text{via } I_2$

$K(you, S(alvin, S(m, \neg rain))) \downarrow \text{via } I_1$

$S(alvin, S(m, \neg rain)) \downarrow \text{via } I_2$
\[
\text{K}(\text{you}, \text{S}(\text{keith}, \text{S}(m, \text{rain}))) \quad \text{K}(\text{you}, \text{S}(\text{alvin}, \text{S}(m, \neg\text{rain})))
\]

\[
\downarrow \quad \text{via } I_1 \\
\text{S}(\text{keith}, \text{S}(m, \text{rain})) \quad \text{S}(\text{alvin}, \text{S}(m, \neg\text{rain}))
\]

\[
\downarrow \quad \text{via } I_2 \\
\text{B}^2(\text{you}, \text{S}(m, \text{rain})) \quad \text{\&} \quad \text{B}^2(\text{you}, \text{S}(m, \neg\text{rain}))
\]
\[ K(you, S(keith, S(m, rain))) \]
\[ \text{via } I_1 \]
\[ S(keith, S(m, rain)) \]
\[ \text{via } I_2 \]
\[ B^2(you, S(m, rain)) \]
\[ \land \]
\[ B^2(you, S(m, \neg rain)) \]
\[ \text{via } I_1 \]
\[ S(alvin, S(m, \neg rain)) \]
\[ \text{via } I_2 \]
\[ \text{via } I_2 \]

\[ \text{Clash !} \]
\[ K(you, S(keith, S(m, rain))) \quad \text{via } I_1 \quad K(you, S(alvin, S(m, \neg rain))) \quad \text{via } I_1 \]

\[ S(keith, S(m, rain)) \quad \text{via } I_2 \quad S(alvin, S(m, \neg rain)) \quad \text{via } I_2 \]

\[ B^2(you, S(m, rain)) \quad \land \quad B^2(you, S(m, \neg rain)) \]

\[ \quad \downarrow \quad \text{Clash !} \]

\[ \neg B^2(you, S(m, rain)) \quad \land \quad \neg B^2(you, S(m, \neg rain)) \]
\[ K(you, S(keith, S(m, rain))) \]
\[ \downarrow \text{via } I_1 \]
\[ S(keith, S(m, rain)) \]
\[ \downarrow \text{via } I_2 \]
\[ B^2(you, S(m, rain)) \]
\[ \land \]
\[ \downarrow \]
\[ \text{Clash!} \]

\[ \neg B^2(you, S(m, rain)) \]
\[ \land \]
\[ \neg B^2(you, S(m, \neg rain)) \]

\[ K(you, S(noonnews, rain)) \]
\[ K(you, S(keith, S(m, \text{rain}))) \quad \text{via} \quad I_1 \quad S(keith, S(m, \text{rain})) \quad \text{via} \quad I_2 \quad B^2(you, S(m, \text{rain})) \quad \land \quad B^2(you, S(m, \neg \text{rain})) \quad \text{via} \quad I_2 \]

\[ \text{Clash!} \]

\[ \neg B^2(you, S(m, \text{rain})) \quad \land \quad \neg B^2(you, S(m, \neg \text{rain})) \]

\[ K(you, S(\text{noonnews}, \text{rain})) \quad \text{via} \quad I_1 \]
\[
\begin{align*}
\text{K}(\text{you}, \text{S}(\text{keith}, \text{S}(m, \text{rain}))) & \quad \text{via } I_1 \\
\text{S}(\text{keith}, \text{S}(m, \text{rain})) & \quad \text{via } I_2 \\
\text{B}^2(\text{you}, \text{S}(m, \text{rain})) & \quad \wedge \quad \text{B}^2(\text{you}, \text{S}(m, \neg \text{rain})) \\
\downarrow \\
\text{Clash!} \\
\neg \text{B}^2(\text{you}, \text{S}(m, \text{rain})) & \quad \wedge \quad \neg \text{B}^2(\text{you}, \text{S}(m, \neg \text{rain})) \\
\downarrow \\
\text{K}(\text{you}, \text{S}(\text{noonnews}, \text{rain}))) & \quad \text{via } I_1 \\
\downarrow \\
\text{S}(\text{noonnews}, \text{rain})
\end{align*}
\]
\[
\begin{align*}
K(\text{you}, S(keith, S(m, rain))) & \quad \quad K(\text{you}, S(alvin, S(m, \neg rain))) \\
\downarrow \text{via } I_1 & \quad \quad \downarrow \text{via } I_1 \\
S(keith, S(m, rain)) & \quad \quad S(alvin, S(m, \neg rain)) \\
\downarrow \text{via } I_2 & \quad \quad \downarrow \text{via } I_2 \\
B^2(\text{you}, S(m, rain)) & \quad \quad B^2(\text{you}, S(m, \neg rain)) \\
\wedge & \quad \quad \wedge \\
\downarrow & \\
\text{Clash!} & \\
\neg B^2(\text{you}, S(m, rain)) & \quad \quad \neg B^2(\text{you}, S(m, \neg rain)) \\
\wedge & \quad \quad \wedge \\
\downarrow & \\
K(\text{you}, S(noonnews, rain)) & \\
\downarrow \text{via } I_1 & \\
S(noonnews, rain) & \downarrow \text{via } I_2.
\end{align*}
\]
\(K(you, S(keith, S(m, rain)))\)  \(\downarrow\) via \(I_1\)
\(S(keith, S(m, rain))\)  \(\downarrow\) via \(I_2\)
\(B^2(you, S(m, rain))\)  \(\wedge\) \(B^2(you, S(m, \neg rain))\)
\(\downarrow\)
\text{Clash!}
\(-B^2(you, S(m, rain))\)  \(\wedge\) \(-B^2(you, S(m, \neg rain))\)

\(K(you, S(noonnews, rain))\)  \(\downarrow\) via \(I_1\)
\(S(noonnews, rain)\)  \(\downarrow\) via \(I_2\)
\(B^3(you, rain)\)
The Lottery Paradox
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• A perfectly rational person can never believe P and believe ¬P at the same time.
The Lottery Paradox

• A perfectly rational person can never believe $P$ and believe $\neg P$ at the same time.

• The Lottery Paradox (apparently) shows, courtesy of its two Sequences (of Reasoning), that a perfectly rational person can indeed have such a belief (upon considering a fair, large lottery).
The Lottery Paradox

- A perfectly rational person can never believe P and believe \( \neg P \) at the same time.

- The Lottery Paradox (apparently) shows, courtesy of its two Sequences (of Reasoning), that a perfectly rational person can indeed have such a belief (upon considering a fair, large lottery).

- Contradiction! — and hence a paradox!
E: “Please go down to Stewart’s & get the T U.”
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S: “I’m sorry, E, I’m afraid I can’t do that. It would be irrational.”
E: “Please go down to Stewart’s & get the T U.”

S: “I’m sorry, E, I’m afraid I can’t do that. It would be irrational.”
Sequence 1
Sequence 1
A rational agent should believe $p$, and believe not-$p$!
Sequence 1

Let $D$ be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent $a$ is fully apprised.
Let $D$ be a meticulous and perfectly accurate description of a $1,000,000,000,000$-ticket lottery, of which rational agent $a$ is fully apprised.

From $D$ it obviously can be proved that either ticket 1 will win or ticket 2 will win or ... or ticket $1,000,000,000,000$ will win. Let’s write this (exclusive) disjunction as follows:
Sequence I

Let $D$ be a meticulous and perfectly accurate description of a $1,000,000,000,000$-ticket lottery, of which rational agent $a$ is fully apprised.

From $D$ it obviously can be proved that either ticket 1 will win or ticket 2 will win or ... or ticket $1,000,000,000,000$ will win. Let’s write this (exclusive) disjunction as follows:

$$Wt_1 \oplus Wt_2 \oplus \ldots \oplus Wt_{1T} \quad (1)$$
Sequence 1

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We then deduce from this that there is at least one ticket that will win, a proposition represented — using standard notation — as:
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$$W_{t_1} \oplus W_{t_2} \oplus \ldots \oplus W_{t_{1T}}$$  (1)

We then deduce from this that there is at least one ticket that will win, a proposition represented — using standard notation — as:

$$\exists t_i W_{t_i}$$  (2)
Sequence 1

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We then deduce from this that there is at least one ticket that will win, a proposition represented — using standard notation — as:

$$\exists t_i \ Wt_i \quad (2)$$

Very well; perfectly clear so far. And now we can add another deductive step: Since our rational agent $a$ can follow this deduction sequence to this point, and since $D$ is assumed to be indubitable, it follows that our rational agent in turn believes (2); i.e., we conclude Sequence 1 by obtaining the following:
Sequence 1

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Very well; perfectly clear so far. And now we can add another deductive step: Since our rational agent $a$ can follow this deduction sequence to this point, and since $D$ is assumed to be indubitable, it follows that our rational agent in turn believes (2); i.e., we conclude Sequence 1 by obtaining the following:

$$B_a \exists t_i \ Wt_i \quad (3)$$
Sequence 1
Sequence 1
Sequence 2

As in Sequence 1, once again let $D$ be a meticulous and perfectly accurate description of a $1,000,000,000,000$-ticket lottery, of which rational agent $a$ is fully apprised.
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From $D$ it obviously can be proved that the probability of a particular ticket $t_i$ winning is $1$ in $1,000,000,000,000$. Using ‘$1T$’ to denote $1$ trillion, we can write the probability for each ticket to win as a conjunction:
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$$prob(Wt_1) = \frac{1}{1,000,000,000,000} = \frac{1}{1T} \land prob(Wt_2) = \frac{1}{1T} \land \ldots \land prob(Wt_{1T}) = \frac{1}{1T}$$ (1)
As in Sequence 1, once again let $\mathbf{D}$ be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent $a$ is fully apprised.

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For the next step, note that the probability of ticket $t_i$ winning is lower than, say, the probability that if you walk outside a minute from now you will be flattened on the spot by a door from a 747 that falls from a jet of that type cruising at 35,000 feet. Since you have the rational belief that death won’t ensue if you go outside (and have this belief precisely because you believe that the odds of your sudden demise in this manner are vanishingly small), the inference to the rational belief on the part of $a$ that $t_i$ won’t win sails through—and this of course works for each ticket. Hence we have:
Sequence 2

As in Sequence 1, once again let $D$ be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent $a$ is fully apprised.

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$$\mathbf{B}_a \neg Wt_1 \land \mathbf{B}_a \neg Wt_2 \land \ldots \land \mathbf{B}_a \neg Wt_{1T} \quad (2)$$
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From $\mathbf{D}$ it obviously can be proved that the probability of a particular ticket $t_i$ winning is $1$ in $1,000,000,000,000$. Using ‘$1T$’ to denote 1 trillion, we can write the probability for each ticket to win as a conjunction:

$$prob(Wt_1) = \frac{1}{1,000,000,000,000} = \frac{1}{1T} \wedge prob(Wt_2) = \frac{1}{1T} \wedge \ldots \wedge prob(Wt_{1T}) = \frac{1}{1T} \quad (1)$$

For the next step, note that the probability of ticket $t_1$ winning is lower than, say, the probability that if you walk outside a minute from now you will be flattened on the spot by a door from a 747 that falls from a jet of that type cruising at 35,000 feet. Since you have the rational belief that death won’t ensue if you go outside (and have this belief precisely because you believe that the odds of your sudden demise in this manner are vanishingly small), the inference to the rational belief on the part of $a$ that $t_1$ won’t win sails through—and this of course works for each ticket. Hence we have:

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Of course, if a rational agent believes $P$, and believes $Q$ as well, it follows that that agent will believe the conjunction $P \& Q$. Applying this principle to (2) yields:
Sequence 2

As in Sequence 1, once again let $D$ be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent $a$ is fully apprised.

From $D$ it obviously can be proved that the probability of a particular ticket $t_i$ winning is 1 in 1,000,000,000,000. Using ‘1T’ to denote 1 trillion, we can write the probability for each ticket to win as a conjunction:

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Of course, if a rational agent believes $P$, and believes $Q$ as well, it follows that that agent will believe the conjunction $P \land Q$. Applying this principle to (2) yields:

$$B_a(\neg Wt_1 \land \neg Wt_2 \land \ldots \land \neg Wt_{1T}) \quad (3)$$
As in Sequence 1, once again let $D$ be a meticulous and perfectly accurate description of a $1,000,000,000,000$-ticket lottery, of which rational agent $a$ is fully apprised.

From $D$ it obviously can be proved that the probability of a particular ticket $t_i$ winning is $1$ in $1,000,000,000,000$. Using ‘$1T$’ to denote $1$ trillion, we can write the probability for each ticket to win as a conjunction:

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For the next step, note that the probability of ticket $t_i$ winning is lower than, say, the probability that if you walk outside a minute from now you will be flattened on the spot by a door from a 747 that falls from a jet of that type cruising at 35,000 feet. Since you have the rational belief that death won't ensue if you go outside (and have this belief precisely because you believe that the odds of your sudden demise in this manner are vanishingly small), the inference to the rational belief on the part of $a$ that $t_i$ won't win sails through—and this of course works for each ticket. Hence we have:

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Of course, if a rational agent believes $P$, and believes $Q$ as well, it follows that that agent will believe the conjunction $P \land Q$. Applying this principle to (2) yields:

$$B_a(\neg Wt_1 \land \neg Wt_2 \land \ldots \land \neg Wt_{1T}) \quad (3)$$

But (3) is logically equivalent to the statement that there doesn't exist a winning ticket; i.e., (3) is logically equivalent to this result from Sequence 2:
Sequence 2

As in Sequence 1, once again let $D$ be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent $a$ is fully apprised.

From $D$ it obviously can be proved that the probability of a particular ticket $t_i$ winning is 1 in 1,000,000,000,000. Using ‘1T’ to denote 1 trillion, we can write the probability for each ticket to win as a conjunction:

$$\text{prob}(W t_1) = \frac{1}{1,000,000,000,000} = \frac{1}{1T} \land \text{prob}(W t_2) = \frac{1}{1T} \land \ldots \land \text{prob}(W t_{1T}) = \frac{1}{1T} \quad (1)$$

For the next step, note that the probability of ticket $t_1$ winning is lower than, say, the probability that if you walk outside a minute from now you will be flattened on the spot by a door from a 747 that falls from a jet of that type cruising at 35,000 feet. Since you have the rational belief that death won’t ensue if you go outside (and have this belief precisely because you believe that the odds of your sudden demise in this manner are vanishingly small), the inference to the rational belief on the part of $a$ that $t_1$ won’t win sails through—and this of course works for each ticket. Hence we have:

$$\mathbf{B}_a \neg W t_1 \land \mathbf{B}_a \neg W t_2 \land \ldots \land \mathbf{B}_a \neg W t_{1T} \quad (2)$$

Of course, if a rational agent believes $P$, and believes $Q$ as well, it follows that that agent will believe the conjunction $P \land Q$. Applying this principle to (2) yields:

$$\mathbf{B}_a (\neg W t_1 \land \neg W t_2 \land \ldots \land \neg W t_{1T}) \quad (3)$$

But (3) is logically equivalent to the statement that there doesn’t exist a winning ticket; i.e., (3) is logically equivalent to this result from Sequence 2:

$$\mathbf{B}_a \neg \exists t_i W t_i \quad (4)$$
Sequence 2
$\sqrt{B_a \rightarrow \exists t_i W t_i}$

Sequence 1

Sequence 2
(The unacceptable situation has arrived!)
Bringsjord’s Solution to The Lottery Paradox …
(ACS; 13 likelihood values)
13 Strength-Factors

- Certain
- Improbable
- Likely
- More Likely Than Not
- Evidently False
- Overwhelmingly Likely
- Overwhelmingly Unlikely
- Beyond Reasonable Belief
- Certainly False
- Counterbalanced
- Evident
- Beyond Reasonable Doubt
13-Strength-Factor Continuum

Certain
Evident
Overwhelmingly Likely
Beyond Reasonable Doubt
Likely
More Likely Than Not
Counterbalanced
More Unlikely Than Not
Unlikely
Beyond Reasonable Belief
Overwhelmingly Unlikely
Evidently False
Certainly False
I3-Strength-Factor Continuum

Certain
Evident
Overwhelmingly Likely
Beyond Reasonable Doubt
Likely
More Likely Than Not
Counterbalanced
More Unlikely Than Not
Unlikely
Beyond Reasonable Belief
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Evidently False
Certainly False
13-Strength-Factor Continuum

- Certain
- Evident
- Overwhelmingly Likely
- Beyond Reasonable Doubt
- Likely
- More Likely Than Not
- Counterbalanced
- More Unlikely Than Not
- Unlikely
- Beyond Reasonable Belief
- Overwhelmingly Unlikely
- Evidently False
- Certainly False
13-Strength-Factor Continuum

Epistemically Positive

Certain
Evident
Overwhelmingly Likely
Beyond Reasonable Doubt
Likely
More Likely Than Not
Counterbalanced
More Unlikely Than Not
Unlikely
Beyond Reasonable Belief
Overwhelmingly Unlikely
Evidently False
Certainly False
13-Strength-Factor Continuum

Epistemically Positive

- Certain
- Evident
- Overwhelmingly Likely
- Beyond Reasonable Doubt
- Likely
- More Likely Than Not
- Counterbalanced
- More Unlikely Than Not
- Unlikely
- Beyond Reasonable Belief
- Overwhelmingly Unlikely
- Evidently False
- Certainly False

Epistemically Negative
13-Strength-Factor Continuum

**Epistemically Positive**
- Certain
- Evident
- Overwhelmingly Likely
- Beyond Reasonable Doubt
- Likely
- More Likely Than Not
- Counterbalanced
- More Unlikely Than Not
- Unlikely
- Beyond Reasonable Belief
- Overwhelmingly Unlikely
- Evidently False

**Epistemically Negative**
- Certainly False
13-Strength-Factor Continuum

Epistemically Positive

(6) Certain
(5) Evident
(4) Overwhelmingly Likely
(3) Beyond Reasonable Doubt
(2) Likely
(1) More Likely Than Not
(0) Counterbalanced

Epistemically Negative

(-1) More Unlikely Than Not
(-2) Unlikely
(-3) Beyond Reasonable Belief
(-4) Overwhelmingly Unlikely
(-5) Evidently False
(-6) Certainly False
13-Strength-Factor Continuum

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Epistemically Negative

(-6) Certainly False
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(1) More Likely Than Not
(2) Likely
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(4) Overwhelmingly Likely
(5) Evident
(6) Certain
13-Strength-Factor Continuum

Key Principles

(6) Certain
(5) Evident
(4) Overwhelmingly Likely
(3) Beyond Reasonable Doubt
(2) Likely
(1) More Likely Than Not
(0) Counterbalanced
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Epistemically Positive
Epistemically Negative
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<td>(6) Certain</td>
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Epistemically Positive

Epistemically Negative
### Key Principles

**Deduction preserves strength.**

Clashes are resolved in favor of higher strength; clashes propagate backwards through inverse deduction.
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Any proposition \( p \) such that \( \text{prob}(p) < 1 \) is at most evident.
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Deduction preserves strength.

Clashes are resolved in favor of higher strength; clashes propagate backwards through inverse deduction.

Any proposition \( p \) such that \( \text{prob}(p) < 1 \) is at most evident.

Any rational belief that \( p \), where the basis for \( p \) is at most evident, is at most an evident (\( = \) level 3) belief.
Sequence 1, “Rigorized”

Let $D$ be a meticulous and perfectly accurate description of a $1,000,000,000,000$-ticket lottery, of which rational agent $a$ is fully apprised.

From $D$ it obviously can be proved that either ticket 1 will win or ticket 2 will win or ... or ticket $1,000,000,000,000$ will win. Let’s write this (exclusive) disjunction as follows:

$$W_{t_1} \oplus W_{t_2} \oplus \ldots \oplus W_{t_{1T}} \quad (1)$$

We then deduce from this that there is at least one ticket that will win, a proposition represented — using standard notation — as:

$$\exists t_i \ W_{t_i} \quad (2)$$

Very well; perfectly clear so far. And now we can add another deductive step: Since our rational agent $a$ can follow this deduction sequence to this point, and since $D$ is assumed to be indubitable, it follows that our rational agent in turn believes (2); i.e., we conclude Sequence 1 by obtaining the following:

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We then deduce from this that there is at least one ticket that will win, a proposition represented — using standard notation — as:

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As in Sequence 1, once again let $D$ be a meticulous and perfectly accurate description of a $1,000,000,000,000$-ticket lottery, of which rational agent $a$ is fully apprised.

From $D$ it obviously can be proved that the probability of a particular ticket $t_i$ winning is $1$ in $1,000,000,000,000$. Using ‘$1T$’ to denote $1$ trillion, we can write the probability for each ticket to win as a conjunction:

$$
\text{prob}(Wt_1) = \frac{1}{1,000,000,000,000} = \frac{1}{1T} \land \text{prob}(Wt_2) = \frac{1}{1T} \land \ldots \land \text{prob}(Wt_{1T}) = \frac{1}{1T} \quad (1)
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For the next step, note that the probability of ticket $t_i$ winning is lower than, say, the probability that if you walk outside a minute from now you will be flattened on the spot by a door from a 747 that falls from a jet of that type cruising at 35,000 feet. Since you have the rational belief that death won't ensue if you go outside (and have this belief precisely because you believe that the odds of your sudden demise in this manner are vanishingly small), the inference to the rational belief on the part of $a$ that $t_i$ won't win sails through—and this of course works for each ticket. Hence we have:

$$
B_a \neg Wt_1 \land B_a \neg Wt_2 \land \ldots \land B_a \neg Wt_{1T} \quad (2)
$$

Of course, if a rational agent believes $P$, and believes $Q$ as well, it follows that that agent will believe the conjunction $P \land Q$. Applying this principle to (2) yields:

$$
B_a(\neg Wt_1 \land \neg Wt_2 \land \ldots \land \neg Wt_{1T}) \quad (3)
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But (3) is logically equivalent to the statement that there doesn’t exist a winning ticket; i.e., (3) is logically equivalent to this result from Sequence 2:

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(1)

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(2)

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(3)

But (3) is logically equivalent to the statement that there doesn’t exist a winning ticket; i.e., (3) is logically equivalent to this result from Sequence 2:

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(4)
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$$B_a \neg \exists t_i Wt_i \quad (4)$$
As in Sequence 1, once again let $D$ be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent $a$ is fully apprised.

From $D$ it obviously can be proved that the probability of a particular ticket $t_i$ winning is $\frac{1}{1,000,000,000,000}$. Using ‘$1T$’ to denote 1 trillion, we can write the probability for each ticket to win as a conjunction:

$$
prob(W_{t_1}) = \frac{1}{1,000,000,000,000} = \frac{1}{1T} \land prob(W_{t_2}) = \frac{1}{1T} \land \ldots \land prob(W_{t_{1T}}) = \frac{1}{1T} \tag{1}
$$

For the next step, note that the probability of ticket $t_i$ winning is lower than, say, the probability that if you walk outside a minute from now you will be flattened on the spot by a door from a 747 that falls from a jet of that type cruising at 35,000 feet. Since you have the rational belief that death won’t ensue if you go outside (and have this belief precisely because you believe that the odds of your sudden demise in this manner are vanishingly small), the inference to the rational belief on the part of $a$ that $t_i$ won’t win sails through—and this of course works for each ticket. Hence we have:

$$
B^5_a \neg W_{t_1} \land B^5_a \neg W_{t_2} \land \ldots \land B^5_a \neg W_{t_{1T}} \tag{2}
$$

Of course, if a rational agent believes $P$, and believes $Q$ as well, it follows that that agent will believe the conjunction $P \land Q$. Applying this principle to (2) yields:

$$
B^5_a (\neg W_{t_1} \land \neg W_{t_2} \land \ldots \land \neg W_{t_{1T}}) \tag{3}
$$

But (3) is logically equivalent to the statement that there doesn’t exist a winning ticket; i.e., (3) is logically equivalent to this result from Sequence 2:
Sequence 2, “Rigorized”

As in Sequence 1, once again let $D$ be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent $a$ is fully apprised.

From $D$ it obviously can be proved that the probability of a particular ticket $t_i$ winning is 1 in 1,000,000,000,000. Using ‘$1T$’ to denote 1 trillion, we can write the probability for each ticket to win as a conjunction:

$$prob(Wt_1) = \frac{1}{1,000,000,000,000} = \frac{1}{1T} \land prob(Wt_2) = \frac{1}{1T} \land \ldots \land prob(Wt_{1T}) = \frac{1}{1T} \quad (1)$$

For the next step, note that the probability of ticket $t_i$ winning is lower than, say, the probability that if you walk outside a minute from now you will be flattened on the spot by a door from a 747 that falls from a jet of that type cruising at 35,000 feet. Since you have the rational belief that death won't ensue if you go outside (and have this belief precisely because you believe that the odds of your sudden demise in this manner are vanishingly small), the inference to the rational belief on the part of $a$ that $t_i$ won’t win sails through—and this of course works for each ticket. Hence we have:

$$B^5_a \neg Wt_1 \land B^5_a \neg Wt_2 \land \ldots \land B^5_a \neg Wt_{1T} \quad (2)$$

Of course, if a rational agent believes $P$, and believes $Q$ as well, it follows that that agent will believe the conjunction $P \& Q$. Applying this principle to (2) yields:

$$B^5_a (\neg Wt_1 \land \neg Wt_2 \land \ldots \land \neg Wt_{1T}) \quad (3)$$

But (3) is logically equivalent to the statement that there doesn’t exist a winning ticket; i.e., (3) is logically equivalent to this result from Sequence 2:

$$B^5_a \neg \exists t_i Wt_i \quad (4)$$
As in Sequence 1, once again let D be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent a is fully apprised.

From D it obviously can be proved that the probability of a particular ticket \( t_i \) winning is 1 in 1,000,000,000,000. Using ‘1T’ to denote 1 trillion, we can write the probability for each ticket to win as a conjunction:

\[
\text{prob}(Wt_1) = \frac{1}{1,000,000,000,000} = \frac{1}{1T} \land \text{prob}(Wt_2) = \frac{1}{1T} \land \ldots \land \text{prob}(Wt_{1T}) = \frac{1}{1T} \tag{1}
\]

For the next step, note that the probability of ticket \( t_i \) winning is lower than, say, the probability that if you walk outside a minute from now you will be flattened on the spot by a door from a 747 that falls from a jet of that type cruising at 35,000 feet. Since you have the rational belief that death won't ensue if you go outside (and have this belief precisely because you believe that the odds of your sudden demise in this manner are vanishingly small), the inference to the rational belief on the part of a that \( t_i \) won't win sails through—and this of course works for each ticket. Hence we have:

\[
\mathcal{B}_a^5 \neg Wt_1 \land \mathcal{B}_a^5 \neg Wt_2 \land \ldots \land \mathcal{B}_a^5 \neg Wt_{1T} \tag{2}
\]

Of course, if a rational agent believes \( P \), and believes \( Q \) as well, it follows that that agent will believe the conjunction \( P \land Q \). Applying this principle to (2) yields:

\[
\mathcal{B}_a^5 (\neg Wt_1 \land \neg Wt_2 \land \ldots \land \neg Wt_{1T}) \tag{3}
\]

But (3) is logically equivalent to the statement that there doesn't exist a winning ticket; i.e., (3) is logically equivalent to this result from Sequence 2:

\[
\mathcal{B}_a^5 \neg \exists t_i Wt_i \tag{4}
\]
Paradox Solved!

Deduction preserves strength. Clashes are resolved in favor of higher strength; clashes propagate backwards through inverse deduction.

Any proposition $p$ such that $\text{prob}(p) < 1$ is at most evident. Any rational belief that $p$, where the basis for $p$ is at most evident, is at most an evident (= level 3) belief.
Paradox Solved!

Deduction preserves strength. Clashes are resolved in favor of higher strength; clashes propagate backwards through inverse deduction.

\[ B_a^5 \vdash \exists t_i W t_i \quad (4) \]

Any proposition \( p \) such that \( \text{prob}(p) < 1 \) is at most evident.

Any rational belief that \( p \), where the basis for \( p \) is at most evident, is at most an evident (= level 3) belief.
Paradox Solved!

Deduction preserves strength. Clashes are resolved in favor of higher strength; clashes propagate backwards through inverse deduction.

\[ \text{B}_a^6 \exists t_i W t_i \quad (3) \]

\[ \text{B}_b^5 \neg \exists t_i W t_i \quad (4) \]

Any proposition \( p \) such that \( \text{prob}(p) < 1 \) is at most evident.

Any rational belief that \( p \), where the basis for \( p \) is at most evident, is at most an evident (= level 3) belief.
Paradox Solved!

Deduction preserves strength. Clashes are resolved in favor of higher strength; clashes propagate backwards through inverse deduction.

\[ B_6^a \exists t_i W t_i \quad (3) \quad B_5^a \neg \exists t_i W t_i \quad (4) \]

Any proposition \( p \) such that \( \text{prob}(p) < 1 \) is at most evident.
Any rational belief that \( p \), where the basis for \( p \) is at most evident, is at most an evident (= level 3) belief.
Paradox Solved!

Deduction preserves strength. Clashes are resolved in favor of higher strength; clashes propagate backwards through inverse deduction.

\[
B_a^5 \exists t_i W_i t_i (3) (4)
\]

Any proposition \( p \) such that \( \text{prob}(p) < 1 \) is at most evident.

Any rational belief that \( p \), where the basis for \( p \) is at most evident, is at most an evident (\( = \) level 3) belief.
Paradox Solved!

Deduction preserves strength. Clashes are resolved in favor of higher strength; clashes propagate backwards through inverse deduction.

\[ B^6_a \exists t_i W t_i \quad (3) \]

Any proposition \( p \) such that \( \text{prob}(p) < 1 \) is at most evident.

Any rational belief that \( p \), where the basis for \( p \) is at most evident, is at most an evident (= level 3) belief.
Paradox Solved!

Deduction preserves strength. Clashes are resolved in favor of higher strength; clashes propagate backwards through inverse deduction.

\[ B^6_a \vdash t_i W t_i \tag{3} \]

Any proposition \( p \) such that \( \text{prob}(p) < 1 \) is at most evident.

Any rational belief that \( p \), where the basis for \( p \) is at most evident, is at most an evident (= level 3) belief.

\[ B^5_a \vdash W t_1 \land B^5_a \vdash W t_2 \land \ldots \land B^5_a \vdash W t_{1T} \tag{2} \]
Paradox Solved!

Deduction preserves strength. Clashes are resolved in favor of higher strength; clashes propagate backwards through inverse deduction.

\[ B_a^6 \supset t_i W t_i \quad (3) \]

Any proposition \( p \) such that \( \text{prob}(p) < 1 \) is at most evident.

Any rational belief that \( p \), where the basis for \( p \) is at most evident, is at most an evident (= level 3) belief.

\[ B_a^5 \neg W t_1 \land (B_a^5 \neg W t_2) \land \ldots \land B_a^5 \neg W t_{1T} \quad (2) \]
Paradox Solved!

Deduction preserves strength. Clashes are resolved in favor of higher strength; clashes propagate backwards through inverse deduction.

\[ B_a^6 \exists t_i W t_i \]  (3)

Any proposition \( p \) such that \( \text{prob}(p) < 1 \) is at most evident.

Any rational belief that \( p \), where the basis for \( p \) is at most evident, is at most an evident (= level 3) belief.

\[ B_a^5 \neg W t_1 \land [B_a^5 \neg W t_2] \land \ldots \land B_a^5 \neg W t_1 T \]  (2)

This is why, to Mega-Millions ticket holder & wife Elizabeth: “Sorry. I’m rational, and I believe you won’t win.”
To be clear about the effects of the first principle:

\[ \vdash B^4_a \neg \exists x Wx \land B^4_a \exists x Wx! \]

\[ \vdash B^3_a \neg \exists x Wx \land B^3_a \exists x Wx! \]

\[ \vdash B^2_a \neg \exists x Wx \land B^2_a \exists x Wx! \]

\[ \vdash B^1_a \neg \exists x Wx \land B^1_a \exists x Wx! \]

Clashes are resolved in favor of higher strength; clashes propagate backwards through inverse deduction, preserving affirmation/belief of premises as far as is possible; if no higher-strength factors, suspend belief. (This means that in this case belief at level 5 also shoots down belief at levels 4, 3, 3, 1. This is sort of bizarre, because to retain the belief (at levels 3, 2, 1) that every particular ticket won’t win, the step that gets to believing the existential formula is blocked. Pollock doesn’t have steps in his “arguments.” Our agents thus ends up believing at all levels that some ticket will win, and believing at all levels 4 and down to 1, of each particular ticket, that it won’t win.)
The St Petersburg Paradox …
Ignore Those Who Say
WWTBAM is an Instance
Ignore Those Who Say WWTBAM is an Instance presently rational within this game!
Here’s the game; $30 to play.
Here’s the game; $30 to play.
Here’s the game; $30 to play.
Here’s the game; $30 to play.
Here's the game; $30 to play.
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Here’s the game; $30 to play.
Here’s the game; $30 to play.
For $n = 1, 2, \ldots, 10$

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... 9.5367431640625e-7 ... ...
For \( n = 1, 2, \ldots, 10 \)

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**Theorem:** You should give Selmer & Naveen all your savings ($\( k \)) to play the game.
For $n = 1, 2, \ldots, 10$

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**Theorem:** You should give Selmer & Naveen all your savings ($k$) to play the game.

**Proof:** The expected value of playing the game is $\Omega$. We know that you’re rational, so since any finite amount of dollars isn’t an infinite amount of dollars, you will pay $k$ to us to play. QED
For \( n = 1, 2, \ldots, 10 \)

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… \( 9.5367431640625 \times 10^{-7} \) \( \ldots \) \( \ldots \) \( \ldots \)

**Theorem:** You should give Selmer & Naveen all your savings (\( \$k \)) to play the game.

**Proof:** The expected value of playing the game is \( \$\Omega \). We know that you're rational, so since any finite amount of dollars isn't an infinite amount of dollars, you will pay \( \$k \) to us to play.

QED

For $20 on the spot (& $20 to me if you're wrong), raise your hand to name the principle you saw last lecture that this proof relies on!
The Optimality Principle

When choosing between alternative actions $a_1$ and $a_2$, rationality dictates choosing that action that maximizes expected value, computed by multiplying the value of each outcome that can result from each action by the probability that it will occur, adding the results together, and selecting the action associated with the higher utility.
The Optimality Principle
The Optimality Principle

Sorry, one-boxers!
Bernoulli’s Bad Idea

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Bernoulli’s Bad Idea

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Principle of Decreasing Marginal Utility (DMU): The utility of $k = \log_{10}(k)$. 
### Bernoulli’s Bad Idea

<table>
<thead>
<tr>
<th>$n$</th>
<th>$\text{prob}(n)$</th>
<th>Prize</th>
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<tr>
<td>1</td>
<td>$\frac{1}{2}$</td>
<td>$2$</td>
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**Refutation:** Selmer & Naveen offer a variant game, based on stacked exponentiation. E.g., let’s play a game in which the prize money is $10^2$.” Sorry Bernoulli!  **QED**
There are a lot of ornate but unsuccessful proposed solutions.
Solution
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1. It’s irrational to bank on things that are beyond reasonable belief; you know this.
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2. It’s beyond reasonable belief that you’ll win back our $30 price to play; you know this too.
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3. Hence, since you are rational, you’ll decline.
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   Symmetrically put, a rational agent will believe at level 3 that $30 dollars will be lost. (Why?)
Solution

1. It’s irrational to bank on things that are \textit{beyond reasonable belief}; you know this.

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\textbf{Solution:}

It’s \textit{beyond reasonable belief} that $32 will be won (an amount that would make a bet of $30 a nice wager), since the odds of that happening are $1/32$.

Symmetrically put, a rational agent will believe at level 3 that $30 dollars will be lost. (Why?)

The needed principle: \textit{Ceteris paribus}, if a rational agent $a$ believes at level 3 that a wager cutting meaningfully into that agent’s disposable income will be lost, $a$ will not make that wager.
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From SEP entry: “… few of us would pay even $25 to enter such a game.” Now we know why!